A Conjecture On Quantum Electrodynamics

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Abstract

The strand conjecture proposes a specific Planck-scale model of nature that derives from an idea popularized by Dirac. The conjecture describes elementary fermions as rational tangles. The results of Battey-Pratt and Racey imply that the tangle model yields the free Dirac equation. Tangle classification implies the observed spectrum of elementary fermions, including unique values for spin, the other quantum numbers, mixing angles and particle masses.

Strands explain the principle of least action. Using the results of Reidemeister, tangle deformations induce exactly three types of interactions. They are local gauge interactions, exchange three types of elementary bosons, and show precisely the known symmetry groups U(1), broken SU(2) and SU(3). Electromagnetism obeys minimal coupling.

The tangles for fermions and bosons allow only the observed Feynman diagrams, without any additions or modifications. The complete Lagrangian of the standard model arises, including that of quantum electrodynamics. Numerous testable predictions arise, including the existence of limit values for electric and magnetic fields.

The conjectured strand process that occurs at QED interaction vertices suggests an ab-initio estimate for the fine structure constant. Particle tangles suggest an explanation for the relation between topology and the perturbative g-factor expansion. The strand conjecture also suggests lower and upper limits for the mass values of the electron and the other leptons.

Keywords: quantum electrodynamics; strand conjecture; tangle model; electron mass; fine structure constant; anomalous magnetic moment.

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1 Introduction

After the successful development of quantum electrodynamics in the middle of the twentieth century, the only two open questions are about the origin of its fundamental constants:

- ▷ What exactly happens at the fundamental interaction vertex of quantum electrodynamics and how does this process determine the fine structure constant?
- ▷ What determines the mass value and thus the full propagator of the electron?

Most approximations for the fine structure constant, since the earliest attempts, have been numerological, and thus not satisfactory [1]. A satisfactory explanation of the fine structure constant must derive it from a unified description of quantum phenomena that includes at least the weak and the strong interaction. One rare attempt is reference [2]. The same unsatisfactory situation holds for the electron mass value. Any proposed explanation of the fine structure or the electron mass can only be correct if the explanation also derives all the other unexplained fundamental constants of the standard model of particle physics: the nuclear coupling constants, the particle masses, and the mixing angles.

In the past hundred years, numerous researchers have sought a unified description of quantum phenomena and of the standard model. So far, no unified proposal has provided any hope that an explanation for the fundamental constants is within reach. In addition, practically all such proposals predicted new effects that failed to show up [3].

The present paper explores a candidate for a unified description of quantum phenomena with four central properties: the candidate model has a simple foundation, agrees with experimental data and with the standard model, predicts no new effects, and allows to calculate the fundamental constants. The emphasis of the following exploration is on the experimental tests and the implications for quantum electrodynamics and its two fundamental constants.

The starting point is the recently proposed *strand conjecture* [4]. The conjecture posits that particles, space and horizons are made of fluctuating and unobservable strands of Planck length radius. Only strand crossings that change orientation, so-called *crossing switches*, are observable; the reason will become clear below. Elementary fermions are modelled as unknotted, so-called *rational* tangles. Interactions are modelled as *deformations* of fermion tangles.

As argued below, the strand conjecture implies the four fundamental interactions, yields the U(1), broken SU(2) and SU(3) gauge symmetries of the gauge interactions, and produces the observed spectrum of elementary bosons and fermions. In appears that the strand conjecture reproduces the Lagrangian of the standard model of particle physics with massive neutrinos [4]. In the microscopic domain, the conjecture predicts that the standard model is valid at all measurable energies, without any modification, any new symmetry, any new particle, or any new dimension. The conjecture predicts the lack of new measurable effects of any kind, at any energy scale.

Despite its agreement with experiments, strands imply the possibility of calculating the fundamental constants of nature, in particular the fine structure constant and the mass values of the

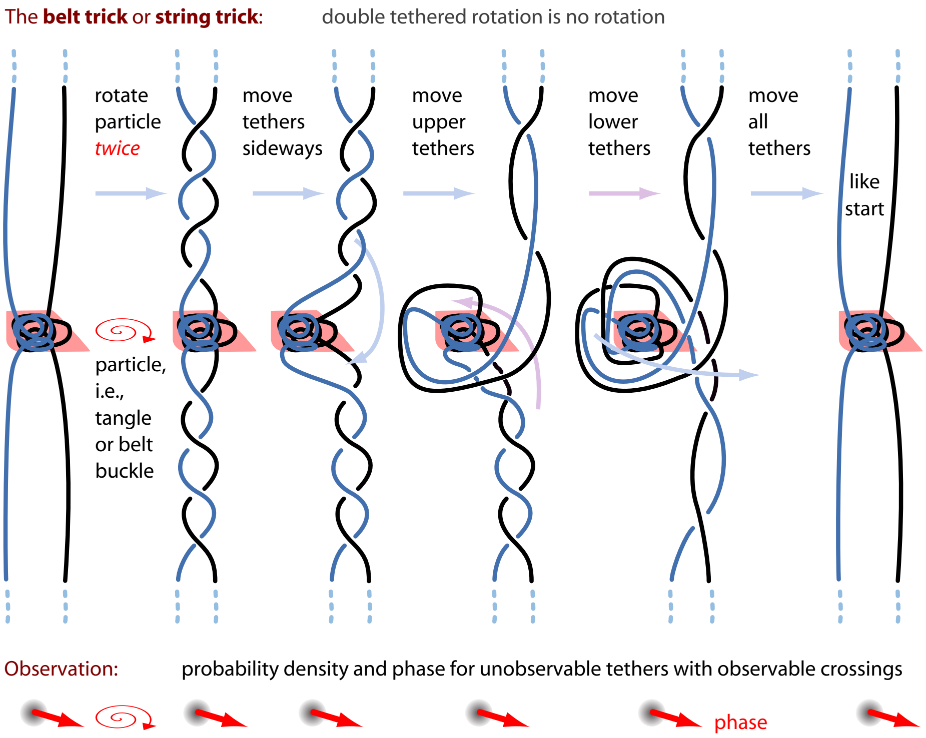


Figure 1: The *belt trick* or *string trick*: a rotation by 4π of a tethered particle, such as a belt buckle or a tangle, is equivalent to no rotation – if the tethers are allowed to fluctuate and untangle as shown. Untangling tethers is not possible after a particle rotation by only 2π: tangles with 4 or more tethers thus show the properties of *spin 1/2* particles. As a result, a tethered particle is able to rotate continuously, without limits. In the strand conjecture, fluctuations lead to a rare, but spontaneous appearance of the trick. The frequency of this spontaneous appearance, together with the ensuing core displacement, determines particle mass (as argued in Section 17).

electron and the other leptons. This possibility of calculation is explored below. The remaining coupling constants and masses, as well as the various mixing angles and phases, were explored in reference [4]. To make the present paper self-contained, the first few sections summarize the strand conjecture. A thorough list of predictions and experiments to test the conjecture is provided.

2 The origin of the conjecture

Bohr used to present quantum theory as consequence of a smallest observable action value $\hbar$ [5]. Dirac then included the maximum energy speed c. From around 1929 onwards, Dirac mentioned the so-called *string trick* or *belt trick* in his lectures. Illustrated in Figure 1, the trick describes the main properties of spin 1/2 as the result of tethered rotation. As Dirac wrote [6], the trick also explains that no spin value below $\hbar/2$ is possible.

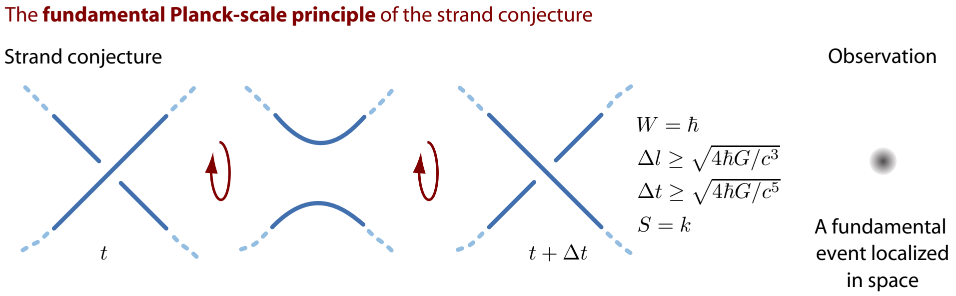


Figure 2: The fundamental principle of the strand conjecture describes the simplest observation possible in nature, a fundamental event. In the strand conjecture, a fundamental event is almost point-like. A fundamental event results from a *skew strand crossing switch* – the exchange of underpass and overpass – at a location in three-dimensional space. The strands themselves are not observable, are impenetrable, and are best imagined as *tubes* having Planck-size radius. The crossing switch defines $\hbar$ as the unit of the physical action W. Both the Planck length and the Planck time arise, respectively, from the smallest and from the fastest possible crossing switch. The fastest crossing switch is discussed in Section 11. The crossing (switch) can also be taken as the strand realization of a qubit.

In nature, spin 1/2 is observed, but particle tethers are not. Dirac's tether proposal provided a first hint that nature might be described with extended constituents that are themselves *unobservable*, but whose crossing switches are observable. In 1980, Battey-Pratt and Racey went further. They showed that the full free Dirac equation could be deduced from unobservable tethers attached to localized masses [7]. Their result suggests that *every quantum effect* can be thought as being due to unobservable extended constituents with observable crossing switches.

3 The strand conjecture and its fundamental principle

The strand conjecture states that everything in nature – matter, radiation, space, and horizons – is made of strands that fluctuate at the Planck scale.

▷ A *strand* is defined as smooth simple curved line – a one-dimensional, open, continuous, everywhere infinitely differentiable subset of $\mathbb{R}^3$ or of a curved 3-dimensional Riemannian space, with trivial topology and without endpoints – that is surrounded by a perpendicular disk of Planck radius $\sqrt{\hbar G/c^3}$ in each point of the line, and whose shape is randomly fluctuating over time.

Strands are thus thin flexible *tubes* having a Planck-size radius. From a viewpoint of mathematics, the definition extends the usual definition that is used in knot theory for ropelength calculations [8, 9] by including shape fluctuations. The definition implies that strands *cannot* interpenetrate or intersect. From a physics viewpoint, strands have *no* observable properties. Even though strands are not observable, their topological tangling are, as will become clear shortly. Strands thus visualize the minimum length $\sqrt{4\hbar G/c^3}$ as the shortest distance between two strand segments. When physical observables are introduced as explained below, strands imply that the minimum length is an unattainable and unobservable limit.

The strand conjecture claims:

- ▷ Crossing switches – the exchange of underpass and overpass – determine the Planck units, and in particular $\hbar$, as illustrated in Figure 2.
- ▷ Though strands are unobservable, *crossing switches are observable*, because of their relation to $\hbar$, c, k and G.
- ▷ Space is a *network* of strands. Horizons are *weaves* of strands. Particles are *tangles* of strands.

The first statement is called the *fundamental principle*. The other two statements follow from the first. In simple terms, the strand conjecture claims that Figure 2 contains all of physics. In particular, the strand conjecture claims that the figure contains quantum electrodynamics. This is argued in the following.

The term *crossing* always implies a *skew* crossing or *apparent* crossing, when drawn in two dimensions: a strand crossing always consists of a *strand overpass* and a *strand underpass*. In three dimensions, strands are thus *always* at a distance, as illustrated in Figure 2 and Figure 3. In particular, crossing *switches* – the exchange of underpass and overpass – always arise via strand deformations, and in no other way. The complete reason that crossing switches are observable – and that nothing else is – is given below, in section 11.

A crossing switch defines a physical event. In the strand conjecture, events are processes. The crossing switch is the most fundamental event and the most fundamental process. All observed processes in nature are *composed* of crossing switches; this includes macroscopic and microscopic motion of matter and radiation, gravitational field evolution and gravitation waves, as well as all measurements.

Physical observables – such as length, mass or field intensities – *emerge* from combinations of crossing switches. Crossing switches occur at crossings. For example, as illustrated in Figure 3, a skew strand crossing allows defining the same properties that characterize a wave function: the geometry of crossings allows defining density, position, orientation, and phase. (The *density* is given by the inverse minimum distance; *position* is the midpoint of the shortest distance segment s; *orientation* and *phase* are defined with suitable cross products and sums of the vector representing s and of the two unit tangent vectors of the strands at the endpoints of s.) Geometrically, a crossing is described by one real number, describing the minimum strand distance or density, and by four angles defining the crossing geometry around the position of the crossing. The geometric parameters of a crossing can be mapped to the parameters of the Pauli wave function – or to (half of) those of a Dirac wave function. In particular, the *phase of the wave function* of a particle arises

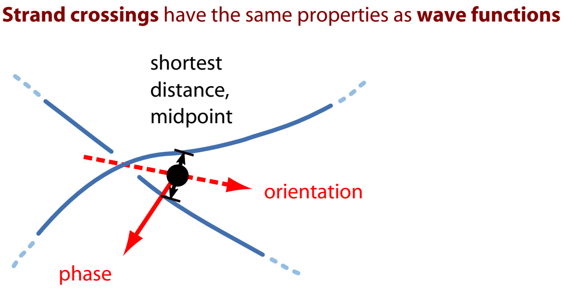


Figure 3: The geometric properties of a skew strand crossing – a strand overpass and a strand underpass – resemble those of wave functions. Both crossings and wave functions allow defining position, orientation, phase and density. For both, the *absolute phase* value around the orientation axis can be chosen freely. In contrast, for both, *phase differences* due to rotations around the axis of orientation are always uniquely defined.

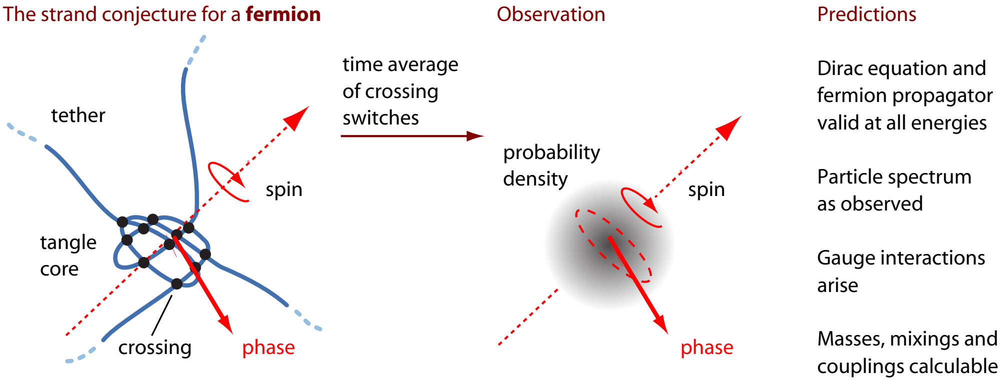


Figure 4: In the strand conjecture, the wave function is due to fluctuating crossings, and the probability density is due to fluctuating crossing switches – both after averaging. The phase of the wave function arises as the vector sum of all the crossing phases. Wave functions due to to crossing strands form a Hilbert space. The tethers – strands that continue up to large spatial distances – lead to spin 1/2 behaviour under rotations and to fermion behaviour under particle exchange. The tethers imply that core rotation and core displacement are related; this allows to define a mass value. The relation between core rotation and core displacement also implies that cores move more slowly than light.

as the sum of all crossing phases in the particle tangle, averaged over the fluctuations. The freedom in the definition of the phase is at the origin of the freedom of gauge choice.

In the strand conjecture, all elementary fermions are *rational* tangles, i.e., unknotted open tangles. (Only rational tangles allow to reproduce the particle transformations that occur in interactions.) Equivalently, elementary fermions are made of unknotted but tangled tethers. For a tangle of fluctuating strands, the average crossing distribution is the wave function, and the average crossing switch distribution is the probability distribution. The connection is illustrated in Figure 4. For a particle tangle, the average phase, the average density, and the two average spin orientation angles define the two complex components of the Dirac wave function Ψ for a particle. For the *mirror* tangle, the corresponding averages define the two complex components of the *antiparticle*.

In his lectures, Dirac used a system equivalent to that of Figure 1 to demonstrate that a *single* tethered core behaves, under rotations, like a spin 1/2 particle. Indeed, a *double* rotation of a tethered core can be undone by rearranging the tethers only; in contrast, this is impossible after a single rotation. Dirac's demonstration can be extended to show that *two* tethered cores also imply that under exchange, tangle cores behave as fermions. Indeed a *double* exchange of tethered cores can be undone by rearranging the tethers only; in contrast, this is impossible after a single exchange. Both results apply independently of the number of tethers, as long as their number is 3 or larger. Videos that visualize both spin 1/2 and fermion behaviour exist on the internet [10].

In summary, tethered particles reproduce spin (intrinsic rotation) and particle exchange (a double translation). The suspicion arises that *every* quantum motion can be described with tethered particles. This is indeed the case.

4 From tethers to the free Dirac equation

This section summarizes how to deduce the Dirac equation from strands.

In 1980, Battey-Pratt and Racey showed [7] that every *tethered* massive quantum particle – thus every little massive sphere with attached strands that leave up to spatial infinity – is described by the Dirac equation for free particles. In other terms, Battey-Pratt and Racey assumed unobservable strands attached to a central mass and derived the Dirac equation. They even wrote Dirac about it, but they got no answer. Unfortunately, Dirac passed away shortly afterwards.

In the strand conjecture, the result of Battey-Pratt and Racey is extended further. The massive particle itself is *also* assumed to be made of strands: an elementary fermion is conjectured to be a (rational) tangle *core*, i.e., the tangled region of a tangle whose tethers reach up to large distances.

In the strand conjecture, a fermion moving freely through space can thus described by a constantly rotating tangle core whose central position is advancing through space. The relation between rotational and translational motion defines the inertial mass of the particle. The free motion of a tangle thus models Feynman's description of a quantum particle as an advancing rotating arrow [11]. The free motion of a fermion implies that advancing tangles with rotating cores are a model for fermion propagators. Strands thus can be seen as visualizing the work of Hestenes [12] on the Dirac equation.

In the strand conjecture, a fermion moving freely through space can also be described with the help of wave functions and probability density: the spatial region of maximum density, i.e., the region with most crossing switches, advances, and at the same time, the phase rotates in space. In the tangle model, the faster the particle tangle rotates and advances, the more its spin is aligned with momentum. Even at the highest rotation frequency possible (the corrected Planck frequency), the translational motion of a tangle is smaller than the speed of light c. Lorentz covariance is ensured. In short, particle tangles do behave like fermion propagators. And indeed, as Battey-Pratt and Racey showed, tethered relativistic particles are described by the free Dirac equation.

In the strand conjecture, strands are not observable, but their crossing switches are. With the fundamental principle, the result [7] of Battey-Pratt and Racey can be rephrased in the following concise way:

▷ The free Dirac equation is essentially a differential version of Dirac's string trick.

Another way to express a central aspect of the connection between the belt trick and the Dirac equation is the following: Dirac's belt trick implies the γ^μ matrices. Exactly like usual quantum theory, also strands imply probabilities, Zitterbewegung, interference, a Hilbert space, contextuality, entanglement, mixing, decoherence, antiparticles and all other quantum effects [18].

A second, equivalent way to understand the appearance of the free Dirac equation from strands is the following. The free Dirac equation

$$i\hbar\gamma^\mu\partial_\mu\psi = mc\psi \tag{1}$$

arises from five basic properties:

1. The action limit given by $\hbar$, yielding wave functions ψ,
2. The energy speed limit for massive particles given by c,
3. A particle mass value m that connects phase rotation frequency and wavelength using the imaginary unit i,
4. The spin 1/2 properties in Minkowski space-time,
5. Particle–antiparticle symmetry, the last two points being described by the γ^μ matrices.

These five properties are necessary and sufficient to yield the free Dirac equation. (The connection of the γ^μ matrices with the geometry of spin was first made about a century ago [13].) The tangle model of particles reproduces these five properties in the following way:

1. All observables are due to crossing switches, which imply a minimum observable action $\hbar$ (see Figure 2) and the existence of a wave function (see Figure 3 and Figure 4),

2. Tangle cores are constrained to advance less than one Planck length per Planck time, thus less than c (see Figure 1),
3. Tangle core rotation connects rotation and displacement and generates a finite mass value m much smaller than the Planck mass (see Figure 1 and Section 16),
4. Tethering reproduces the spin 1/2 properties for rotation, exchange and boosts, and thus introduces the γ^{μ} matrices (see Figure 1),
5. Tangle and mirror tangle correspond to particle and antiparticle.

Both in nature and in the strand conjecture, the inability to observe action values below $\hbar$ leads to wave functions and probability densities. Both in nature and in the strand conjecture, the inability to observe speed values larger that c leads to Lorentz invariance and the relativistic energy–momentum relation. Both in nature and in the strand conjecture, together with the mass and the spin 1/2 properties due to tethers, the γ^{μ} matrices and the Dirac equation for a free particle arise [14, 15, 16]. Electromagnetic fields will be included below.

The strand conjecture also explains the existence of quantum motion in a third, more general way. Modern physics has shown that all motion can be described with the principle of least action. When the principle is applied, the Lagrangian describes the way to determine the value of the action. In the strand conjecture, *action* is the number of crossing switches. The principle of least action then simply becomes the *principle of fewest crossing switches.* (This statement resembles Schwinger's quantum action principle.) In modern physics, the usual statement is: motion minimizes action. In the strand conjecture, the corresponding statement is: motion minimizes crossing switches. For fermions, after suitable spatial averaging, this general idea leads to the free Dirac Lagrangian and to the free Dirac equation.

A fourth argument for the validity of the Dirac equation uses the derivation by Lerner [17], which is based on two properties. First, the definition of spin using strands implies that the spin current is *conserved.* Second, the definition of spin using strands also implies the *Lorentz covariance* of spin, i.e., the proper behaviour under rotations and boosts. (This second property is also shown explicitly by Battey-Pratt and Racey [7].) Together, as Lerner showed, these two properties imply the Dirac equation.

In summary, strands deduce the Dirac equation from the fundamental principle. Equivalently, the Dirac equation results from the behaviour of crossings in fluctuating tangles. In particular, the usual expressions for the usual fermion propagator follow from strands. In short, strands visualize how the quantum of action $\hbar$ leads to the free Dirac equation.

As a result, strands predict the lack of deviations from relativistic quantum theory. (Radiation fields from the gauge interactions are only added below; the statement remains valid when interactions are included.) Strands yield no exceptions at any measurable energy – strands just yield to small effects at the Planck scale. *Finding a situation or an energy scale for which the Dirac equation is not valid would falsify the strand conjecture.*

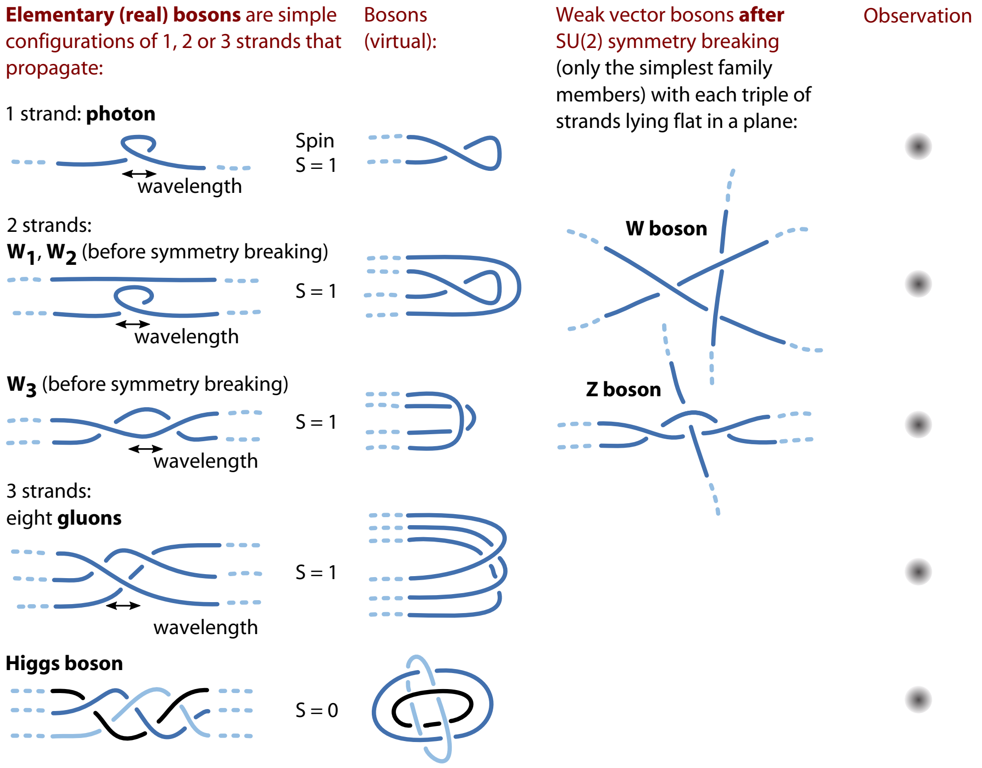


Figure 5: This overview shows the conjectured tangles for every elementary boson. The tangles are made of one, two or three strands. For each boson, the advancing tangle determines the spin value and the propagator. Spin 1 boson tangles rotate when propagating. Photons and gluons are massless, and are described by exactly one tangle each. The W, Z and Higgs have mass, and thus have additional, more complex tangles in addition to the one shown here (see text). No further elementary bosons are predicted to exist.

In addition to the Dirac equation, the fundamental principle of the strand conjecture implies that every Planck unit (corrected by changing G to $4G$) is an insurmountable *limit* to physical observables in the quantum domain [18]. More precisely, the strand conjecture predicts the lack of any trans-Planckian effects. Therefore, *observing an elementary particle whose energy is larger than* $\sqrt{\hbar c^5/4G} = 6.1 \cdot 10^{18}$ GeV *would falsify the strand conjecture.*

The Planck limits are common – though often neglected – knowledge. But the description of quantum theory with strands also implies *one new result.* If elementary particles are rational tangles, then their spectrum, their interactions, their masses and all their other particle properties are not free, but are *fixed* by their tangle structure. This can be checked.

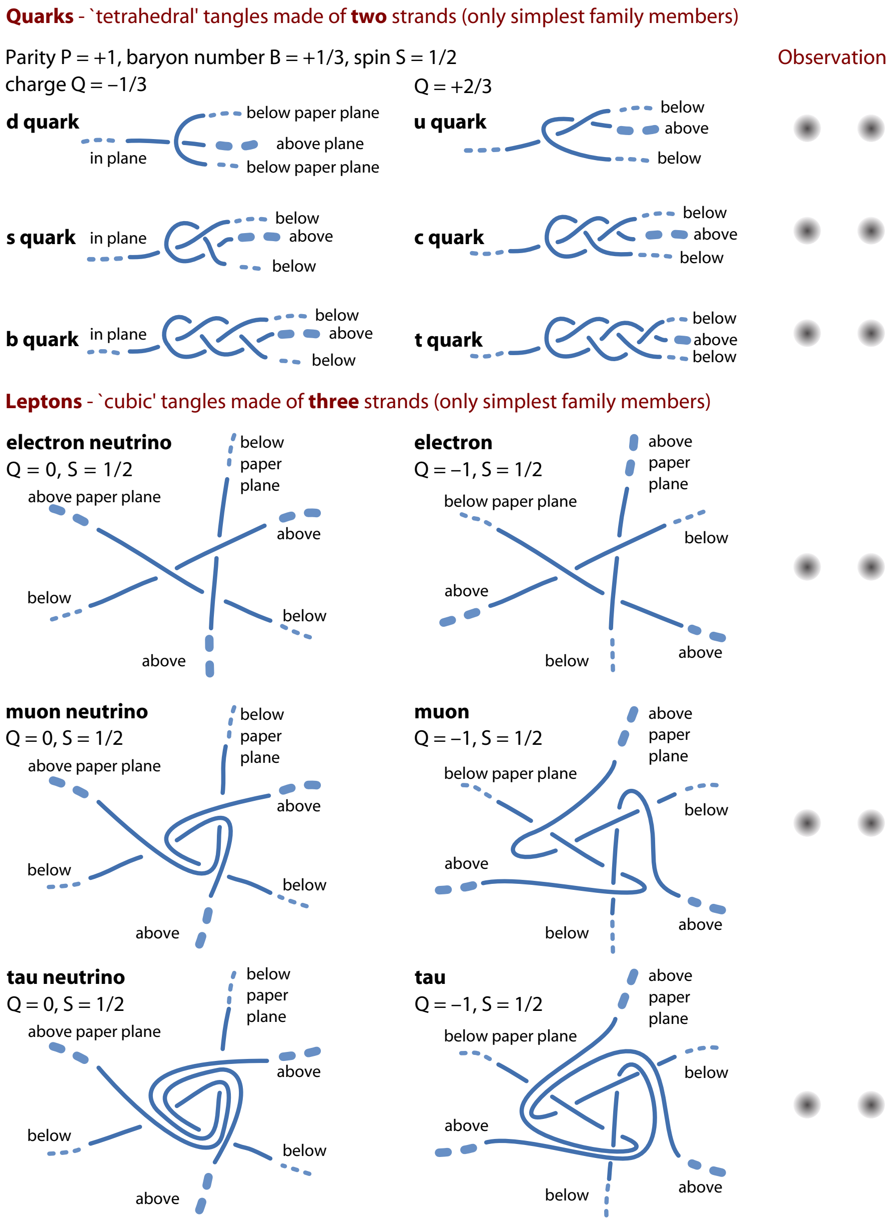


Figure 6: The overview shows the simplest conjectured tangles for each elementary fermion. They are made of two or three strands. Elementary fermions are rational, i.e., unknotted tangles. This structure leads to Higgs coupling, as illustrated in Figure 9. Tangles generate positive mass values and exactly three generations. At large distances from the tangle core, the tethers of the quarks follow the axes of a tetrahedron. At large distances from the core, the tethers of leptons follow the three coordinate axes. Neutrino cores are simpler when observed in three dimensions: they are simply twisted triples of strands. All massive particles have additional, more complex tangles in addition to the one shown here (see text). No additional elementary fermions appear.

5 Predictions about the particle spectrum and about particle structure

This section summarizes how strands lead to the observed spectrum of bosons and fermions. The details were already explored elsewhere [4].

Elementary bosons can consist of one, two or three strands. More strands imply composite systems. The Reidemeister moves suggest that one-stranded bosons correspond to photons, two-stranded bosons to the W_1, W_2 or W_3, and three-stranded bosons to gluons. After symmetry breaking, when two-stranded boson tangles incorporate a vacuum strand, they yield the three-stranded W and Z bosons. The complete overview of boson tangles is given in Figure 5. No additional elementary boson appears possible. Photon and gluon tangles are massless, because they can rotate unhindered by tethers, whereas the W and the Z boson have mass. *The discovery of additional gauge bosons would falsify the tangle model.*

The Higgs boson is a braid made of three strands. For all massive particles, Higgs braids can be added to the tangle core. All massive particles – fermions or bosons – are thus described by an infinite *family* of tangles that contain a simple core, that core plus one Higgs braid, that core plus two braids, etc. The mass value is influenced by this – single or multiple – Higgs boson addition, as illustrated in Figure 9. That figure also shows that the Higgs couples to itself; it is thus massive. Because no addition of a Higgs braid to cores of massless elementary particles is possible, massless elementary particles are described by a *single* tangle. *The discovery of additional massive Higgs bosons would falsify the tangle model.*

Elementary fermions can consist of two or three strands. One-stranded particle tangles cannot have spin 1/2 nor have mass because the belt trick is not applicable to them. Two-stranded fermions are quarks, three-stranded fermions are leptons. The simplest specific tangles are given in Figure 6. All massive elementary particles have additional tangles: each one are described by an infinite *family* of tangles that contain the simplest core, that simplest core plus one Higgs braid, the simplest core plus two braids, etc. Both quarks and leptons are limited to three generations by the coupling to the Higgs (and the three-dimensionality of space). The quark tangle assignments reproduce the quark model of hadrons [4, 18], including the correct retrodiction of which mesons violate CP; they also reproduce all meson and hadron mass sequences. The lepton tangle assignments and the quark tangle assignments reproduce the weak interaction. Particle mixing is explained in reference [4]. The neutrino assignments explain their handedness and their small mass. Additional elementary fermions are not possible. *The discovery of additional fermions would falsify the tangle model.*

The tangles of the elementary particles also determine their parities (from their mirror behaviour and their tangle core rotation), their spin (from the rotation behaviour of their tangle core), their baryon and lepton number (from the number and spatial structure of tethers), and all their other flavour quantum numbers (from the quark content, i.e., from the core topology). The topological and geometric origin of strong and weak charge is explored in reference [18]. Electric charge is explored in detail below. *The discovery of forbidden values of quantum numbers or of the non-conservation of baryon number or lepton number would falsify the tangle model.*

In summary, in the strand conjecture, tangle classification leads to the fermion and boson spectrum observed in nature. Every observed quantum number is due to a *topological* property of particle tangles. (In contrast, the fundamental constants are due to (averaged) *geometric* properties of tangles.) The appearance of the gauge groups from the boson structure is summarized in the next section. *The discovery of any new elementary particle – such as axions, anyons, supersymmetric partners, or any new dark matter particle – or the discovery of any new quantum number would falsify the tangle model.*

As a note, *discovering any substructure in elementary* particles *that differs from tangles of strands – such as preons, knots, or any other localised or extended substructure – would invalidate the strand conjecture.* As a further note, it is possible that some fermion tangles are wrong, but the strand conjecture as a whole is still correct. In particular, the tangles assigned to the leptons need critical scrutiny.

6 Predictions about gauge interactions

This section summarizes earlier results [4] showing that interactions are *tangle core deformations.* This connection is illustrated in Figure 7. Deformations of a localized tangle core modify the phase of the corresponding particle. For example, an externally applied magnetic field modifies the phase of an electron wave function through the absorbed virtual photons.

Deformations of three-dimensional objects are described by gauge groups. In 1926, Reidemeister showed that every tangle core deformation is composed of three basic types: *twists, pokes* and *slides* [19]. Together, they are now called the first, second and third *Reidemeister moves*. The moves have a property that is not widely known [4]:

▷ Tangle core deformations – given by Reidemeister moves – determine the observed gauge groups U(1), broken SU(2) and SU(3).

In particular, the gauge group U(1) arises because twists, the first Reidemeister move, can be generalized to arbitrary angles and concatenated. Also, a double twist can be rearranged to no twist at all, so that the non-trivial topology of U(1) arises. Electric charge is defined in Section 9 as 1/3 of the (signed) sum of chiral crossings. Electric fields are volume densities of virtual photons, i.e., of twists. Magnetic fields are flow densities of twists. Only massive tangles can be electrically charged. *Discovering a massless and electrically charged elementary particle would falsify the strand conjecture.*

The gauge group SU(2) arises because pokes, the second Reidemeister move, can be seen as rotations by the angle π around the three coordinate axes; they form an SU(2) algebra. The generalization of these rotations to arbitrary angles yield the full SU(2) group. Strands imply that only massive fermions can exchange weak bosons. Due to the tangle structure of particles, SU(2) *breaking* arises, and so does maximal parity violation: parity violation occurs because the core rotations due to spin 1/2 interfere with the core deformations due to the group SU(2) of the weak

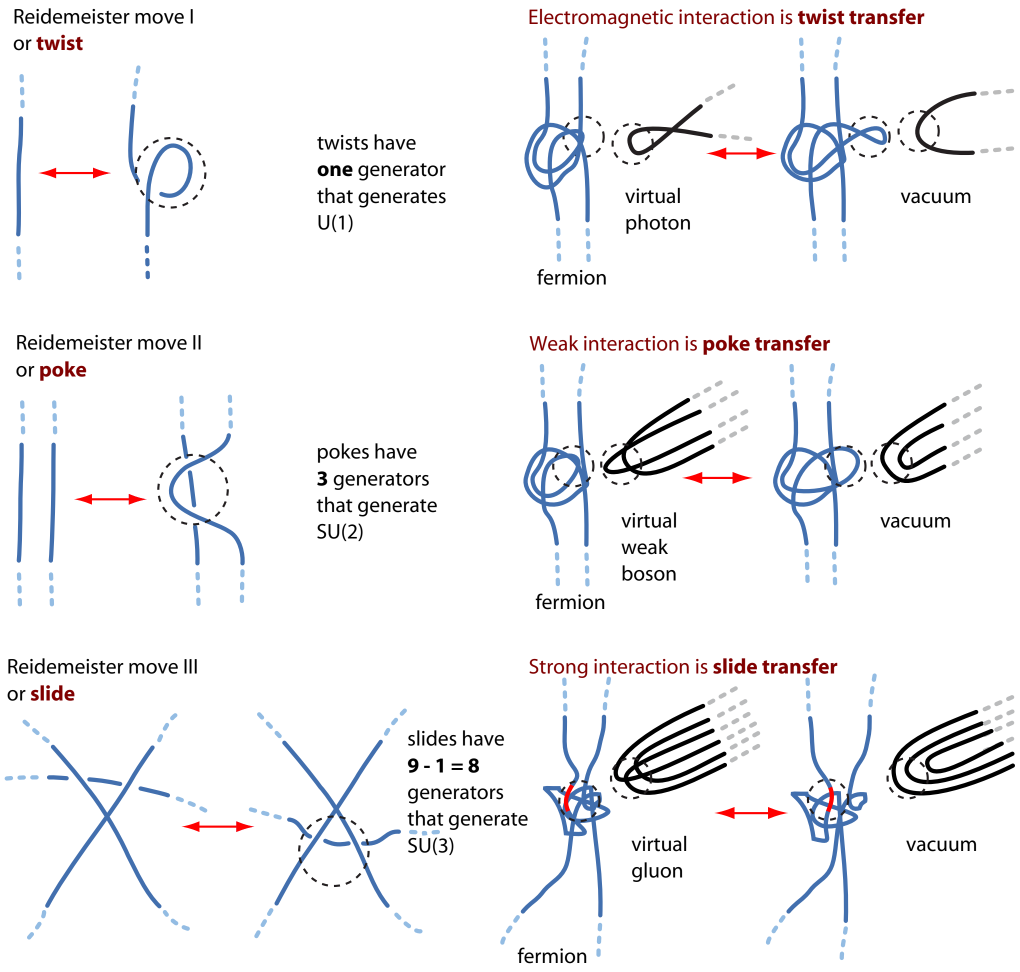


Figure 7: The three Reidemeister moves classify the possible deformations of tangle cores [19]. The moves also determine the generators of the observed gauge interactions, determine the generator algebra, and thus fix the three gauge groups [4, 18]. Every group generator rotates the region enclosed by a dotted circle by the angle π. The full gauge group arises by generalizing these local rotations to arbitrary angles.

interactions [4, 18]. *Discovering deviations from the known weak interaction properties would falsify the strand conjecture.*

The gauge group SU(3) arises because slides, the third Reidemeister move, reproduce the algebra of the eight generators of SU(3). This is the main result of the previous paper [4]. The full gauge group SU(3) arises because slides can be seen as local rotations by π, and these rotations can be generalized to arbitrary angles. CP violation does not and cannot occur in the strand conjecture for the strong interaction. Color charge is given by the orientation of the three-ended side of a quark tangle in space. Color fields are densities of virtual gluons. *Discovering deviations from the known strong interaction properties would falsify the strand conjecture.*

As a note, strands also explain the usual gauge group representation for each elementary particle. *Discovering any particle with a different representation behaviour would falsify the strand conjecture.*

As a second note, strands thus explain why complex numbers, quaternions and octonions each play a role in one gauge interaction. *Discovering that any other, unrelated number field describes any observed interaction would falsify the strand conjecture.*

As a third note, the visualization of the three gauge interactions agrees with and deepens the ideas of Boudet [20].

Because there are only three Reidemeister moves, strands predict the lack of any other gauge interaction. *Discovering a new or a larger gauge group, or corresponding bosons, would falsify the strand conjecture.*

7 Predictions about the standard model

In the strand conjecture, all Feynman diagrams of the standard model with massive neutrinos are recovered. This result from reference [4] is summarized in Figure 8 and Figure 9. No other vertices and no other propagators arise. Therefore, combining

- The particle spectrum deduced from tangles (illustrated in Figure 6 and Figure 5),
- The Dirac equation and the corresponding propagator for each massive particle deduced from tangles (see Section 4),
- The Feynman diagrams due to tangles (see Figure 8 and Figure 9),
- The boson Lagrangians with the corresponding boson propagators (deduced for electromagnetism in Section 9 and for nuclear interactions in reference [4]), and
- The fundamental constants – masses, mixing angles and couplings – deduced from tangles (reference [4] and Section 17),

the *complete Lagrangian* of the standard model arises. The standard model thus appears to result from the fundamental principle of the strand conjecture. Equivalently, the standard model thus appears to arise directly from the Planck scale.

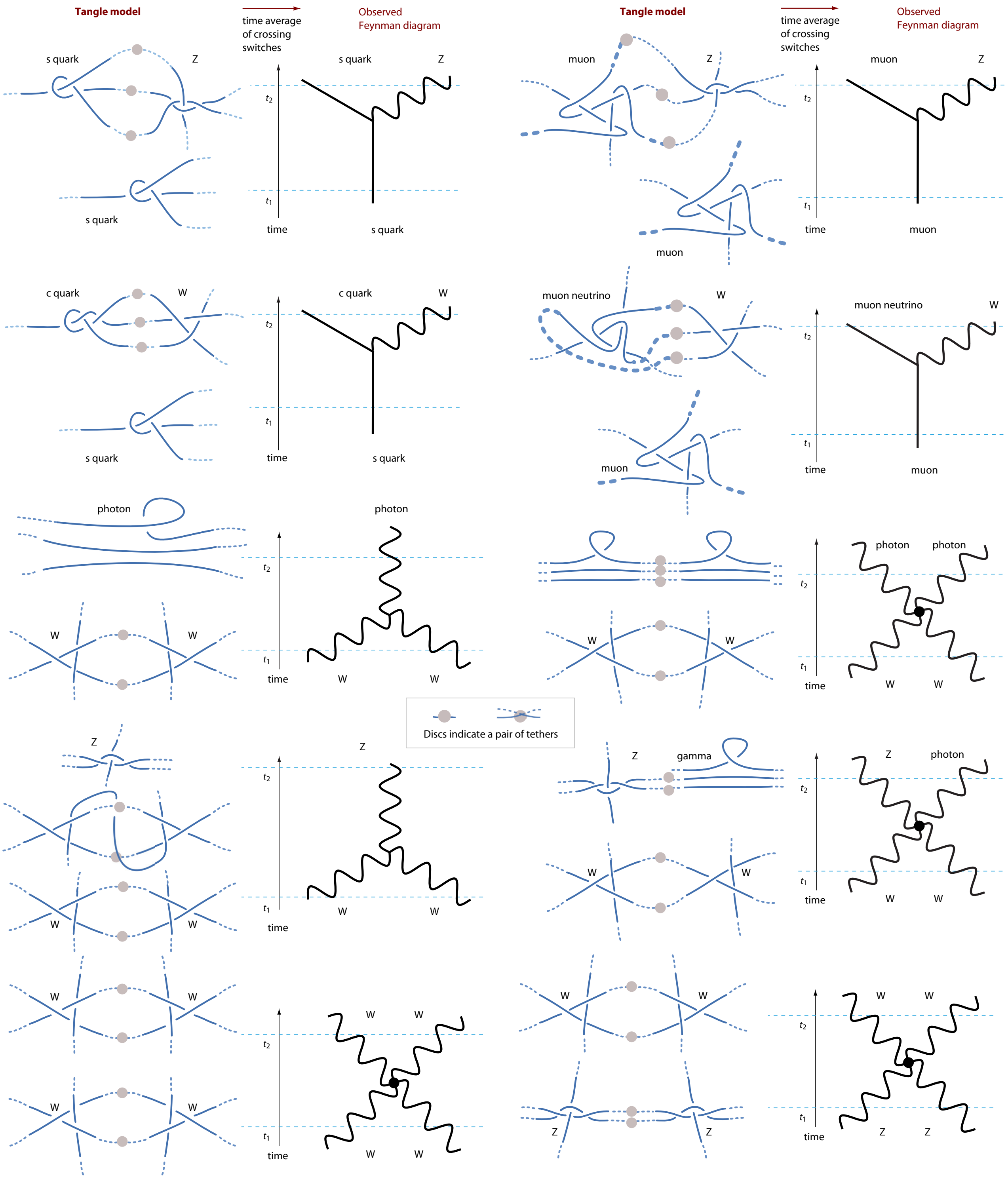


Figure 8: The interaction vertices allowed by fermion and boson topologies imply the complete Lagrangian of the standard model (part one).

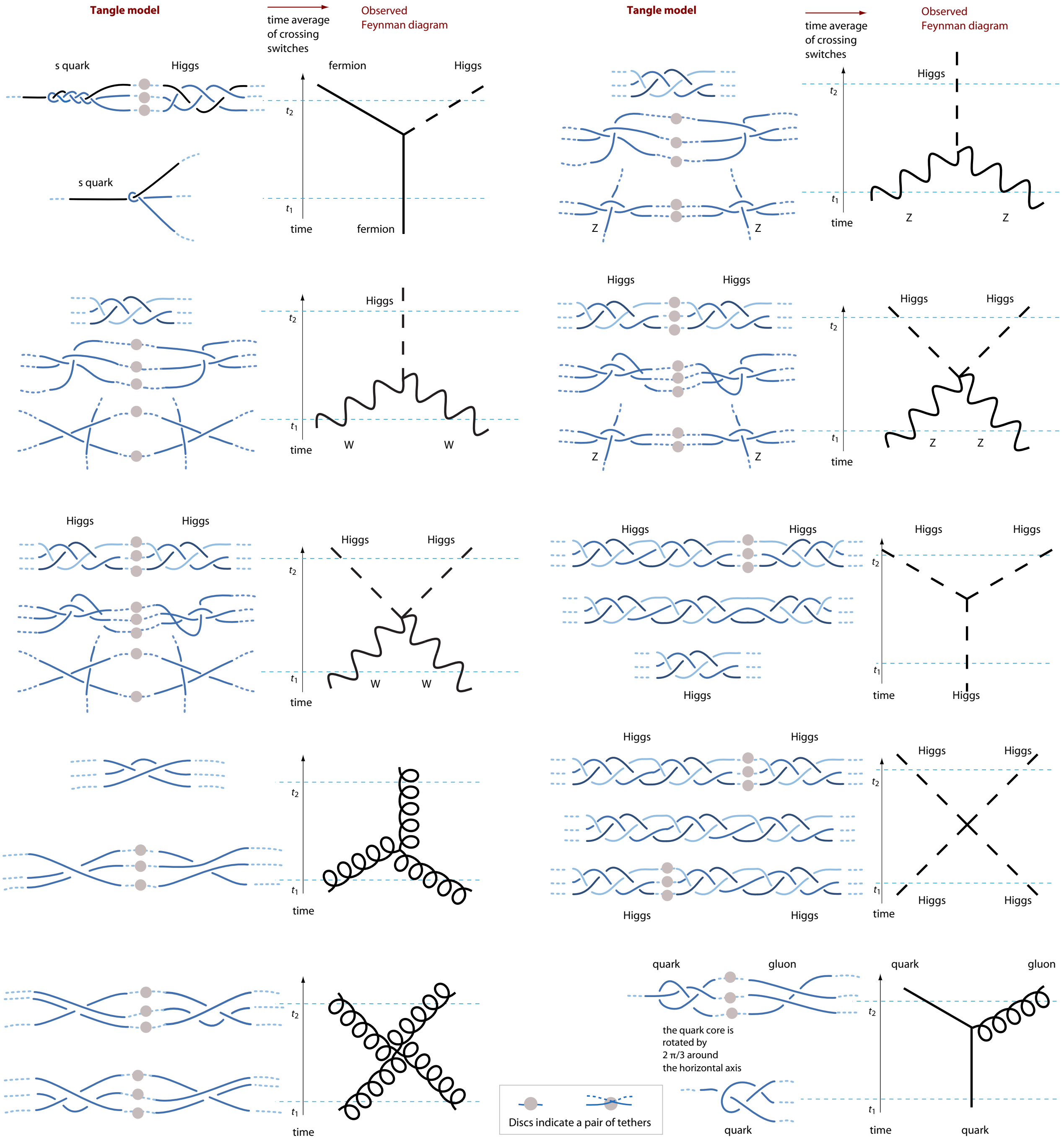


Figure 9: The interaction vertices allowed by fermion and boson topologies imply the complete Lagrangian of the standard model (part two).

A second, related way to deduce the Lagrangian of the standard model from strands does not make use of Feynman diagrams:

1. Rational tangles determine the *fermion* spectrum – exactly three generations of quarks and leptons – with the observed particle properties: spin, charges, representations, other quantum numbers, and masses (see Figure 6).
2. Rational tangles imply that fermion mixings arise and are described by the usual phases and angles (see [4]).
3. Free fermions tangles are described by free Dirac Lagrangians (see Section 4).
4. Tangles determine the *boson* spectrum with the observed particle properties – spin, charges, representations and masses (see Figure 5).
5. Gauge bosons tangles imply that free gauge bosons are described by the usual free field Lagrangians (see Figure 5 and Figure 7).
6. Tangle deformations imply that particle interactions are local, simply coupled, renormalizable, have the usual – unbroken or broken – gauge symmetries, obey the conservation of quantum numbers and show unique couplings (see Figure 7).
7. The *Higgs* tangle implies that the Higgs is massive, has spin 0 and is described by its usual Lagrangian (see Figure 5).
8. The Higgs tangle explains the Yukawa mass terms by braid addition inside tangle families (see also Figure 9).

In total, particle physics appears to be described by the standard model Lagrangian. These results can be summarized:

▷ The standard model results from tangles.

This connection is unambiguous. It arises because of the limited number of rational tangle families and because of the limited number of gauge interactions. Above all, the connection can be tested: the tangle model predicts that *there is* no *physics beyond the standard model with massive Dirac neutrinos.* This is the central prediction of the strand conjecture in the domain of high energy physics.

If any interaction, such as grand unification, any symmetry, such as supersymmetry, any gauge boson or any fermion that differs *from the standard model is observed, the strand conjecture would be falsified.* More strictly: *if any single prediction from the strand conjecture turns out to be incorrect, the conjecture is falsified.*

In the following, specific predictions for quantum electrodynamics are deduced. They are all based on the assumption that the standard model derives from strands. *If the standard model turns out to be false or even incomplete, the strand conjecture and the following sections are equally false.*

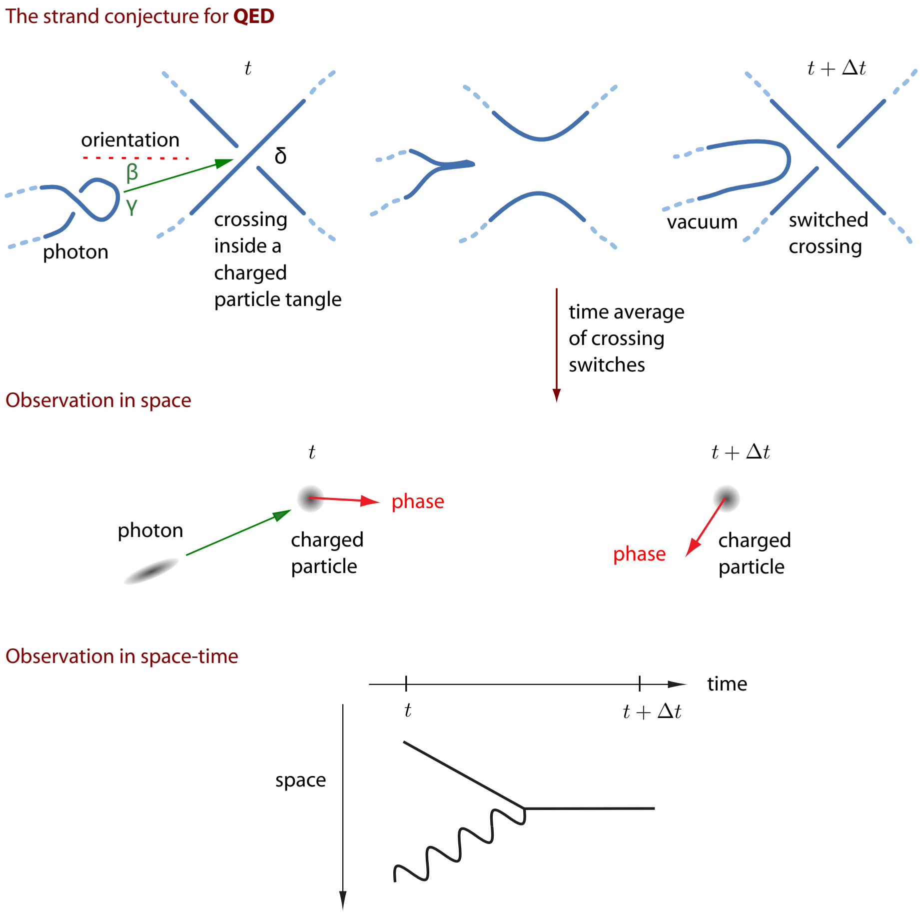
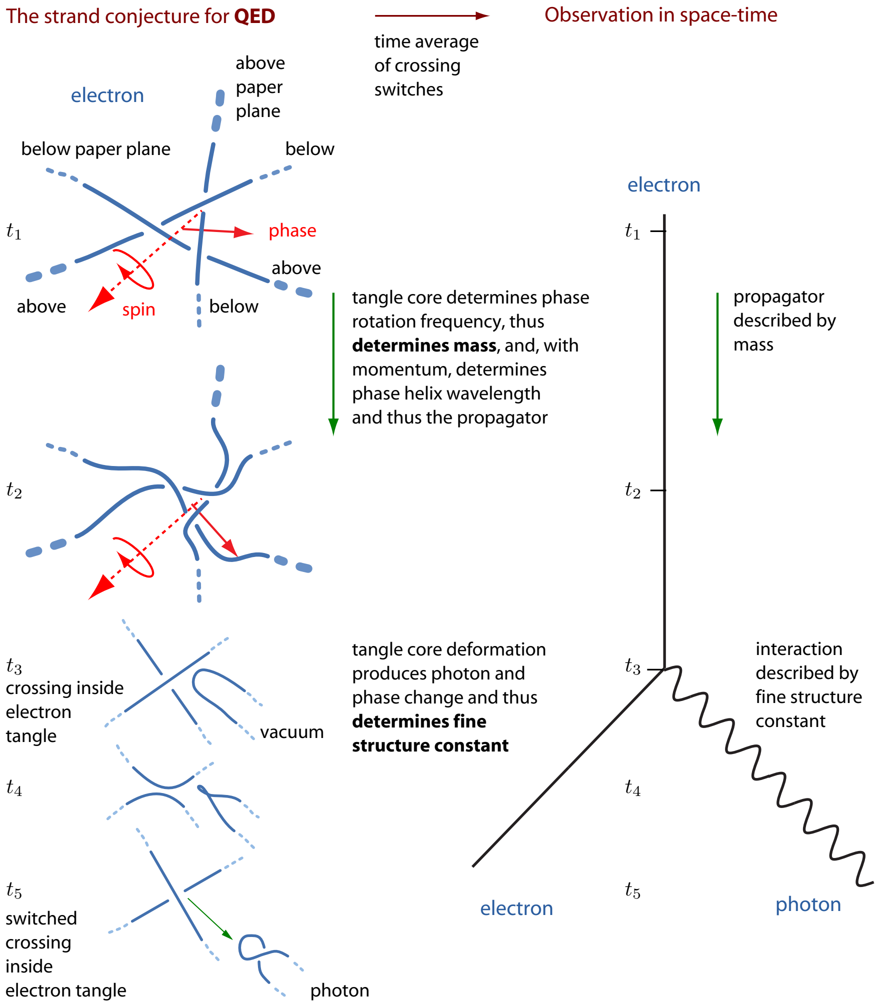


Figure 10: The geometry for the basic process of quantum electrodynamics (QED) is illustrated. Top: the absorption of a photon by a strand crossing, i.e., by a tangle region carrying the charge $e/3$, at Planck scale. Centre: the corresponding observation at usual scales. Bottom: the corresponding Feynman vertex.



tangle core determines phase rotation frequency, thus **determines mass**, and, with momentum, determines phase helix wavelength and thus the propagator

propagator described by mass

tangle core deformation produces photon and phase change and thus **determines fine structure constant**

interaction described by fine structure constant

Figure 11: All of QED is illustrated in one picture. In the strand conjecture, the electron mass and the fine structure constant are determined by the electron tangle (here the simplest family member) and its shape change under fluctuations. Like every particle mass, the electron mass is fixed by the average belt trick frequency during propagation. Like every coupling constant, the fine structure constant is determined by the average phase change in the charged particle core (electron in this case) due to emission of a boson (a photon in this case).

8 The strand description of quantum electrodynamics

Strands imply that the electromagnetic interaction is due to the *first Reidemeister move:*

> ▷ The electromagnetic interaction is the – partial or complete – switch of a skew strand crossing in a charged tangle core. The crossing switch is accompanied by the absorption or emission of a photon twist.

The strand description at the basis of quantum electrodynamics (QED) is illustrated in Figure 10 and in Figure 11. When a photon is absorbed, it transfers its twist to a crossing that is part of a tangle core of a charged particle. The photon (partially) switches the charged tangle crossing and thereby loses its own twist; as a result, it becomes a vacuum strand and the photon effectively disappears. At the same time, the phase of the charged tangle core changes, due to the crossing switch that occurs in the particle tangle.

In the corresponding photon *emission* process, a vacuum strand acquires a twist from a tangle core switch. Again, due to the crossing switch in the particle tangle, the phase of the charged tangle core changes. In other terms, both the absorption and the emission of a photon change the phase of a charged tangle.

In the strand conjecture, a (real) photon is a propagating twist, as illustrated in Figure 5. Photons, being untangled, are *massless*. Photons have *spin 1* because their core is invariant after a rotation by 2π. Photons, being massless twists, and have exactly *two helicity states.* Photons advance through vacuum in a way that resembles a localised corkscrew on a strand advancing in a "mattress" that is provided by the vacuum – with the difference that the corkscrew can also "jump" from a strand to a neighbouring one. Photons have a rotating phase, zero charge(s) and infinite lifetime. *Observing any deviation from the photon propagator would falsify the strand conjecture.*

During the crossing–twist transfer – i.e., the electromagnetic interaction – the *phase* of the charged particle *changes.* This connection reproduces the general observation that in nature the phase of wave functions can change in only two ways: either by propagation (as described by the free Dirac equation) or by interaction (as described by the Feynman vertices).

The emission and the absorption of a photon occur via the removal or addition of a twist. *Full* twists – rotations by the angle π are illustrated on the top left of Figure 7. Full twists can be generalized to *partial* twists with arbitrary rotation angles. And partial twists can be concatenated: their angles can be added. In addition, a *double* full twist can be undone be moving the tethers and is thus equivalent to *no* twist at all. Together, these properties imply that the concatenation of any two partial twists by the angles α and β can be represented by

$$\mathrm{e}^{i\alpha}\mathrm{e}^{i\beta} = \mathrm{e}^{i(\alpha+\beta)} \quad . \tag{2}$$

Twists thus define the group U(1).

Because the crossing–twist transfer – i.e., the electromagnetic interaction – arises in a volume of a few cubic Planck lengths, the interaction is effectively *local*. Because crossing–twist transfer arises in a finite volume of extremely small size, there are *no issues* with divergences or renormalization. Because strands have a small but finite diameter, a regularization of quantum electrodynamics arises at the Planck scale, and a Landau pole does not arise. *Observing any deviation from quantum electrodynamics would falsify the strand conjecture.*

As will be argued now, because twist transfer is related to tangle chirality, electromagnetic interaction is related to electric charge.

9 Predictions about electric charge and classical electrodynamics

In nature, a particle is electrically *neutral* if its phase does not change when absorbing random photons. In nature, a particle is electrically *charged* if its phase changes in a preferred direction when absorbing random photons.

In the strand conjecture, the tangle cores of all *neutral* particles are *topologically achiral*, i.e., they are equal to their mirror image in the minimal crossing projection. As a result, neutral particles show no average phase change when they are hit by random photons. In contrast, *electrically charged* particles have *topologically chiral* tangle cores. Chiral cores differ from their mirror image in the minimal projection. Indeed, chiral cores have a preferred rotation direction when they absorb random photons: they are electrically charged.

The strand definition of electric charge implies that charge has two signs, is quantized, is conserved, and emits virtual photons. Electric charges of particles and antiparticles are predicted to be of exactly the same value, but of opposite sign. And electric charge is predicted to arise only in particles with non-vanishing mass. All this agrees with observation.

Electric charge is a consequence of tangle chirality. In all interactions and Feynman diagrams, including those shown below, chirality is conserved. As a result, *electric charge is conserved.* (This also implies that the charge density and the probability density of fermions is conserved.)

All electric charges move *slower than light,* because in the strand conjecture, only massive tangles can be electrically charged. Electric charges thus differ from photons.

In the strand description of electrodynamics, the electric field E is the *volume density of twists.* The magnetic field B is the *twist flow density* [18].

In nature, a static electric charge emits virtual photons. In the strand conjecture, a fluctuating chiral tangle emits twists whose ends are attached to the fermion tangle. Such twists represent virtual photons and are continuously emitted into or transferred to the surrounding vacuum. The electric field – the virtual photons – around a static charge is illustrated in Figure 12. As a result of the vanishing mass of photons, *Coulomb's law holds.*

Established mathematical arguments now allow deducing an important result. Whenever

1. Electric charge is conserved, i.e., obeys the continuity equation (in the strand conjecture, this occurs due to the topological definition of charge),
2. All electric charges move strictly slower than light, in Minkowski space-time (in the strand conjecture, this is intrinsic to the tangle model of massive elementary particles), and

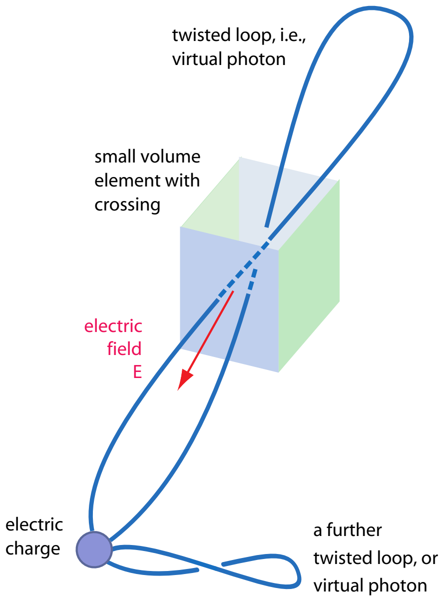


Figure 12: Electric fields – collections of twisted loops, i.e., of virtual photons – arise randomly around a (point) electric charge and lead to Coulomb's law. Both for the classical case of a point charge and for the quantum mechanical case – an electric charge due to a tangle forming a probability density – the figure also illustrates *minimal coupling*.

3. Coulomb's law is valid (automatic in the strand description of electromagnetism as it is due to Reidemeister 1 moves),

the inevitable consequence [21, 22] is that

▷ Maxwell's equations hold.

In summary, the strand conjecture implies and predicts classical electrodynamics. The Lagrangian of classical electrodynamics, including minimal coupling and the Lagrangian of the free electromagnetic field, thus follow from strands. However, there is one limitation.

In the strand conjecture, all physical observables are due to tangle crossing switches. And all strands have a minimum effective diameter. As a result, all physical observables are predicted to have limit values. For electric and magnetic fields, the limits are $E \leq c^4/4Ge = 1.9 \cdot 10^{62}\,\mathrm{V/m}$ and $B \leq c^3/4Ge = 6.3 \cdot 10^{53}\,\mathrm{T}$. (Similar limits apply to the field values of the nuclear interactions.) In fact, using the smallest electric charge $e/3$, one could argue that the field limits are three times larger. In any case, so far, even the largest observed field values, from particles to magnetars, are several orders of magnitude smaller than these limits. *Observing a field value beyond these corrected Planck limits would falsify the strand conjecture.*

In the tangle model, electric charge is a *topological property* of tangle cores. As explained in an earlier publication [4], every crossing in the minimal projection of a particle tangle leads to an electric charge $+e/3$ or $-e/3$. This assignment leads to the observed charge values for all elementary particles. In particular, this assignment thus explains why the charge of the proton is observed to be exactly equal, within measurement precision, to the charge of the positron. This charge equality is not explained in the standard model; in contrast, the tangle model explains it, because electric charge is a topological quantity, independent of the particle type. The tangle model seems to be the first explanation for the equality of proton and positron charge in the research literature. *Discovering an exception to electric charge quantization in multiples of $e/3$ – such as a charge $e/2$ or a millicharged particle – would falsify the strand conjecture.* (A corresponding prediction can be made for the nuclear charges.)

The explanation of electric charge quantization is consistent with an additional prediction of the strand conjecture. The tangle model of virtual photons led to the strand description of electric and magnetic fields. The strand description of electromagnetic fields in turn implies the *lack of magnetic charge* in nature. The strand conjecture does not allow the construction of magnetic charge. *The discovery of a magnetic monopole or of a dyon would falsify the strand conjecture.*

In summary, *observing any deviation from Maxwell's equations, at any energy or scale, would falsify the strand conjecture.*

10 The coupling between matter and the electromagnetic field

Figure 12 illustrates the coupling between matter and electromagnetism. First of all, the strands in the figure show that the coupling is proportional to the charge: higher charge values emit more virtual photons. In an absorption process, higher charge values absorb more virtual photons. The coupling to the electromagnetic field is thus proportional to the charge q.

Secondly, the absorbed or emitted photon strand changes the *phase* of a charge. More photons have a larger effect. A detailed exploration shows that the coupling is proportional to the potential.

Thirdly, because twists are exchanged, the energy of a charge is changed by the value of the *scalar potential* times the charge. And because of twist exchange, the momentum of charge – defined by the shape of its probability density – is changed by the *vector potential* times the charge. These properties define minimal coupling.

Equivalently, strands and twist exchanges visualize and realize the freedom to chose the phase of a tangle; this was illustrated in Figure 3 and Figure 10. The freedom of choosing the phase leads to an U(1) gauge freedom. Strands thus imply U(1) gauge invariance. In particular, strands also illustrate that the coupling to the electromagnetic field is *equivalent* to gauge invariance: they are due to the same geometric effects.

Strands can thus be seen as visualizing the ideas of Baylis and of Hestenes [23, 24, 12]: the *electromagnetic field* is defined by the spacetime rotation rate that it induces on a charge. Strands realize this definition with the help of twist exchange.

In short, strands imply *minimal coupling.* This property is valid both classically and quantum mechanically. As a result, strands reproduce the full Lagrangian of quantum electrodynamics. In particular, strands reproduce the *propagators* of photons and elementary charges, as well as the basic *interaction vertex* of quantum electrodynamics. *Observing a deviation from minimal coupling, at any energy or scale, would falsify the strand conjecture.*

11 Electromagnetism, measurements and minimum time

Strands explain the electric charge of particles from their tangle topology. Strands explain the origin of Maxwell's equations. But strands do more: they explain the fundamental principle itself.

The fundamental principle – illustrated in Figure 2 – defines all observations, all measurements and all observables as due to crossing switches. As discussed above, the basic QED process illustrates that *crossing switches are observable precisely because they couple to electromagnetic fields*. Every observation process and every measurement device – for measuring length, time, mass or any other physical observable – use electromagnetic fields. The use of electromagnetic fields is often forgotten – for example when reading the position of a pointer of a weighing scale – but it is essential in every measurement. Without electromagnetism there are no measurements. Every observation, every measurement, and every comparison with a standard are made using electromagnetism. For example, all seven base units of the international system of units (SI) – and thus all units – are indeed defined and realized with electromagnetic means of observation. As another example, all human senses – even hearing – are electromagnetic. Every measurement and every observation is electromagnetic. Strands make this point forcefully, at the most fundamental level. Even though there is no realistic chance to do so, it can be said: *discovering any non-electromagnetic observation or measurement would falsify the strand conjecture.*

The coupling of crossing switches to electromagnetism also explains why a minimum time arises in the fundamental principle. A crossing switch could, in principle, take an arbitrary short time. But such a crossing switch would not and does not couple to the electromagnetic field: a photon wavelength shorter than a (corrected) Planck length is not possible. Such a ultra-rapid crossing switch would not be observable; it would not have any physical effect. In short, only crossing switches that take longer than a (corrected) Planck time have physical relevance. *Discovering any effect whatsoever that is due to time intervals shorter than the minimum time would falsify the strand conjecture.*

12 Predictions about electric dipole moments

The tangle model and the photon absorption process illustrated in Figure 10 imply that the charge 'units' $e/3$ or $-e/3$ inside an elementary particle are (at high energy) at average distances of the order of the Planck length. In particular, the electron tangle contains three charge units of the same sign. For all elementary particles, strands imply that the *intrinsic* electric dipole moment d is at most four times the Planck length times the charge unit e, thus

$$d \lesssim e\,4\,l_{\mathrm{Pl}} \approx 0.6 \cdot 10^{-34}\,e\,\mathrm{m} \quad . \tag{3}$$

The intrinsic dipole values predicted by the tangle model for elementary particles – either zero or negligibly small – are valid *provided* that the tangles of Figure 5 and Figure 6 are correct.

In the strand conjecture, *additional* electric dipole moments arise because charges of opposite sign occur in the perturbation expansion, i.e., through strand fluctuations. These additional electric dipole moments occur in the same way as in the standard model. Strands thus predict the same electric dipole moment values as the standard model, where sizeable electric dipole moments arise only from operators of higher order. The dipole values predicted by the standard model and those predicted by the tangle model are still many to several orders of magnitude smaller than the experimental limits, of which the best is for the electron: $d_e < 1.1 \cdot 10^{-31}\, e\,\mathrm{m}$ [25]. Hopefully, future experiments will allow stricter tests. For example, values for the dipole moment considerably larger than expression (3) are predicted by supersymmetric models. *Discovering a sizeable electric dipole moment for electrons or other elementary particles would falsify the strand conjecture.*

13 Predictions about the fine structure constant

In quantum electrodynamics, the fine structure constant can defined in the following way:

▷ The (average) change of phase induced by the emission or absorption of a photon by a particle of unit electric charge determines the square root of the fine structure constant.

In the strand conjecture, the definition is the same; only the tangle model for particles is added. The definition can also be generalized to the nuclear interactions.

Because the emission or absorption of a photon occurs at a skew strand crossing, the tangle model of QED explains how a charged particle can have a spread-out wave function and nevertheless can behave as (almost) point-like in interactions. The wave function is due to the tangle fluctuations of the complete tangle, which is spread out in space. In contrast, the electromagnetic interaction occurs at a single crossing, which is effectively point-like. The same applies to the nuclear interactions.

Because the emission or absorption of a photon occurs via the removal or addition of a skew strand crossing, the value of the fine structure constant is determined by the geometry of the strand process. The same applies to the nuclear interactions.

Because the strand conjecture reproduces all known Feynman diagrams, quantum field theory is predicted to remain valid at all observable energy scales. In particular, all three effective gauge coupling constants run with energy, for the same reason as they do in quantum field theory. *Any experimental deviation from the running of the coupling constants – including the discovery of a new energy scale – would falsify the strand conjecture.*

Strands imply that all coupling constants, including the fine structure constant, are *fixed, unique, calculable* and smaller than 1. In particular, the fine structure constant is predicted to be *constant* over time and space – despite occasional opposite claims. The same is predicted for the nuclear interactions. In addition, the strand conjecture predicts that the fine structure constant and the nuclear coupling constants are *the same* for all particles and for all antiparticles. *Any experiment disproving the particle-independence, time-independence or position-independence of the coupling constants would falsify the strand conjecture.*

14 Estimating the fine structure constant

The switch of a tangle crossing illustrated in Figure 10 allows calculating the fine structure constant α. The figure shows the projection along the shortest distance s of a tangle crossing. In the neighbourhood of the shortest distance, each strand is parallel to the paper plane. Let δ be the angle between the two strands in this projection. The direction perpendicular to the paper plane is best imagined as the axis of a sphere whose north pole is above the paper and whose south pole is below it. The paper plane then is the equatorial plane. The photon incidence angle β shown in Figure 10 is a *longitude* on this sphere; it can vary from $-\delta/2$ to $+\delta/2$. The other photon incidence angle γ is the angle from the incident photon direction to the paper plane; it thus corresponds to a *latitude* and varies from $-\pi/2$ to $+\pi/2$.

When a photon approaches a tangle core, it twists the part of the crossing surrounding it. The details of the photon incidence determine the *probability* p that a crossing switch takes place. The details also determine the *value* ν of the induced phase change. Both quantities can be estimated from geometry. The result given below is more precise than the result deduced previously [4].

The following arguments are based on the geometry of Figure 10 and Figure 13. If the paper plane is taken to be perpendicular to the shortest distance of a crossing, the orientation axis lies *in* the paper plane, as illustrated in Figure 13. The phase due to a crossing with angle δ is best described by a vector oriented perpendicularly to the orientation axis, as illustrated in Figure 3 and Figure 13. The freedom of choice of gauge allows to take as phase vector any such vector.

The geometric setting now allows estimating the important quantities. In the first step, the *value* ν of the phase change around the orientation axis that is due to a crossing switch needs to be estimated. The general contribution of a strand crossing to the total tangle phase is estimated to be $\sin\delta$. A switch that occurs due to a photon incident along the orientation axis of the crossing reverses the crossing phase from the original value to its opposite; the phase change is thus $\nu = 2\sin\delta$. This is the value by which the phase of the total tangle changes when the absorbed photon arrives *precisely* along the orientation axis. For a general photon incidence, described by the angles β and γ, the induced crossing switch is only *partial*. The *approximate* value for the phase change is expected to be

$$\nu \approx 2\sin\delta\cos\beta\cos\gamma \quad . \tag{4}$$

This approximation for the general case completes the first step.

In the second step, the *total probability* p that a photon induces a crossing switch must be estimated. The probability that a photon induces a crossing switch will vanish for a photon arriving along the poles of the crossing. In other terms, perpendicularly to the paper plane, $p(\gamma = \pm\pi/2) =$

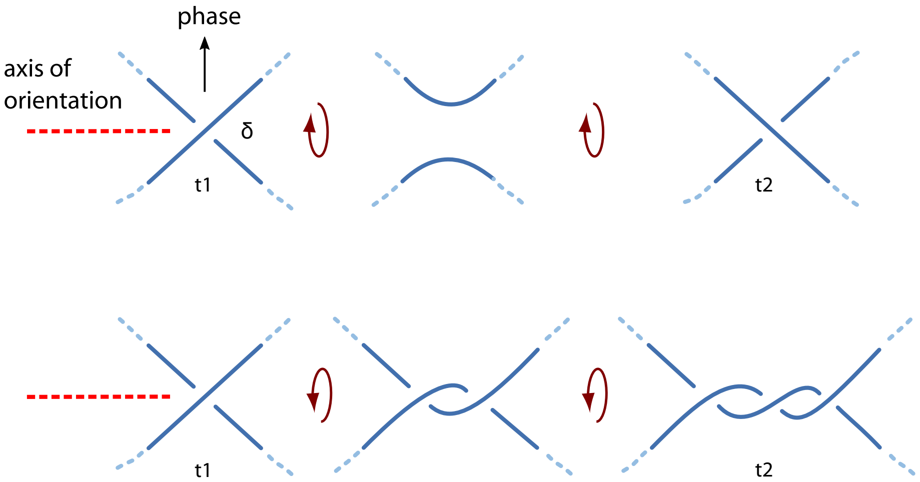


Figure 13: Two possible effects of photons on a crossing are illustrated. Top: the photon can decrease a twist, untwisting it, and continue until the twist changes sign. Bottom: the photon can increase a twist by the same amount. In the strand conjecture, the different probabilities of these two processes appear to be at the basis of the value of the fine structure constant (see Section 14).

0. In addition, the probability for a crossing switch vanishes for photons arriving perpendicularly to either of the two strands. Furthermore, the switch probability is expected to be highest for the case $\gamma = \beta = 0$, i.e., for symmetrical incidence. For such a symmetrical incidence, the switch probability p is easiest to estimate. The probability varies with the crossing angle δ and with the direction of the induced deformation. Photons can either change the crossing into one of the opposite sign, or change the crossing into a double one, as illustrated in Figure 13. The probabilities for the two processes differ. *Decreasing* the twist is more probable than *increasing* the twist, because the available (configuration) volume for a twist decrease is larger than for a twist increase. The difference between the twist-decreasing probability p_d and the twist-increasing probability p_i yields the total probability value $p = p_d - p_i$ that is sought. Both probabilities can be approximated.

One task is the determination of the *twist-decreasing* probability p_d. This probability will depend on the complement of the spherical angle spanned by the two strands when they are untwisted. In addition, the probability p_d will depend on the ratio between the photon wavelength λ and the minimum strand distance s. The effect should yield a pre-factor 1 for $\lambda > s$; the pre-factor should decrease to 0 for smaller wavelengths. After averaging over all values for λ, in case of symmetric incidence, the twist-decreasing probability will obey $p_d < (\cos\delta/2)^2$.

The previous reasoning has now to be generalized to a general angle of photon incidence. The *approximate* twist-decreasing probability p_d for a crossing switch is expected to be $p_d \approx \cos\theta_1 \cos\theta_2$, where θ_n is the angle between the strand n and the direction of photon incidence.

These approximations yield the probability p_d of a twist-decreasing crossing switch for all possible geometries. The angles θ_n are determined by the scalar products $\cos\theta_1 = (\cos(\delta/2), \sin(\delta/2), 0)\cdot$

$(\cos\beta\cos\gamma, \sin\beta\cos\gamma, \sin\gamma)$ and $\cos\theta_2 = (\cos(\delta/2), -\sin(\delta/2), 0)\cdot(\cos\beta\cos\gamma, \sin\beta\cos\gamma, \sin\gamma)$. They yield an approximate probability

$$p_d \approx (\cos(\delta/2)\cos\beta\cos\gamma)^2 - (\sin(\delta/2)\sin\beta\cos\gamma)^2 \ . \tag{5}$$

The other task is the determination of the *twist-increasing* probability p_i. Like p_d, also the probability p_i will depend on the angle δ and on the strand distance s. A precise relation between p_i and p_d can probably only be determined with computer simulations. However, Figure 13 suggests a crude approximation. In the case of a twist *increase*, the crossing is an obstacle to the approaching photon. The probability for a twist increase might thus be *about half* the probability of a twist decrease: $p_i \approx (0.5 \pm 0.1)\, p_d$. Taking this relation as approximation for *all* incidence geometries implies

$$p \approx (0.5 \pm 0.1)\, p_d \ . \tag{6}$$

This completes the second step of the calculation.

The third and final step of the calculation of the fine structure constant is the averaging over all possibilities. First of all, the calculation requires averaging the phase change for an absorbed photon times the respective probability. The average has to take place, first of all, over all photon incidence angles β and γ. This requires the use of the spherical surface element $(1/4\pi)\cos\gamma$. Secondly, the calculation requires averaging over all strand crossing configuration angles δ – using the probability density for strand angles given by $\sin\delta$. Thirdly, an average over all photon polarizations is needed; it introduces a factor 1/2. Finally, multiplication by 3 gives the fine structure constant for a full unit charge, i.e., for a tangle core with *three* crossings. This completes the third and last step.

Combining the three calculation steps, the estimate for the electromagnetic coupling constant becomes

$$\sqrt{\alpha} \approx \frac{3}{8\pi} \int_{\delta=0}^{\pi/2} \int_{\beta=-\pi/2+\delta/2}^{\pi/2-\delta/2} \int_{\gamma=-\pi/2}^{\pi/2} p\, \nu\, \sin\delta\, \cos\gamma \, \mathrm{d}\gamma\, \mathrm{d}\beta\, \mathrm{d}\delta \ . \tag{7}$$

Inserting the above approximate expressions for ν, p_d and p, gives $\sqrt{\alpha} \approx 0.09 \approx \sqrt{1/126}$. This result most probably is a lucky hit. Estimating the total error of the approximations to be 30% yields

$$\frac{1}{278} \lesssim \alpha \lesssim \frac{1}{69} \ . \tag{8}$$

At low energy, the experimental value is $1/137.03599914(3)$, and at at Planck energy the standard model prediction is $1/110(5)$. Given the crudeness of the approximations, result (8) still seems *too good to be true.*

To improve the calculation, first of all, the geometric approximation needs to be improved. It might well be possible that research on link geometry, such as reference [26], will be of help. Indeed, part of Figure 13 can be seen as depicting what researchers studying geometric link shapes call ‘simple clasps’. Next, the calculation must be corrected for the admixture from the weak interaction. Indeed, the exchange of (half of) the weak W_3 boson tangle is similar to the exchange of a photon twist.

On the positive side, the Planck scale model for the basic QED diagram remains promising: it reproduces all qualitative aspects of quantum electrodynamics. The approximate value for the fine structure constant is ab initio, unique, constant, and equal for all particles of unit charge. *If a future, more precise calculation of α disagrees with measurements, the strand conjecture would be falsified.*

15 Predictions about the g-factor and the anomalous magnetic moment

The exploration of the magnetic dipole moment of elementary particles has fascinated many. A stone near Schwinger's grave shows his formula for the *g-factor*

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi} \tag{9}$$

which is correct to first order in the fine structure constant α. The ratio $g/2$ is defined as

$$\frac{g}{2} = \frac{\mu/e}{S/m} \tag{10}$$

with the help of the magnetic (dipole) moment μ, the electric charge e, the spin S and the mass m. In the meantime, the anomalous magnetic moment $\frac{g}{2} - 1$ has been measured to a precision better than 10^{-9} [27] and calculated, with comparable precision, up to order α^5 [28].

In the tangle model of elementary particles, spin is due to the rigid rotation of the tangle core. Spin is thus related to the rotation of the *mass*, whereas the magnetic moment is related to the rotation of the *charge*. In the strand conjecture, the ratio $g/2$ can thus be seen as the ratio between two rotation frequencies. This allows deducing a number of conclusions and tests.

To order α^0, the tethered core rotates *rigidly*. In this case, the rotation of the tangle charge is exactly due to the rotation of the tangle core. Now, the mass value of a particle is due to the motion of the tethers. Due to the belt trick, the motion of the tethers occurs with *half* the frequency of the core. Thus, $g/2$ is equal to 1 in the strand conjecture. If particles were not tethered, the result would not arise. In short, strands imply that the g-factor, to order α^0, is equal to 2 for all charged particles. This applies to all those charged systems whose rotating cores make use of the belt trick, independently of the spin value. (The result thus does *not* apply to a macroscopic charged metal sphere, for example. In this case $g = 1$, because the belt trick plays no role in the rotation.) So far, the conclusion agrees with experiment [29] and theory [30]: to order α^0, all charged fermions, but also the W boson and all charged black holes have a g-factor of 2.

In the tangle model, an expression for the g-factor to order α^1 arises when in addition to rigid core rotation, also the deformations of *one* strand segment *inside the core* are taken into account. In the language of quantum electrodynamics, the expression arises when one virtual photon is emitted and reabsorbed. In the language of strands, the simplest deformation of the rotating tangle core occurs when one twist, i.e., one virtual photon, is emitted and reabsorbed. This twist emission

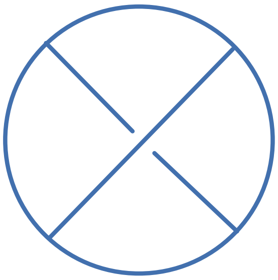


Figure 14: This topological configuration, following Broadhurst and Kreimer, implies that the irrational number $\zeta(3)$ arises in the coefficient of α^2 for the g-factor. The strand conjecture produces the *same* configuration, if the circle is seen as representing the tangle core (or its rim), and if the two spokes – the two strand segments being deformed – are seen as the two virtual photon (twists) that leave the core and point towards the reader.

and reabsorption leads to a phase change of the core. Due to this deformation, when a tangle core rotates due to the belt trick, an effective, additional, small electrical rotation occurs. Like in quantum electrodynamics, the importance of the effect is described by the electromagnetic coupling. Using the definition of the fine structure constant given above, in section 13, leads to a simple result: To order α^1, the effect of the core deformation by one virtual photon is an additional phase jump angle given by the fine structure constant α. Translated to the rotation situation, this implies that $g/2$ is larger than 1 by the ratio between α and the full rotation angle 2π. Strands thus imply Schwinger's formula (9) to order α^1.

One day it should be possible to determine in a similar way the expression for the g-factor to order α^2. The expression was first calculated (correctly) by Petermann [31]. The calculation requires to take into account in particular the situation that *two* virtual photons (i.e., two twists) can also tangle *around each other*. Already a long time ago, Broadhurst and Kreimer pointed out a connection between topology and the g-factor [32, 33]. They argued that specific knots imply the appearance of specific zeta values in the expression for the g-factor at higher order in α. In particular, they argued that the coefficient $\zeta(3) = \sum 1/n^3$, which appears at order α^2, is due to the specific topological configuration illustrated in Figure 14. This configuration indeed arises when two virtual photons are emitted by an electron tangle.

Following Broadhurst and Kreimer, generalizing the configuration of Figure 14 to more spokes is expected to lead to $\zeta(5)$, which arises at order α^3 [34]. And indeed, such a generalized configuration arises in the strand conjecture when more strand segments in the core are deformed, i.e., when more virtual photons play a role. The tangle model for electrons and photons thus might provide an underlying explanation for this research direction and for the relation between topology and the g-factor. The appearance of zeta values might be due to the integrations over the different geometric configurations in space of the twisted loops that correspond to virtual photons.

In summary, the tangle model suggests the following conjecture: the perturbative expression for the g-factor arises due to the effects of tangle topology on the multiple integrals that appear in the calculation. This conjecture seems to be supported by the appearance of certain irrational numbers in the coefficients of the perturbative g-factor expansion. If strands agree with the full expression for the g-factor also at higher orders, the tangle model would be confirmed. Whether strands might help in *simplifying* perturbative calculations cannot be said, at present. But it can be said that the finite number of strands in the finite number of lepton tangles leads to a single vertex for quantum electrodynamics, and thus to it renormalizability. As a result, *discovering a contradiction between the tangle model and the g-factor expansion would falsify the strand conjecture.*

16 Predictions about elementary particle masses

In the strand conjecture, the fundamental constants are emergent properties. Also the mass of an elementary particle is emergent.

Mass is energy divided by c^2, and energy is action per time. In the strand conjecture, every crossing switch produces a quantum of action $\hbar$. The mass value of a fermion at rest, in units of the corrected Planck mass, is thus given by the average number of crossing switches that occur per corrected Planck time. If the tangle core at rest would keep a constant orientation and phase, the average number of crossings observed over time would vanish. The mass value of a fermion is therefore due to the frequency of the spontaneous belt trick that appears due to strand fluctuations [4]. This relation implies predictions that can be tested, even before any mass values are calculated.

The strand conjecture implies a simple result about inertial and gravitational mass. On the one hand, the belt trick generates a displacement and thus relates rotation and displacement. In the Dirac equation, this relation is described by inertial mass. Figure 1 gives a (pale) impression of this connection. On the other hand, the double tether twists generated by the belt trick correspond to virtual gravitons; the belt trick thus also determines gravitational mass. And because both mass values are due to the same mechanism in the strand conjecture, inertial and gravitational mass are intrinsically *equal*.

Strands also predict that the mass values of all elementary particles – due to the respective belt trick frequencies – are *positive*, *fixed*, *unique* and *constant* in time and space, across the universe. The mass value of a tangle is positive, and not vanishing, because the probabilities for spontaneous tangle rotations in two opposite directions differ. The difference is due to the lack of symmetry of tangle cores with respect to rotation: tangle shapes are helical. (Only the simplest tangle for the down quark, shown in Figure 6, is not helical; its other family members are, however.) Mass values for particles and antiparticles, i.e., for tangles and mirror tangles, are predicted to be *equal*. For a system of several particles that are non-interacting, the total mass is predicted to be the *sum* of the particle masses.

The probability for a belt trick is low. In particular, the probability for the belt trick is much lower than one crossing switch per corrected Planck time. The strand conjecture thus predicts that mass values m for elementary particles are much lower than the (corrected) Planck mass:

$$m \ll \sqrt{\hbar c/4G} = 6.1 \cdot 10^{27}\,\mathrm{eV} \quad . \tag{11}$$

The inequality agrees with experiment. According to the strand conjecture, the low probability for the belt trick is therefore the main reason that elementary particle masses are small compared to the Planck mass. The main mass hierarchy is explained.

Strands also imply that particle mass values depend on the tangle structure of the particle: more complex tangles – the number of tethers being equal – have larger mass. In the case of the leptons, strands thus predict

$$m_e < m_\mu < m_\tau \text{ and } m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \quad . \tag{12}$$

The latter prediction, on the normal mass ordering of neutrinos, should be testable in the coming years. The strand conjecture does not appear to allow anomalous neutrino mass ordering. *Any deviation from any one of the mentioned predictions on particle masses – such as non-normal neutrino mass ordering or particle masses that vary across the universe or vary during its history – would falsify the strand conjecture.*

In the strand conjecture, particle tangles, and in particular the cores of tangles, get tighter and *flatter* at higher energy. The flattening, a result of relativity, will influence the frequency of the belt trick and change the mass value. High energy thus leads to a *running* of particle mass. In particular, a flatter tangle core will behave like a more complex core. Strands thus suggest that lepton mass values should increase slightly with energy. This is indeed the case in the standard model, where the mass values of leptons change less than about 10% up to Planck energy [35]. The running should be calculable with computer simulations. *Any deviation from the mass running calculated with strands from experiment would falsify the strand conjecture.*

There is a slight chance that the mass running calculated with the standard model might deviate from the mass running of the tangle model. In this case, experiments could test the strand conjecture directly.

17 Estimating the mass of the electron and the other leptons

In the strand conjecture, the mass value of a fermion is determined by the frequency of the spontaneous belt trick that appears due to strand fluctuations. The spontaneous belt trick leads to an average of crossing switches per time. This yields an average action value per time, which defines an energy and thus a mass value.

The frequency of the spontaneous belt trick can be estimated. At present, this requires three approximations.

In the strand conjecture, every massive particle is represented by a *family* of tangles. As mentioned in Section 5, the family members differ by the number of Higgs braids they contain. In the following, the calculation only takes into account the tangle of the *simplest* family member. The effect of the other family members – the Yukawa terms – is neglected in the following.

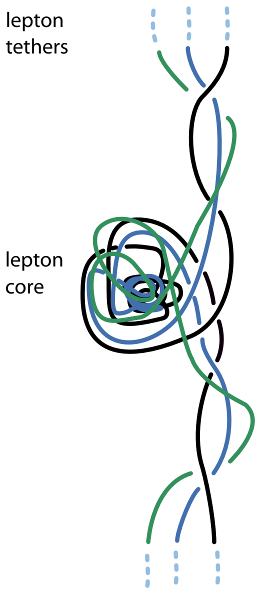


Figure 15: This drawing schematically illustrates the *most improbable* strand configuration during the belt trick of a lepton, thus of a tangle made of three strands. The configuration allows deducing mass limits (see Section 17).

Secondly, the simplest mass calculation is for a tangle *at rest* that rotates all the time in one direction only. Relativistic effects are thus ignored, and so are effects of the helicity of tangle shape.

The third approximation is the assumption that *tight* tangles are representative for average tangles. In the strand conjecture, tangles can be tight or loose. The belt trick occur in all cases. For extremely loose tangles, the frequency of the belt trick is expected to be independent of the tangle size. For tight tangles, the tube-like character of the tangle will be felt. Taking tight tangles as representative ignores the running of mass with energy.

Using tight tangles to determine mass values implies that the diameter of strands is not neglected – as it was up to this section. In other words, gravity is not neglected in the following. This is as expected: determining particle mass indeed requires taking into account both quantum and gravitational effects.

Using the three mentioned approximations, estimating mass values of elementary particles is simplified. In the strand conjecture, mass is given by the average number of crossing switches (times $\hbar$) that occur per minimum time (divided by c^2). A mass estimate for an elementary tangle requires estimating the probability of the spontaneous belt trick, i.e., of spontaneous tethered rotation [4]. However, estimating the probability of for the spontaneous appearance of the belt trick turns out to be *a difficult problem.* The research literature does not contain any hint towards a solution. Researchers on polymers, on fluid vortices, on cosmic strings, on string theory, on superfluids, and on statistical knot theory do not have explored the topic yet. The following ideas should thus be seen as first tentative steps into a dark room.

The belt trick takes several steps – illustrated in Figure 1 – in which the tethers follow specific configurations in space. To estimate the probability of the belt trick, it is best to focus on the most improbable configuration that is occurring during the motion. In Figure 1, this is the fifth configuration from the left, when the tethers wrap around the tangle core. For the leptons, whose tangles consist of three strands, the most improbable tangle configuration is illustrated in Figure 15. An estimate of the probability for this tangle configuration appears possible.

During the belt trick, the most improbable configuration arises through core rotation and tether deformations. The combination results in a frequency f for the spontaneous belt trick. Each belt trick implies an average number n of crossing switches. This yields, in corrected Planck units

$$m \approx f \cdot n \ . \tag{13}$$

This expression applied to every tethered structure whose core rotates, including black holes. Indeed, for a Schwarzschild black hole, the second factor – large in this case – plays the main role [4]. In contrast, for an elementary particle, both factors – both small in this case – are expected to play a role.

An estimate of the frequency or probability for the most improbable tether configuration requires to estimate the length of the tethers involved. At the Planck scale, a fluctuating strand segment of length l realizes a specific configuration in space – specified within a minimum length for each sub-segment of minimum length l_{min} – with a probability p

$$p \approx \mathrm{e}^{-l/l_{\text{min}}} \ . \tag{14}$$

Here it is assumed that the effective temperature at the Planck scale, which describes the fluctuations of strands, is the highest possible, i.e., the (corrected) Planck temperature. In Planck units, the probability p yields a frequency f. Expressions (13) and (14) allow estimating lower and upper limits for the masses of leptons.

The first step is to derive a *lower limit* for the lepton mass. In this case, the factor n is assumed to have the lowest possible value. The value of n is due to the tangle core and to the tethers. For the simplest case of an electron neutrino, the lowest value will be of order 1. In order not to forget it, the value 2 is taken as representative. This is the fourth approximation.

For a lepton with six tethers, the frequency of the most improbable configuration, shown in Figure 15, appears to be

$$f \approx \left(\mathrm{e}^{-l_{\text{add}}/l_{\text{min}}}\right)^6 \cdot O(1) \ . \tag{15}$$

Here, l_{add} is now the additional length in each tether that is required to go round the core and to produce the twists illustrated in Figure 15. The exponent 6 is due to the six tethers. The factor $O(1)$ is of order 1 and describes the number of rotation axes and the number of ways that the tethers are separated in the belt trick. In Figure 15, the separation during the belt trick is into two sets of three tethers each; sets of two and four, or one and five are also possible. The factor O(1) takes into account the various options. Again, a value of 2 is taken as representative. This is the fifth approximation.

For the smallest possible lepton core diameter, corresponding to the electron neutrino shown in Figure 6, the total length $l_{\rm add}$, for a *tight* core, is about 12 ± 3 minimum lengths. Given the difficulty to calculate the length $l_{\rm add}$, the value has been determined with actual ropes. This is the sixth and last approximation.

Taken together, a lower mass limit $m_{\rm ll}$ for leptons – and thus for neutrinos – arises, given by

$$\frac{m_{\rm ll}}{\sqrt{\hbar c/4G}} \approx ({\rm e}^{-12})^6 \cdot 2 \cdot 2 = 2.2 \cdot 10^{-31} \quad \text{or, equivalently} \quad m_{\rm ll} \approx 1\,{\rm meV} \ . \tag{16}$$

The corrected Planck mass $\sqrt{\hbar c/4G} = 6.1 \cdot 10^{27}$ eV was used to recover the units used in particle physics. However, this lower limit on neutrino mass rests on six approximations. The approximations, in particular the uncertainty for the length $l_{\rm add}$, generate an estimated error of a factor 100, so that the lower limit $m_{\rm ll}$ should be written as

$$100\,\mu{\rm eV} \ \gtrsim \ m_{\rm ll} \ \gtrsim \ 100\,{\rm meV} \ . \tag{17}$$

The error is *large*. But the lower mass limit is not yet in contrast with data. In the coming decades, a mass value for the neutrinos might be measurable, for example using the KATRIN experiment [36]. *A contradiction between improved neutrino mass calculations and experiment would invalidate the strand conjecture.*

An *upper mass estimate* for leptons can also be deduced. It requires to take into account the size and shape of the tangle core. For a core of non-negligible size, the factor n in expression (13), counting the crossing switches for each belt trick, becomes important. The factor depends both on the volume of the core and on the number of tethers.

During the belt trick, every crossing inside the core and every crossing resulting from the tethers wrapping around the core will lead to additional crossing switches. The factor n thus describes three effects. First, it describes the crossing switches due to the tethers and the core (segments) rotating *against each other* during the belt trick. Second, it describes all crossing switches *inside the core* between two belt tricks. Third, it describes all crossing switches *among the tethers* that occur between two belt tricks.

A lepton tangle core can be approximated as a sphere of diameter d, measured in units of the minimal length. The number of crossing switches due to the first effect depends on the volume times the length of each tether; thus it increases as $(d^4)^6$. The second effect increases as d^6. The third effect increases as the length of each tether wrapping around the core, thus in total as d^6. Together, a rough estimate thus yields

$$n \approx d^{36} \ . \tag{18}$$

For the largest lepton core, the tangle geometry of Figure 6 leads to the estimate $d \approx 4 \pm 1$. Experiments with real ropes lead to a ratio $l_{\rm add}/l_{\rm min} \approx 14 \pm 5$. This yields an upper lepton mass $m_{\rm ul}$ bound given by

$$\frac{m_{\rm ul}}{\sqrt{\hbar c/4G}} \approx ({\rm e}^{-14})^6 \cdot 2 \cdot 4^{36} = 3 \cdot 10^{-15} \quad \text{or, equivalently} \quad m_{\rm ul} \approx 20\,{\rm TeV} \ . \tag{19}$$

However, the errors on the upper limit are so large that the value of m_{ul} is *useless*. For completeness, the experimental mass of the most massive lepton, the tau, is $1.8\,\mathrm{GeV}$ and corresponds to $3 \cdot 10^{-19}$ corrected Planck masses. The experimental mass of the electron, $0.5\,\mathrm{MeV}$, corresponds to $0.8 \cdot 10^{-22}$ corrected Planck masses.

18 Discussion

The upper and lower lepton mass limits deduced above are *disappointing*. The mass estimates have large error bars, essentially because the problem is a composition of several mathematically challenging issues. The specific tangle topology for each lepton was not taken into account yet. The running with energy and the effects of the other tangles in each lepton tangle family (i.e., the effect of coupling to the Higgs) were neglected. As a result, the upper mass limit is not precise and differs from the experimental electron mass by many orders of magnitude. Also the lower mass limit is so vague that it cannot be compared to experiments yet.

Despite the disappointing mass limits, two encouraging aspects remain. Possibly for the first time, the tangle model promises to calculate mass values ab initio. The mass values are unique, constant over time and space, positive, equal for particles and antiparticles, equal to the gravitational mass and running with energy. The mass hierarchy between neutrinos, the charged leptons and the Planck mass is explained – without additional assumptions. More precise estimates of lepton masses, in particular of the electron mass, are possible either with efficient computer simulation programs or with improved analytical approximations that take tangle topology into account. This challenge remains open. *The failure to reproduce, with more precise calculation methods, any of the observed lepton mass values would falsify the tangle model.*

The other encouraging aspect of the strand conjecture is the potential to determine, using the *same* model, the fine structure constant, again ab initio. The value for the fine structure constant is unique, constant over time and space, running with energy, and equal for all particles. Because there are fewer approximations in this case, the possibility for a precise determination appears much closer. And also this challenge remains open. *The failure to reproduce, with more precise calculation methods, the observed value of the fine structure constant α would falsify the tangle model.*

19 Outlook

The tangle model has many unusual aspects. The model is *counter-intuitive:* it requires to get used to the idea that every particle in nature is tethered. The tangle model proposes a *microscopic model for quantum theory,* despite the failure of all past attempts in this domain. The model arises *directly from Planck-scale physics,* without any intermediate structure. The tangle model describes events, interactions, physical processes and the standard model with *simple pictures.* The model implies that all dynamics in nature, and in particular the complete Lagrangian of the standard model, can be described *algebraically*. The tangle model deduces that the standard model is *consistent.* The model predicts that the standard model is *valid at all energies.* The tangle model predicts that the high energy region will yield *no new, unknown phenomena.* The model predicts that the only aspect of nature that is presently unknown is the *origin of the fundamental constants.*

Despite the many unusual aspects, the strand conjecture agrees with the standard model of particle physics. In particular, the conjecture also agrees with all experiments so far.

Despite the unusual aspects and its incompleteness, the strand conjecture for the origin of the standard model remains appealing. The strand conjecture describes nature with a single and simple principle: the conjecture is elegant. Also, the conjecture does not predict spectacular effects: the conjecture is modest. The conjecture cannot be modified without destroying the whole structure: the conjecture is genuine and consistent. In addition, if just one conclusion or just one prediction drawn from the conjecture is wrong, it must be abandoned: the conjecture is thus both fragile and sincere. Finally, the similarity of crossing switches and qubits makes the conjecture intriguing. The conjecture thus incorporates the main aspects of attractiveness.

So far, experiments in QED are not able to reach Planck scales. This implies that the proposed tangle structure of particles cannot be tested directly. However, indirect tests are possible. The lack of measurable deviations of any kind from the standard model is predicted. Comparison with direct calculations of the perturbative expansion of the g-factor appear possible. Observations in the non-perturbative domain of QED, in particular near the electric or magnetic field limits, would allow even stricter tests.

At present, the strictest tests for the strand conjecture seem to be calculations of the particle masses and of the fine structure constant. The estimates proposed above are not yet precise enough to allow definite statements in favour or against the conjecture. Nevertheless, the strand conjecture is one of the few proposals promising to predict the neutrino mass values before their actual measurement. This work aims at encouraging more research on calculating the frequency of the belt trick that is induced spontaneously by strand fluctuations, and the frequency of photon absorption with its effects. In the case that the future, more precise calculations disagree with experiments, the strand conjecture would be falsified. If, instead, the calculations agree with experiments, a pretty result would ensue: the origin of the colours observed in nature would finally be explained.

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