Maximum force: a simple principle encompassing general relativity

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The theory of special relativity is based on the existence of a maximum speed in nature, and all its results can be deduced from this limit value. It is argued that in a similar way, general relativity can be based on a maximum force in nature, given by $c^4/4G$, and that all its results can be deduced from this limit value. Some new experimental tests of general relativity are proposed. The approach makes general relativity accessible to secondary school students.

A. Introduction

The development of general relativity has proceeded in a complex way. Albert Einstein needed many years to deduce it from several guiding ideas that included the principle of general covariance, the equivalence principle, the principle of correspondence, the principle of general relativity and Mach's principle. David Hilbert reduced the derivation to a few months by using the least action principle. In the following it is argued that general relativity could have been derived in an even simpler way, namely with a limit statement. Special relativity started when the speed limit in nature was taken as a basic principle from which all its consequences were deduced. Both the present author [1, 3] and independently Gary Gibbons [2] have proposed that general relativity can be summarised in a similar statement: *There is a maximum force in nature:*

$$F \leq \frac{c^4}{4G} = 3.0 \cdot 10^{43} N \quad . \tag{1}$$

To show the usefulness of this approach, three sets of arguments need to be given. First of all, one has to gather evidence that this value is indeed the highest force value observed in nature. Secondly, one has to show that this value is a limit value for all possible and imaginable situations. Finally, in order to elevate it to a principle of nature, one has to show that general relativity indeed follows from this limit force.

B. The origin of the claim

A maximum force claim produces a certain unease, as there is a long tradition in physics to avoid the use of the concept of 'force'. Heinrich Hertz wanted to erase the concept from physics, and purposely wrote an influential textbook on classical mechanics without the concept. The fathers of quantum theory then dropped the term 'force' completely from the vocabulary. Finally, general relativity eliminated the concept of 'gravitational force'. However, despite these historical tendencies, the above principle does make sense. In the following, the usual definition of force as the change of momentum with time or as the change of energy with distance is used.

The force limit has also an appealing quality. In nature, forces can be measured. A measurement is a comparison with a standard. A comparison of forces is only possible if there is a common force standard valid for all situations and all systems. The maximum force provides such a standard. A maximum force thus explains why forces can be measured at all.

The expression for the maximum force bound contains both the speed of light c and the constant of gravitation G; it thus qualifies as a statement from relativistic gravitation. The origin of the claim is simple. The value of the force limit is the energy of a Schwarzschild black hole divided by its diameter. (The origin of the factor 4 in the limit – or equivalently, the use of diameter instead of any other multiple of the radius – is chosen to recover the inverse square law of universal gravity at low speeds and curvatures.) In Schwarzschild black holes, i.e. black holes that are neither charged nor rotating, the diameter is related to the mass by $L = 4Gm/c^2$.[4] For a general object in nature, one has $L \geq 4Gm/c^2$. This relation can easily be transformed into the above force limit.

The value of the force limit also appears when the so-called 'surface gravity' of a black hole is multiplied by its mass. The surface gravity is the gravitational acceleration of a falling test object at the surface of a black hole divided by the red-shift factor. Both quantities diverge when approaching the horizon, but their ratio remains finite.

In short, a maximum force value is equivalent to stating that black holes are the most concentrated form of matter or energy, and that in this most concentrated form the mass is proportional to the diameter. One notes that no other type of matter approaches the limit density. In particular, other types of black holes – rotating, charged or both – do not beat the limit.[4]

The maximum force principle is claimed to be valid for *any* type of observer; it does not play a role whether this observer is inertial, with high or low velocity relative to the system under observation, freely falling or even strongly accelerated. For example, the Lorentz contraction has no effect on a spherical black hole, as spheres are transformed to spheres. The mass concentration limit cannot be overcome by changing to a rapidly moving ob-

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server.

The next step is to check the maximum force claim with the experimental data.

C. Experimental tests

It is straightforward to check that no physical system in everyday life shows forces that even come close to the maximum force limit.

The next domain to check is the microscopic world. But even the large accelerations that particles feel in collisions inside the sun, in cosmic ray absorption or in the most powerful accelerator collisions always lead to forces much smaller than the force bound. The same is valid for neutrons in neutron stars, quarks inside protons or particles near black holes.

On the other end of the scale, in the astronomical domain, forces between stars or galaxies are always smaller than the limit, as are forces in their interior. Not even the interactions between any two halves of the universe exceeds the limit, whatever physically sensible division between the two halves is taken. (The term 'physically sensible division' will be defined below; otherwise, exceptions to the claim *can* be found. The reader might enjoy searching for such an exception.) Astronomers have also failed to find regions of space-time whose curvature is large enough to allow exceeding the force limit. Not even the recent observations of black holes have brought to light forces larger than the limit value or objects smaller than the corresponding black hole radius. Nevertheless, the force limit is clearly testable; it provides a simple test of general relativity that might be realized with high precision measurements at gravitational wave detectors or in binary pulsars. A variation of this test, making use of general relativity's relation between distance and energy errors, has already been proposed and discussed in a previous work.[3]

D. Tests with Gedanken experiments

The discussion of the force limit can be compared to the discussion of the speed limit. To be convincing, one does not only need to show that the speed of light limit holds for all observations; one also needs to show that no *imagined* experiment can beat it. Thus the force limit has to be checked in the same way, for all possible Gedanken experiments.

The simplest attempt is to try accelerating an object with a force larger than the maximum value. However, a first limit is provided by special relativity: the acceleration of an object is limited by its length. Indeed, at a distance given by c^2/a in direction opposite to the acceleration a, a *horizon* appears: nothing behind that distance can interact with the body. In other words, an accelerated body breaks at the latest at that point; the proper force on a body of mass M and length L cannot exceed $Ma = Mc^2/L$. To be observable, the body must remain larger than a black hole; inserting the size of a black hole one finds that the force limit remains unbeaten.

One can also try to generate a higher force in a *static* situation, for example by pulling two ends of a rope in opposite directions. One can even assume that an unbreakable rope does exist. To produce a force exceeding the limit value, a large (elastic) energy has to be stored into the rope, entering from the ends. When the tension in the rope is increased to higher and higher values, more and more (elastic) energy must be stored in smaller and smaller distances. To exceed the force limit, one would need to store more energy per distance than a black hole can hold. A horizon appears, and either the rope breaks or the pulling systems detach from the rope. This happens at the latest at the value given by the force limit.

Instead of systems that pull with wires or other material devices, one can study systems where radiation (both massless or massive) provides the push or pull. However, the arguments hold in exactly the same way, independently of whether photons, gravitons, neutrinos or other particles are used.

Also more concrete Gedanken experiments do not beat the limit. For example, an infinitely high tower does not generate a high enough force on its foundations; calculating its weight by taking into account its decrease with height yields a finite value that never reaches the force limit. If one continually increases its density one has to take into account the tower will change into a black hole. But even the attraction between the earth and a black hole or between two attracting black holes does not yield higher force values, since two black holes cannot come closer than the sum of their horizon radii. The maximum attraction between them – if one imagines it being compensated by a spring between them – is never larger than the force limit. In a completely different domain, also the theoretical search for systems that produce (naked) singularities has not been successful.[4]

Force is the change of momentum with time. A force limit implies a maximum momentum change per time. One can thus search for a way to *stop* a moving physical system so abruptly that the maximum force is exceeded. The non-existence of rigid bodies in nature, already found in special relativity, makes a completely sudden stop impossible. But special relativity provides no limit on the stopping time. However, the inclusion of gravity does provide such a limit. Stopping a moving system implies a transfer of energy. The energy density in nature cannot exceed the mass density of a black hole times c^2. Stopping a particle or a system in too abrupt a way would require more energy to be transferred over the stopping distance than can be put into a black hole of that size.

Similarly, if a rapid system is *reflected* instead of stopped, a certain amount of energy needs to be stored , though for a short time. During that time the energy has to be taken over by the stopping system, and then returned to the system being reflected. Also in this case the stored energy per stopping distance cannot be larger than the energy of a black hole with the corresponding diameter. This argument applies when stopping a macroscopic system such as a galaxy and equally when stopping a microscopic system such as a high-energy particle. The force limit thus cannot be overcome by stopping or reflecting a system with high momentum. In fact, in all Gedanken experiments the maximum energy density of a black hole results in a maximum force. (The converse also holds.)

The study of black hole thermodynamics shows that mass concentrations with higher density than black holes would contradict the principles of thermodynamics.[4] In black hole thermodynamics, surface and entropy are related; reversible processes that reduce entropy could be realized if physical systems could be compressed to smaller values than the black hole radius.

Composing velocities by adding their magnitudes is not possible in special relativity. Also in the case of force such a naive sum is incorrect. If textbooks on relativity had explored the behaviour of force vectors under addition with the same care as they explore the addition of velocity vectors, the force bound would have appeared much earlier in the literature. In all situations where the force limit is challenged, an event horizon appears which makes it impossible to exceed the limit. Any force implies momentum flow, and any momentum flow implies an energy flow. This energy flow can never exceed the black hole mass–energy density in space, and thus also the corresponding momentum flow in time. More details on this issue will be given below.

A force limit, like a size limit, implies that point particles do not exist. Every system, also every elementary particle, must be larger than its corresponding gravitational radius. So far, this prediction is not contradicted by observations, as the predicted sizes are unmeasurably small. However, an even stronger size bound for elementary particles will be given below.

As mentioned above, apparent transgressions of the force limit between two systems can be constructed when one calculates the attraction of any two systems of matter that come arbitrarily close, such as two pieces of a single electron or a quark. However, it will become clear soon that the term 'arbitrarily close' is not physically sensible, thus resolving the apparent paradox.

One also notes that all Gedanken experiments mentioned so far do not allow to exceed the force limit even if the observations are made by a rapidly moving or a strongly accelerating observer.

There is another way to challenge the force limit. Since physical power is force times speed, and since nature provides a speed limit, multiplication of the force bound with c yields an equivalent principle: there is a maximum power in nature. The limit value is given by

$$P \leq \frac{c^5}{4G} = 9.1 \cdot 10^{51} W \quad . \tag{2}$$

The power bound is equivalent to $1.2 \cdot 10^{49}$ horse powers. This limit can be understood intuitively by noting that every engine produces *exhausts*, i.e. some matter or energy that is left behind. For a car engine it is a certain amount of hot gases; for a water turbine it is the slowly moving water leaving the turbine; for a rocket it is the matter ejected at its back end; for a photon rocket or a an electric motor it is electromagnetic energy. If the power of an engine gets close to the limit value, the exhausts increase dramatically in mass–energy. For extremely high exhaust masses, the gravitational attraction from these exhausts prevents further acceleration of the engine with respect to its own exhausts. The maximum power bound thus expresses that there is a built-in braking mechanism in nature. Even the acceleration of a mass pulled by an unbreakable and massless wire (assuming such a wire would exist) is impossible above the force limit, as the engine that pulls the wire runs into the same limitations. Thus the power limit holds independently of whether the engine is mounted inside the accelerating body or outside, on its track.

Like the force limit, also the power the limit must be checked with observations. The luminosity of a star, a quasar or a gamma ray burster can indeed get near the power limit; however, no violation has ever been observed.[4] The sum of all star outputs in the universe does not beat the limit. Also the brightest sources of gravitational waves, merging black holes, do not exceed the power limit. Einstein's expression for the power of gravitational radiation emitted by a gravitational system is bound from above by the power limit.[4] The only system that might saturate the limit is the universe as a whole, in the case that light, neutrino and gravitational wave output are added together. Again, the power limit value can be tested by measuring the output of the universe to a high precision. This provides another test of general relativity.

The power limit is often discussed in textbooks, though maybe not in its full generality. The maximum bound on power, i.e. on energy per time, is claimed here to be valid for any energy flow through any physical surface whatsoever. The stress is on 'physical'. A surface is physical if (a) it does not cross a horizon, if (b) none of its parts is localised more exactly than the minimal length possible in nature. The minimum length (and the corresponding maximum curvature) will be introduced below. If a surface is not physical, counter-examples to the power or force limits *can* be found; however, these counter-examples make no statements about nature. (Ex falso quodlibet.)

The discussion of the power limit shows how to state the force limit more clearly. The flow of momentum per time through *any physical surface whatsoever* cannot exceed the force limit. The size of the surface is not limited! Even dividing the universe into two and shooting as many black holes as possible from one half of the universe to the other does not allow to exceed the force limit. Every time one approaches the force limit, horizons appear that prevent exceeding it.

So far, no counter-example to the power limit has been observed or imagined, neither by choosing a physical surface running across the whole universe, nor by choosing surfaces around elementary particle reactions. Apart from astrophysical sources, one of the few possible systems that comes close to the limit – without beating it – is the final stage of black hole evaporation. Checking that all sources do not exceed the maximum power provides further tests of general relativity.

In summary, nature shows three equivalent limits: the force bound, the power bound and the mass concentration bound. All three are satisfied both in observation and in theory. The Gedanken experiments show that the bound are the tightest ones possible. All three limits are open to future tests and further Gedanken experiments. In any case, we can now turn to the final part of the argument.

E. Deducing general relativity

In order to elevate the maximum force to a physical principle, it is not sufficient that it is a valid limit value in nature. In addition, the full theory of general relativity must be deduced from it. This deduction can be split into several logical steps. First of all, one needs to show that the force limit implies the existence of gravitational attraction between any two systems containing mass–energy, and that in everyday life this attraction follows the inverse square law of universal gravity.

Any physical system can be defined only if it interacts with observers in a different way than its environment does. Since vacuum is always a possible environment, any physical system must interact with one of the non-gravitational interactions; otherwise an observer would not be able to localise and define it. If two systems that would not attract each other through gravitation would exist, one could imagine a simple situation. Two bodies of mass M move on straight parallel lines in opposite directions, like cars on the two lanes of a street. This would be possible at constant speed v and distance $2R$ between the two lanes. Without mutual gravitational attraction, the bodies could pass an observer at rest between them, located on the white line separating the lanes, without ever deviating from their straight paths, whatever the distance $2R$ might be. This would mean that the bodies would be able to get as close as desired. In that case the non-gravitational interaction between the outer parts of the two bodies could (a) exceed any limit and (b) lead to higher concentrations of energy than in black holes. In other words, unlimited force or density values can only appear when gravitation is neglected.

The next step is to explore what happens when the force between any two bodies is limited. If bodies would not interact at all, a maximum force value would not exist. On the other hand, a maximum force implies that there is an interaction whatever the distance between bodies. Obviously, interactions must decrease with distance. Now, in nature matter appears in aggregates. Observations show that there are attractive interactions in nature. In other words, a force limit implies that all distant bodies attract each other.

The next step is to show that a finite maximum force implies the well-known inverse square expression of universal gravitation. To see this, exploring a simple planetary system in the case of small velocities and small forces is most productive. A simple planetary system of size L consists of a (small) satellite circling a central mass M at a distance $R = L/2$. Small velocity implies $aL \ll c^2$ and small force implies $\sqrt{4GMa} \ll c^2$ for the system and all its components. Since the system has only one characteristic speed, the two expression aL and $\sqrt{4GMa}$ must be equal, yielding $a = 4GM/L^2 = GM/R^2$. Low speeds and low forces thus imply that the inverse square law describes the interaction between the satellite and the central mass. The strength of gravity thus results directly from the maximum force. (In fact, the converse is also correct, as is easily checked; this explains the factor 4 in the force limit.)

The result can be visualized also in another way. If the gravitational attraction between a central body and a satellite would be stronger than it is, black holes would be smaller than they are and the maximum force limit could be exceeded. If on the other hand, gravitation would be weaker than it is, a fast and accelerating observer would not be able to determine that the two bodies interact. In summary, a maximum force of $c^4/4G$ implies universal gravity.

The next logical step is to show that a maximum force also implies the rest of general relativity. In particular, it must imply the existence of black holes of the correct size. In fact, this is already implicit in what was said above. An additional argument is the following. Any system with a surface gravity of $c^4/4G$ has a simple property: seen from an observer at infinity, the gravitational red-shift of the motion of the falling body is infinitely high. An observer at infinity notes that time seems to stop for the falling body. But this is the (simplest) definition of an event horizon. A system surrounded by a horizon is a black hole. In short, a force limit implies the existence of horizons and of black holes. In addition, the size of black holes is exactly as expected.

Obviously, space and space-time is *curved* near gravitational systems with horizons. Therefore, the existence of a maximum force implies both the curvature of space and space-time. This result can also be deduced in the usual way, by using the limit speed in nature and applying it at different heights above a gravitating body. As a result, the sum of the angles in a triangle above a mass is not equal to two right angles. More precisely, the value of the curvature can be deduced from the size of black holes. In short, a maximum force implies that flat space and flat space-time are not compatible with gravitational attraction. Since the value of the maximum force is the surface gravity of a black hole times it mass, the values of curvature and of the angle sum deduced from the maximum force principle are identical to those of general relativity. In short, a maximum force tells space-time how to curve.

To show the full equivalence between the maximum force principle and general relativity, it is sufficient to show their equivalence for a static, spherical mass–energy distribution. The statement is most well-known from Feynman's lectures of physics, where he states that all of general relativity can be deduced from the excess radius value for a spherical mass, provided that this relation is generalized to all possible observers.[18] If the curvatures agree in the simple case, they agree for all other cases, including dynamic situations. The full field equations of general relativity can then be deduced. Thus it is not necessary to repeat this deduction here. Nevertheless, a few additional arguments can help making the case.

All motion due to relativistic gravitation is described by the principle of maximum aging and the principle of least action. If forces in nature could be infinite in magnitude, time would stop for an object subject to such a force. In that case these variational principles could not be used to describe the object's motion. Only a finite maximum force gives sense to the variational principles. Without a maximum force, horizons would not be barriers. This connection implies that falling matter moves along geodesics only because nature shows an upper force limit. A maximum force tells matter how to fall. Also here the maximum force limit implies general relativity.

The maximum power is also connected to the existence of gravitational waves. Imagine several fast black holes colliding at high speed. At first sight, it seems that the energy per time entering the collision region could be made arbitrary large by colliding an arbitrary number of rapidly moving black holes. However, the emission of gravity waves (and the formation of event horizons) avoids that the power limit is exceeded. The same effect takes place when two ordinary objects collide. Even the fastest moving observer watching the collision must measure an incoming power smaller than the upper bound, for every region of space-time. This is only possible if part of the kinetic energy is radiated away in form of gravitational waves. This argument is as simple as the usual argument based on the effects of retarded fields.[5] A maximum force implies the existence of gravitational waves.

The maximum force principle fully contains the strong equivalence principle. Therefore it eliminates all alternative gravity which do not fulfil it. The famous Brans–Dicke theory is thus not the correct description of nature. It violates the maximum force limit and differs from general relativity for high curvatures.

In summary, the main effects of general relativity can be deduced from the maximum force principle, and vice versa. The experimental predictions for universal gravity, for the curvature value resulting from mass–energy and for the motion due to curvature are identical. The two approaches are thus equivalent. The maximum force limit encompasses all of general relativity.

F. Selected consequences of the force bound

The power limit implies that the highest luminosities are only achieved when systems emit energy at the speed of light, as otherwise the formation of a black hole cannot be avoided. The sources with highest luminosity must therefore emit entities without rest mass, such as gravitational waves, electromagnetic waves or gluons.

The power bound is also of interest if applied to the universe as a whole. Together with the finite age and size of the universe, it explains why the sky is dark at night: all stars in the universe, taken together, cannot emit more light than the power limit specifies.

The maximum force in nature is compatible and completely equivalent to general relativity and includes universal gravity. Nevertheless, it allows to draw some conclusions more easily than before. In particular, physics can now be seen as making three simple statements on the motions found in nature: [3]

$$\begin{aligned}
\text{quantum theory on action:} \quad & S \geq \frac{\hbar}{2} \\
\text{special relativity on speed:} \quad & v \leq c \\
\text{general relativity on force:} \quad & F \leq \frac{c^4}{4G}
\end{aligned} \tag{3}$$

These statements can be taken as a summary of a large part of twentieth century physics. (The deduction of quantum theory from the quantum of action dates from the founding times of quantum theory. It is summarized in reference [6].) The limits (3) are valid for all physical systems, whether composed or elementary, and are valid for all observers. As is intuitively expected, relativity poses upper limits while quantum theory poses lower limits.

If the fundamental limit for speed, the limit for force and the limit for physical action are combined, *one obtains a limit for every physical observable.* Several such limits are discussed in the literature, though always with incorrect prefactors. For example, the existence of a smallest measurable distance and time interval of the order of the Planck values are explored in quantum gravity and string theory.[7–15] A largest curvature has been discussed in quantum gravity.[16] Usually, these arguments are based on limitations of measurement apparatuses tailored to measure the specific observable under study.[17] With help of the force limit, the observable limits can be deduced in a new and direct way. Apart from a numerical factor, for every physical observable this limit corresponds to the Planck value. The numerical factors result from expressions (3) and are not found in the literature until now. (The limit values are deduced from the commonly used Planck values simply by substituting G with $4G$ and $\hbar$ with $\hbar/2$.) All limits are discussed in detail elsewhere.[3] These limit values are the true natural units of nature. In fact, the most aesthetically pleasing solution is to redefine the usual Planck values for every observable to these extremal values by absorbing the numerical factors into the respective definitions. In other words, *the natural unit or (corrected) Planck unit of a physical observable is at the same time its limit value.*

The existence of a smallest measurable distance and a shortest time interval implies that no surface is physical or 'physically sensible', if any of its elements requires a localisation in space-time to dimensions smaller that the minimum length in nature. Only through this condition the problems with the force and power limits mentioned above are eliminated. In particular, the term 'arbitrarily close' is not physically sensible, as mentioned above. The discussion here is similar to the discussion of Bekenstein's entropy bound.[19] Bousso provided some counter-examples to the maximum entropy claim that rely on the boundary surface having 'infinitely' small details, such as an infinitely sharp zig-zag shape.[20] Those counter-examples are not 'physically sensible' in view of the above results and thus not valid counter-examples.

Obviously, a minimum length implies that space, time and space-time are not continuous. The reformulation of general relativity and quantum theory presented here makes this point especially clear. The issue is thus a direct consequence of the unification of quantum theory and general relativity. No other assumption is needed.

The minimum area in nature is $2\hbar G/c^3$. That is twice the traditional, uncorrected Planck area of $\hbar G/c^3$. This means that the correct entropy–area relation for black holes should be $S/S_{\min} = A/2A_{\min}$. The factor 2 replaces the factor 4 that appears when the usual, uncorrected Planck area is used. The maximum entropy bound is changed accordingly.[19]

Since the maximum force bound is claimed to be true for all types of forces, the result also implies that electrical charges cannot be used to exceed the force limit. Indeed, this connection allows to deduce the detailed properties of charged black holes.

The force bound is also respected by the electromagnetic and nuclear interactions of single elementary particles. The three limit statements (3) result in a (corrected) Planck energy limit $E \leq \sqrt{2G\hbar/c^3}$ (valid for a single elementary particle only) [3] and in a (corrected) Planck distance limit $d \geq \sqrt{\hbar c^5/8G}$; together, these limits imply the force limit. Also the non-gravitational interactions between elementary particles thus cannot exceed the force limit.

The force bound does not depend on electrical or nuclear constants. Thus it implies that all interactions become *unified* at large values. Failure to do so would allow to distinguish interactions even in extremal situations, and thus to add the corresponding forces, in contrast with the force limit.

The force limit also implies that even if elementary particles meet and interact, their paths must be *curved.* More precisely, the force limit implies that the path curvature is limited in value, so that the paths of colliding particles cannot be zig-zag lines. Indeed, such zig-zag paths have never been observed. Even though zig-zag paths are usually drawn in Dyson–Feynman diagrams, gravity does not allow them. This is a hint for the more complex structure of nature expected at highest energies; this structure is expected and explored in string theory and quantum gravity. In those domains the maximum force value is related to the maximum string tension.[2]

As mentioned above, a maximum force limit implies that even elementary particles are *not* pointlike. Together with quantum theory, this bound can be sharpened. Due to the minimum length, elementary particles are thus predicted not only to be larger than their own Schwarzschild radius; they are predicted to be larger than the corrected Planck length. Detecting an elementary particle size is a further test of the force limit and of general relativity.

The force limit will also be of interest in an axiomatic formulation of general relativity. At the same time, it should have obvious applications in the teaching of the field. The force limit brings general relativity to the level of undergraduates and secondary schools.

Finally, the maximum force principle might clarify the validity of the strict Machian view of nature. A force is a quantity describing the interaction between two physical systems; it is a *relative* quantity. A maximum force principle thus gives an absolute limit for a relative quantity. It seems that the Machian point of view, despite its unfashionable status, is strengthened by this principle.

In summary, it was shown that the statement of a maximum force limit in nature allows to deduce the main effects of general relativity. As a side result, precise limit values for all observables in nature have been deduced. The approach is testable by real experiments and by Gedanken experiments. It simplifies general relativity to a bare minimum. At the same time, it clarifies that space and space-time are not continuous and that elementary particles are not point-like.

[1] It might be that the first published statement of the principle was in the 1999 edition of the free on-line physics textbook by C. Schiller, *Motion Mountain – A Hike Beyond Space and Time Along the Concepts of Modern Physics*, p. 272. It can be downloaded at http://www.motionmountain.net. Many arguments given here can already be found there. Discussions of the approach can be found in various usenet discussion groups over the subsequent years.

[2] G.W. Gibbons, *The maximum tension principle in general relativity*, Foundations of Physics **32**, pp. 1891–1901, 2002, or http://www.arxiv.org/abs/hep-th/0210109.

[3] C. Schiller, *Maximum force and minimum distance: physics in limit statements*, http://www.arxiv.org/abs/physics/0309118.

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[5] J.A. Wheeler, *A Journey into Gravity and Space-time*, Scientific American Library & Freeman, New York, 1990, p. 187. When the two bodies fall towards each other, they are accelerated by the retarded fields. When they depart again they gain velocity as described by the retarded fields. This time delay, which is due to the finite energy velocity c found in nature, has a simple effect: the departing bodies have less energy than the incoming ones. The lost energy is radiated away as gravity waves.

[6] For a pedagogical summary, see C. Schiller, *An appetizer – quantum theory for poets and lawyers* http://www.motionmountain.net/C-5-QEDA.pdf; part of reference [1], pp. 530–543.

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[16] A. Ashtekar, *Quantum geometry in action: big bang and black holes*, http://www.arxiv.org/abs/math-ph/0202008.

[17] For a pedagogical summary see C. Schiller, *Does matter differ from vacuum?* http://www.motionmountain.net/C10-QMGR.pdf or C. Schiller, *Le vide diffère-t-il de la matière?* in E. Gunzig & S. Diner, editeurs, *Le vide – Univers du tout et du rien – Des physiciens et des philosophes s'interrogent*, Les Éditions de l'Université de Bruxelles, 1998. An older English version is also available as C. Schiller, *Does matter differ from vacuum?* http://www.arxiv.org/abs/gr-qc/9610066.

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