Testing a Conjecture on Cosmology and Dark Energy

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Abstract

The consequences of the strand conjecture are explored on cosmological scales. The starting point is the realization that strands fluctuating at the Planck scale appear to explain both the Lagrangian of the standard model of particle physics, with massive neutrinos and PMNS mixing, and, at sub-galactic distances, the Hilbert Lagrangian of general relativity. Both Lagrangians arise without any modification, with particle masses, mixing angles and coupling constants that are unique and calculable.

On cosmological scales, fluctuating strands lead to many effects that can be tested against observations. Above all, the formation of an expanding cosmological horizon is predicted. The resulting horizon temperature, horizon luminosity and horizon entropy are compatible with observations. Inside the horizon, the strand conjecture predicts the appearance of empty space, particles and black holes. In particular, the conjecture yields a natural model for matter, radiation and dark energy. The three predicted density values are compatible with observations. The dark energy density is predicted be small and constant over time. The value w in the equation of state of dark energy is predicted to be negative. Inflation is found not to have occurred. No contradictions between the strand conjecture and observations are found.

Keywords: general relativity; cosmological constant; dark energy; strand conjecture.

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1 The quest for the origin of dark energy

The physical origin of dark energy is a subject of intense research. Unraveling its details is important for astronomy and for particle physics.

The so-called *strand conjecture* proposes a fundamental principle that describes nature at the Planck scale. The conjecture describes all physical systems as made of the same extended components. In particular, space, particles and horizons are described as made of fluctuating strands. The strand conjecture appears to reproduce both general relativity and the standard model of particle physics, without any deviations. In both domains, the conjecture makes numerous predictions that allow testing it. So far, no contradictions between experiment and the strand conjecture arose. In the domain of fundamental physics, the conjecture appears to be complete: no observation is unexplained. In particular, strands appear to explain and determine the constants of the standard model – masses, mixing angles and coupling constants. Only this last result justifies the exploration of strands in the domain of cosmology.

As argued in the following, applying the strand conjecture to cosmology yields specific experimental predictions, in particular about dark energy. The predictions are deduced step by step, proceeding with care. They all follow from the fundamental principle. The topic of dark matter will be covered in a subsequent paper.

2 The origin of the strand conjecture

The strand conjecture goes back to the so-called *string trick* or *belt trick* that Dirac used in his lectures. From around 1929 onwards, Dirac explained spin 1/2 as result of tethered rotation, even though he never published anything about it [1]. Tethers were the first hint that nature might be built from unobservable extended constituents. Several decades later, in 1980, Battey-Pratt and Racey understood that also the complete Dirac equation could be deduced from unobservable extended constituents whose crossings can be observed [2]. Then, in 1987, Kauffman conjectured a direct relation between the canonical commutation relation – and thus $\hbar$ – and a crossing switch of such unobservable tethers [3]. This connections were rediscovered in the early twenty-first century. Building on these results, crossing switches of unobservable extended constituents were found to describe the complete standard model of particle physics, including the gauge groups, the particle spectrum, masses, mixing angles and coupling constants [4, 5]. It thus appeared that *every quantum effect* can be described as being due to unobservable extended constituents whose crossings can be observed.

In addition, the surface dependence of black hole entropy [6, 7] and the discovery of maximum power and force [8–15] led to deduce all black hole properties and Einstein's field equations from crossing switches of unobservable extended constituents [4]. It thus appeared that *every gravitational effect* can be thought as being due to unobservable extended constituents. Because the term 'string' had acquired a different meaning in the meantime, the alternative term *strand* appeared more appropriate.

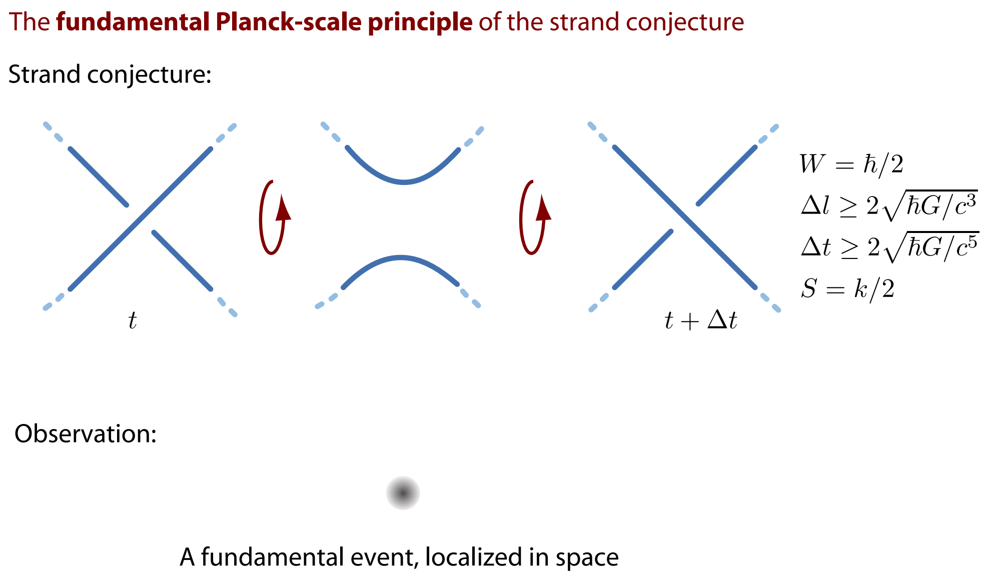


Figure 1: The fundamental principle of the strand conjecture defines the simplest observation in nature, the almost point-like fundamental event. Every event results from a *skew strand crossing switch*, at a given position in three-dimensional space. The strands themselves are not observable; they are impenetrable and best imagined with Planck radius. The crossing switch defines all fundamental constants. The double Planck length limit and the double Planck time limit arise, respectively, from the smallest and from the fastest crossing switch possible. (See also Appendix A.) In the following, cosmology is deduced from the fundamental principle.

3 The strand conjecture

In the strand conjecture all physical systems in nature – matter, radiation, space and horizons – are built from *fluctuating strands*. Strands are defined as smooth one-dimensional curves embedded in three-dimensional space with a bending radius everywhere larger than the Planck length. This definition leads to the strand conjecture, to be detailed in the following:

- ▷ Space is a strand *network*. Horizons are strand *weaves*. Particles are strand *tangles*. Strands are unobservable; however, crossing switches of strands are. Crossing switches determine the Planck units, as illustrated in Figure 1.

Strands have no observable properties: they have no colour, no tension, no mass, no energy. It is easiest (but not fully correct) to imagine strands as having Planck-size radius. Strands cannot interpenetrate; they never form a *real* crossing. When the term ‘crossing’ is used in the present context, only the two-dimensional projection shows a crossing. In three dimensions, strands are

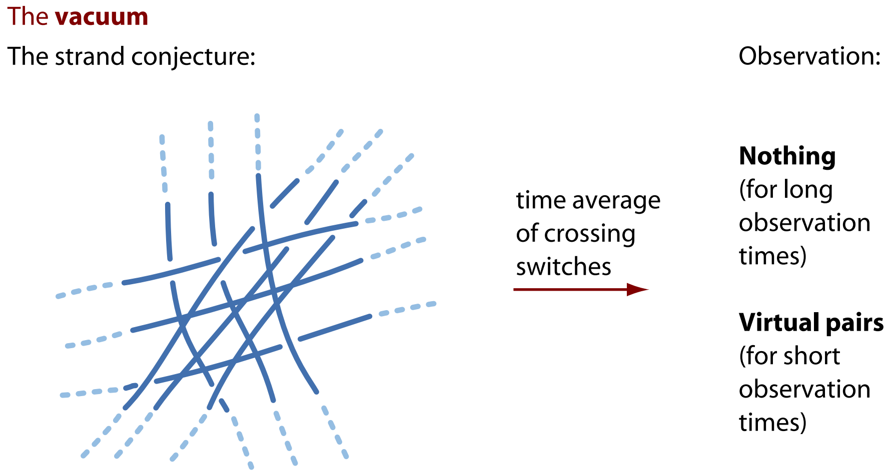


Figure 2: An illustration of the strand conjecture for a flat vacuum: for sufficiently long time scales, the lack of crossing switches leads to a vanishing energy density; for short time scales, particle–antiparticle pairs, i.e., rational tangle–antitangle pairs, arise.

always at a distance. Crossing *switches* only arise via strand deformation, as illustrated in Figure 1. This allows stating:

▷ In the strand conjecture, *all physical observables* – including action, energy, velocity, momentum, mass, length, surface, volume, force, entropy, all field intensities and quantum numbers – arise from strand crossing switches.

In short: all physical observables *emerge* from strands. The following sections give a short summary on how crossing switches of fluctuating strands produce general relativity and quantum effects. Then the implications and consequences of crossing switches in the domains of cosmology are explored. First however, some conceptual issues are clarified.

4 Measurements and minimum time

The fundamental principle can be used to deduce the three known gauge interactions from the three Reidemeister moves [4,5]. The three Reidemeister moves are called twists, pokes and slides. The electromagnetic arises from twist exchange. This result explains the fundamental principle: in twist exchange, the sign of a crossing changes orientation. And in nature, all measurements and all observations are due to the electromagnetic interaction. This is exactly what the fundamental principle tells: all measurements and all observations are due to *crossing switches*.

The fundamental principle appears to allow arbitrary fast crossing switches. This would imply that no minimum time arises and is defined. However, an arbitrary fast crossing switch does not yield an electromagnetic signal: no electromagnetic wave can have a wavelength shorter than

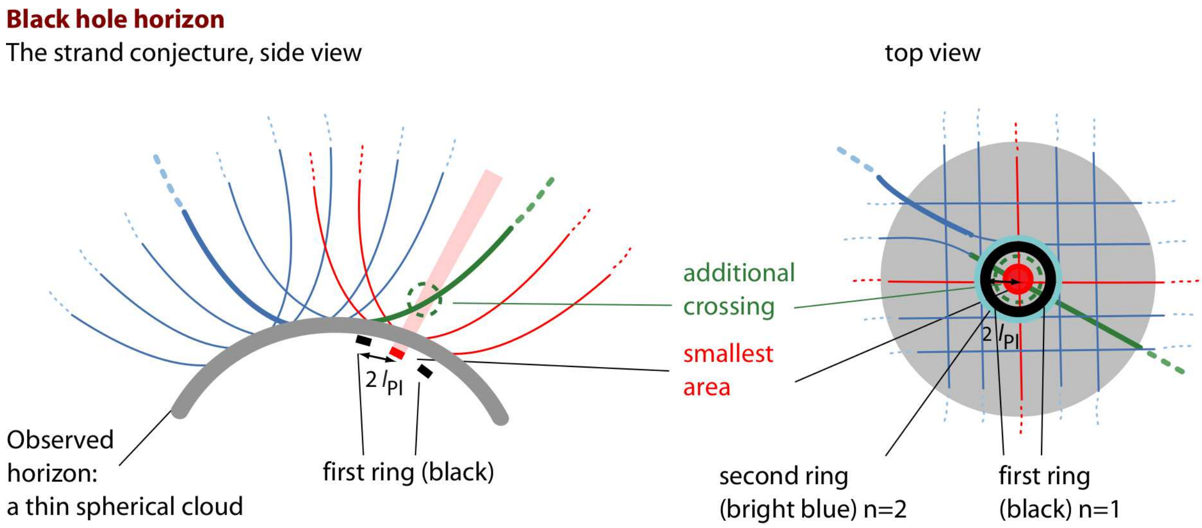


Figure 3: The strand conjecture for a Schwarzschild black hole: the black hole horizon is a cloudy or fuzzy surface produced by the crossing switches of the strands woven into it. Due to the additional crossings on the side of the observer, the number of microstates per smallest area is larger than 2.

a Planck length. Therefore, an arbitrary fast crossing switch is not observable. Only crossing switches that take more than one (corrected) Planck time are observable.

5 From strands to black holes and their horizons

This section summarizes how strands lead to black holes and all their properties [4, 16, 17].

The strand conjecture posits that horizons are one-sided weaves of fluctuating strands. *One-sided* means that all strands leave the horizon on the observer side. A simplified illustration of a black hole is given in Figure 3, both as a cross section and as a top view. All strands arrive from far away, are *woven* into the horizon, and leave again to far away. If strands are imagined as having Planck radius, the strand weave is *maximally tight*.

Maximally tight weaves allow determining the *energy* of a spherical horizon. Energy E has the dimension action per time. Because every crossing switch is associated with an action $\hbar$, the black hole horizon energy is found by determining the number $N_{\rm cs}$ of crossing switches per unit time. Crossing switches propagate across the horizon weave. For a maximally tight weave, the propagation speed is one smallest possible crossing per one shortest possible switch time: switch propagation thus occurs at the speed of light c. In the time T that light would take to circumnavigate a black hole horizon of radius R, all crossings of the horizon switch, yielding the black hole energy

$$E = \frac{N_{\text{cs}}}{T} = \frac{c^4}{4G} \frac{4\pi R^2}{2\pi R} = \frac{c^4}{2G} R \ . \tag{1}$$

Strands thus reproduce the known relation between energy and radius of a Schwarzschild black hole.

A maximally tight weave also determines the number of microstates per black hole horizon area. Figure 3 shows that, for each circular or ring area that contains just one crossing, the effective number N of microstates *above* that area is larger than 2. This excess occurs because of the neighbouring strands that sometimes cross *above* that smallest area. The additional crossing probability depends on where the neighbouring strand leaves the black hole horizon; this yields

$$N = 2 \ + \ \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ... + \frac{1}{n!} + ... = \mathrm{e} = 2.718281... \tag{2}$$

In this expression deduced from strands, the term 2 is due to the two options at the very bottom of the smallest area. The next term $1/2!$ arises from the strand leaving the neighbouring ring shown in Figure 3. The subsequent terms are due to the subsequent rings.

Expression (2) states that $N = \mathrm{e} > 2$ is the average number of strand microstates for each smallest area, i.e., for each corrected Planck area $A_{\text{cPl}} = 4\, G\hbar/c^3$. Each corrected Planck area on a black hole horizon thus contains *more* than 1 bit of information.

The total entropy of the black hole is

$$S = k \ \ln N_{\text{total}} \ \ , \tag{3}$$

where k is the Boltzmann constant and N_{total} the total number of microstates of the horizon. The full horizon area A is composed of corrected Planck areas. The product of the number of states for every corrected Planck area yields the total number of microstates:

$$N_{\text{total}} = N^{A/A_{\text{cPl}}} \ \ . \tag{4}$$

Inserting the result (2), due to strands, yields

$$N_{\text{total}} = \mathrm{e}^{A/(4\, G\hbar/c^3)} \ \ . \tag{5}$$

This total number of horizon microstates can then be inserted into expression (3) for the entropy. The resulting value for the black hole entropy is

$$\frac{S}{k} = \frac{A}{4\, G\hbar/c^3} \ \ . \tag{6}$$

This is the expression discovered by Bekenstein [6].

The ratio of black hole energy and twice the entropy determines the *temperature:*

$$T_{\text{BH}} = \frac{\hbar c}{4\pi k} \frac{1}{R} \ \ . \tag{7}$$

In short, strands thus reproduce all thermodynamic properties of black holes. For example, black hole evaporation arises from strands or tangles that detach from the horizon.

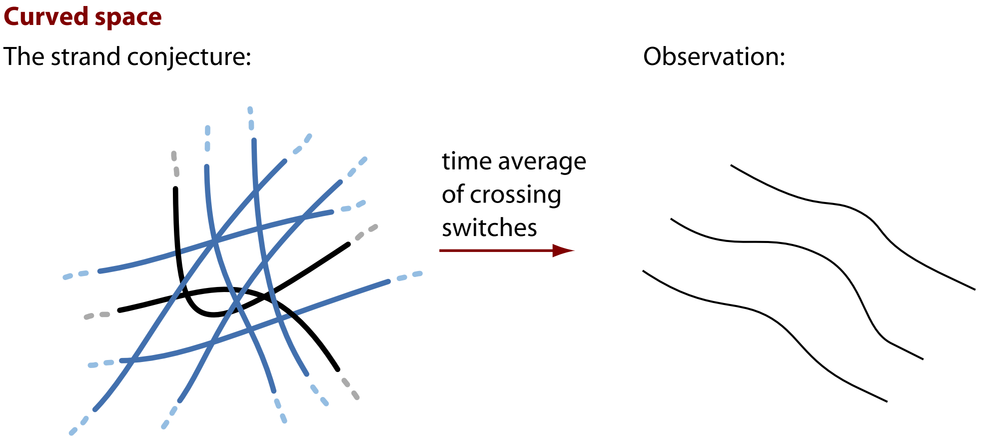


Figure 4: An illustration of the strand conjecture for a curved vacuum. The strand configuration is intermediate between that of a horizon and that of a flat vacuum. The black strands differ in their configuration from those in a flat vacuum: they are tangled. The configuration implies that such a region of space has non-vanishing curvature, energy, entropy and temperature.

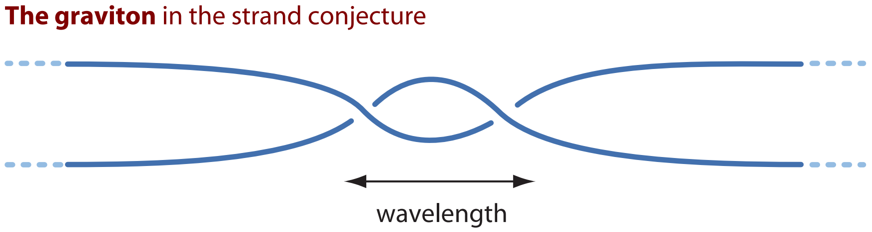


Figure 5: The strand conjecture for the graviton: a twisted pair of strands automatically has spin 2, boson behaviour, and vanishing mass. A gravitational wave is a coherent superposition of a large number of gravitons.

6 From strands to general relativity

This section summarizes how strands lead to general relativity [4, 16].

In 1995, Jacobson showed [18] that the thermodynamic properties of black holes imply Einstein's field equations of general relativity. He started with the entropy–area relation $S = A\,kc^3/4G\hbar$, the temperature–acceleration relation $T = a\,\hbar/2\pi kc$, and the relation between heat and entropy $\delta Q = T\delta S$. He introduced them into the relation

$$\delta E = \delta Q \quad , \tag{8}$$

which is valid *only* in case of a horizon, and derived the first principle of horizon mechanics

$$\delta E = \frac{c^2}{8\pi G} a\, \delta A \quad . \tag{9}$$

The left-hand side can be rewritten, using the energy–momentum tensor T_{ab}, as

$$\int T_{ab} k^a \mathrm{d}\Sigma^b = \frac{c^2}{8\pi G} a\, \delta A \quad , \tag{10}$$

where $\mathrm{d}\Sigma^b$ is the general surface element and k is the Killing vector that generates the horizon. The right-hand side can be rewritten, using the Raychaudhuri equation, as

$$\int T_{ab} k^a \mathrm{d}\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab} k^a \mathrm{d}\Sigma^b \quad , \tag{11}$$

where R_{ab} is the Ricci tensor that describes space-time curvature. This equality between the integrals implies

$$T_{ab} = \frac{c^4}{8\pi G} \left(R_{ab} - (R/2 + \Lambda)\, g_{ab} \right) \quad , \tag{12}$$

where R is the Ricci scalar and the cosmological constant Λ appears as an undetermined constant of integration. These are Einstein's field equations of general relativity.

In short, the field equations result from *thermodynamics of space*. Like every horizon and every black hole, also curved space is a thermodynamic system. Indeed, curved space and horizons can be transformed into each other by change of coordinate system. These results can be summarized by stating that space is *made* of microscopic degrees of freedom and curvature and gravity are *due* to microscopic degrees of freedom. Figure 4 gives an impression of curved space, and Figure 5 of the graviton. Given that strands lead to the thermodynamic properties of black holes, strands realize Jacobson's argument [18], as well as his later thoughts [19]: strands produce the field equations of general relativity [4, 16].

7 From strands to quantum theory and particle physics

This section summarizes how strands lead to quantum field theory and particle physics.

As explained in detail in previous publications [4, 5, 20], strands allow deducing quantum theory and the standard model of particle physics. In flat space, the fundamental principle, together with Dirac's belt trick, implies that *tangled* fluctuating strands describe particles and wave functions. The wave function is the time-average of strand crossing density. For fluctuating *rational*, i.e., unknotted tangles, the crossing switch density yields the probability density. Tangles also yield the Hilbert space, the quantum phase, interference, and the freedom in the definition of phase. Tangles imply spin 1/2 and, above all, Dirac's equation. The general connection is illustrated in Figure 6. Tangles also reproduce fermion behaviour and entanglement. Tangles are fully equivalent to quantum theory. No extension or deviation arises. The tendency to keep the number of crossing switches as low as possible, the *principle of fewest crossing switches*, leads to

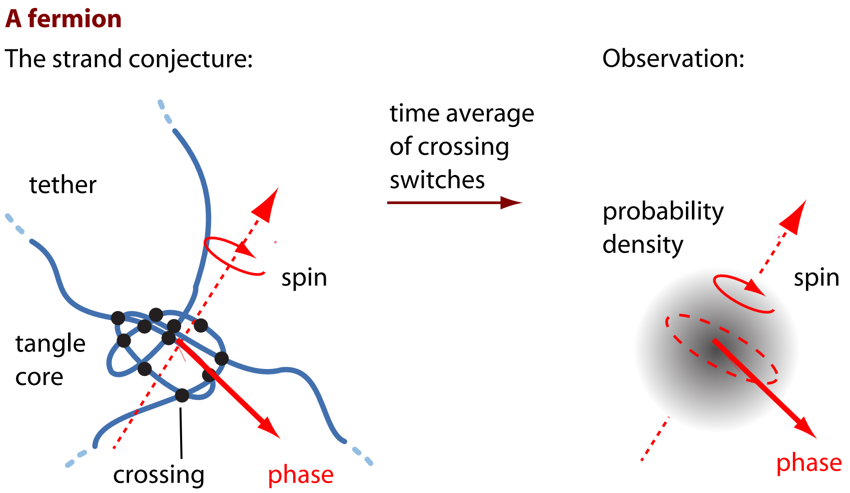


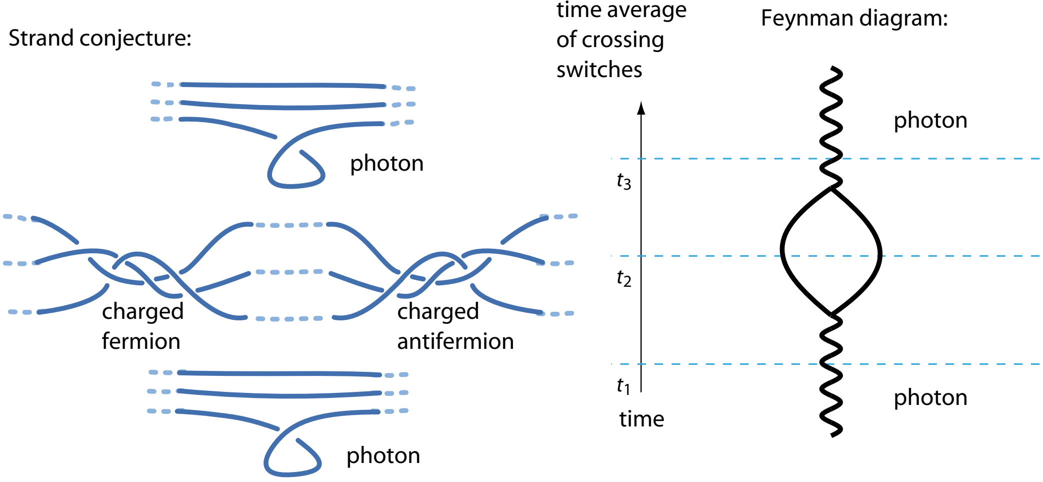
Figure 6: In the strand conjecture, the wave function and the probability density are due, respectively, to crossings and to crossing switches at the Planck scale. The wave function arises as the time average of *crossings* in fluctuating tangled strands; a Hilbert space also arises. The probability density arises as the time average of the *crossing switches* in a tangle. The tethers – connections that continue up to large spatial distances – generate spin 1/2 behaviour under rotations and fermion behaviour under particle exchange. The tangle model ensures that fermions are massive and move slower than light.

the principle of least action. Strands thus *visualize* quantum theory: every quantum effect is due to crossing switches. And every crossing switch is a quantum effect.

Rational, i.e., unknotted tangles reproduce the known spectrum of elementary particles [4, 5, 20]. Each massive elementary particle is represented by an infinite family of rational tangles made of two or three strands. The family members differ only by the number of attached braids. Each braid corresponds to a Higgs boson. Three generations for quarks and for leptons arise. W, Z and Higgs bosons are massive. Tangles for the massless gauge bosons, the photon and the gluons, also arise. Rational tangles also describe particle mass: more complex tangles imply higher mass. Particle mixing is described. Strands reproduce the quark model, including the mass sequence of mesons and hadrons, the CP violation and the mixing of mesons.

Interactions are due to deformations of tangle cores. Such deformations can be classified. This classification is based on the Reidemeister moves and yields the gauge groups U(1), broken SU(2), and SU(3). No other gauge group nor any combined gauge group is possible. Emission of particles, and particle–antiparticle creation and annihilation also arise. These results produce all Feynman diagrams of the standard model – including the examples illustrated in Figure 7. Due to topological reasons, additional elementary particles, additional gauge groups, and further Feynman diagrams are explicitly *excluded*. Perturbative quantum field theory, including quantum electrodynamics and quantum chromodynamics is reproduced completely.

Virtual particle-antiparticle pair



Fermion-antifermion annihilation

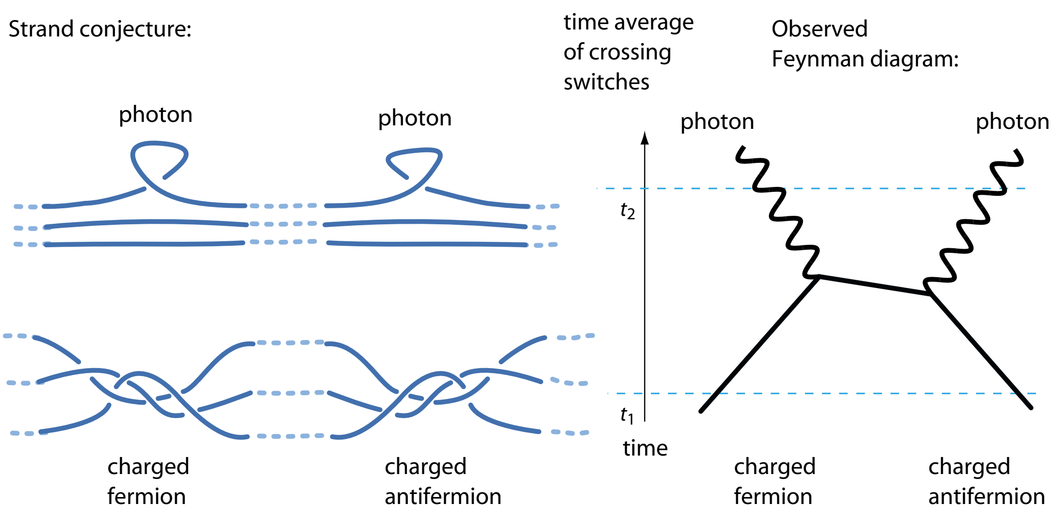


Figure 7: An illustration of two Feynman diagrams of QED in the tangle model.

In short, the strand conjecture reproduces all observations in high-energy physics and provides numerous predictions for experiments [4, 5, 20]. Strands allow calculating the fundamental constants of the standard model: masses, mixing angles and coupling constants. Discovering a single effect beyond the standard model would falsify the strand conjecture.

8 Strand predictions about local physics: particles, gravity, and Planck limits

A number of predictions deduced from the strand conjecture in previous papers [4, 5, 16, 20] are important in the following. They concern observations at sub-galactic scales.

Pr. 1 In the domain of high energy physics, strands predict the lack of deviations from the the standard model of particle physics with massive Dirac neutrinos with PMNS mixing. In the strand conjecture, all particles are represented by *rational* tangles of strands. This tangle model of quantum particles reproduces the known elementary particles and interactions without any additions or modifications.

The tangle model leaves no room for the axion, for WIMPs, the inflaton, or for any other non-standard elementary particle or field conjectured in the past. Finding a new elementary particle or *any* effect *beyond the standard model* of particle physics with at least two massive Dirac neutrinos would *falsify* the tangle model and the strand conjecture. All this is not in contrast with observations so far [21].

Strands also predict the lack of new gauge symmetries, of any other symmetries, or of new energy scales. A future discovery of any such property would *falsify* the strand conjecture. Strands do not allow such additional structures in particle physics. So far, none has been observed.

Pr. 2 In the domain of gravitation, the strand conjecture predicts that local power and luminosity values are limited by $c^5/4G$, local force values are limited by $c^4/4G$, local mass flow rates are limited by $c^3/4G$, and mass to length ratios are limited by $c^2/4G$. In short, strands predict the *lack* of measurable deviations from general relativity at *sub-galactic* distances. These predictions, with the factor 4, agree with all observations and calculations so far. A future discovery of any *deviation from the field equations,* i.e., from the Hilbert action of general relativity, at sub-galactic scales would *falsify* the strand conjecture.

Pr. 3 In the domain of quantum gravity, strands predict that no length and no effect of scales smaller than the corrected Planck length $\sqrt{4G\hbar/c^3}$ can be observed. Similarly strands predict that no time intervals and no effect of time intervals smaller than the corrected Planck time $\sqrt{4G\hbar/c^5}$ can be observed. Elementary particle energies are predicted not to exceed the corrected Planck energy $\sqrt{\hbar c^5/4G}$. These predictions explicitly include the factor 4. A future discovery of any *trans-Planckian* effect would *falsify* the strand conjecture.

Pr. 4 Strands imply that all fundamental constants of particle physics – coupling constants, particle masses and mixing angles – are *constant* over time and space. This agrees with observation, despite occasional claims of the contrary. Finding a *variation* of the fundamental constants would *falsify* the strand conjecture.

Pr. 5 Strands imply that all fundamental constants of particle physics – coupling constants, particle masses and mixing angles – can be *calculated.* Finding a *difference* between the observed and the calculated values of the fundamental constants would *falsify* the strand conjecture.

In particular, strand imply that elementary particle masses are much smaller than the Planck mass $\sqrt{\hbar c/4G}$. Strands thus explain the ratio 10^{42} between the electric and gravitational force between an electron and a positron. Strands imply the SU(3) symmetry of the strong interactions and the quark model. The masses of the baryons and the properties of the strong nuclear interaction explain – in principle – the fusion of hydrogen to helium.

Pr. 6 In the tangle model of particles, electric charge is represented by *topologically chiral* tangles [4, 20]. All particle tangles are free of magnetic charge; such tangles are not possible. Strands thus predict the *lack of magnetic monopoles* in nature. Strands thus solve the magnetic monopole problem, because the tangle model has no room for them. This agrees with observations. Finding a magnetic monopole would *falsify* the strand conjecture.

Pr. 7 Strands predict that electric and magnetic fields are *limited* via maximum force $c^4/4G$ and smallest electric charge $e/3$ by the expression $E \leq 3c^4/4Ge = 5.7 \cdot 10^{62}\,\mathrm{V/m}$ and $B \leq 3c^3/4Ge = 1.9 \cdot 10^{54}\,\mathrm{T}$. The factor 4 is part of the prediction.

Pr. 8 Strands also predict the lack of cosmic strings, wormholes, regions of negative energy, and domain walls.

Pr. 9 In 1967, Sakharov showed that the observed matter–antimatter asymmetry in the universe requires three properties of particle interactions [22]. In modern terms, the three properties are (1) violation of baryon number conservation, (2) C and CP violation, and (3) a non-equilibrium situation during the expansion of the universe. In the strand conjecture, like in all cosmological models, the third property holds automatically. The second property also holds, as discussed in a previous paper [4]: strands predict a unitary mixing matrix with a CP phase for quarks and one for leptons. Also the first property holds; adding or taking a strand from a tangle is a non-perturbative process that does not conserve baryon number. In addition, strands imply that there might be a further effect: the tangling of the universal strand at the very beginning breaks chirality. All this together yields a baryon—antibaryon asymmetry.

In particular, the first property, baryon non-conservation, is realized by the tangle model of quarks and leptons: rearranging the strands of a quark and combining it with a vacuum strand, allows forming a lepton [4]. This non-perturbative process – different from the perturbative processes described with Feynman diagrams – appears to model baryon non-conservation with strands. This description should help resolving the ongoing discussion whether the standard model is sufficient or not to explain the observed baryon–antibaryon asymmetry, and the relative importance of baryogenesis and leptogenesis. The answer should arise when performing simulations of fluctuating strands. If such simulations disagree with observations, the strand conjecture is falsified.

Pr. 10 In the strand conjecture, both gravity and particle physics are possible only in *3 dimensions.* Finding evidence for more or fewer dimensions would *falsify* the strand conjecture.

In short, all these predictions can be condensed in the general prediction that there is no unknown fundamental physics in nature. This prediction agrees with all known experiments on microscopic, macroscopic and astrophysical scales. Building on this correspondence between the strand conjecture and observations, it makes sense to explore the consequences of strands in the domain of cosmology.

9 The main observations about the cosmos

At night, the sky is dark. The observation has several well-known reasons. First, there is a *maximum age* in nature, about $t_0 = 13.8(1) \cdot 10^9\,\mathrm{a} \approx 0.43\,\mathrm{Es}$ [23]. Secondly, precise experiments show that also observable distances are limited: the universe is enclosed by a *cosmological (particle) horizon* at a finite distance. Thirdly, stars and galaxies re surrounded by vacuum that allows to see the dark sky. Fourthly, observations also show that, on average, all matter recedes from all other matter, and that the recession speed increases with distance; the universe is *expanding*. Therefore, light from distant, ancient sources is red-shifted.

In an expanding universe, the *cosmological (particle) horizon* is defined as that surface inside which something has already been observed [24]; the cosmological horizon is thus defined by the observable photons that are arriving from the big bang. The cosmological horizon and all matter inside it are increasing in distance with time. The *expansion of the universe* is confirmed by all observations so far. Also all consequences deduced from the expansion – such as the cosmic background radiation and the details of nucleosynthesis – agree with observations.

In ΛCDM cosmology, the distance to the present cosmological particle horizon is measured to be about $3.3(1) \cdot ct_0$ [23], or $14.4\,\mathrm{Gpc}$. The cosmological horizon is observed to recede at speeds *larger* than the speed of light. In other words, the cosmological horizon is larger than the Hubble radius. Egan and Lineweaver estimated the *entropy* of the cosmological horizon to be around $10^{122}\,k$ [25]. They estimated the entropy *inside* the observable universe to be around $10^{103}\,k$, not taking into account dark energy.

In ΛCDM cosmology, inside the universe, matter, radiation and space are expanding. The expansion rate, the so-called *Hubble constant* H_0, is measured using star distances and red shifts. The modern value is $71(3)\,\mathrm{km/(s\,Mpc)}$ or, in SI units, $2.3(1)/\mathrm{Es}$ [23]. The uncertainty is large, because, in 2020, the value deduced from the cosmic background radiation $67.4(5)\,\mathrm{km/(s\,Mpc)}$ and the value deduced from more local measurements $73.8(1.1)\,\mathrm{km/(s\,Mpc)}$ differ by 10% [26].

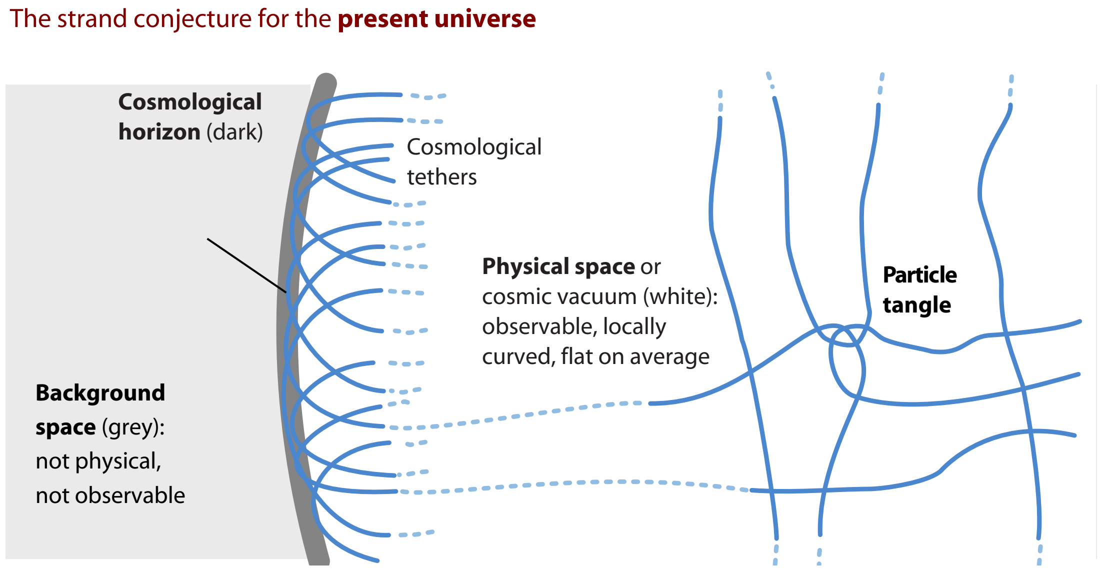


Figure 8: In the strand conjecture, the universe is limited by a cosmological (particle) horizon, as schematically illustrated here. Physical space (white) matches background space (grey) only inside the horizon. Physical space thus only exists *inside* the cosmic horizon.

This difference is subject of intense research. The small but measurable acceleration of the expansion is explored below, from Section 17 onwards.

At large scales, the universe is observed to be *homogeneous* and *isotropic*. The observed mass-energy density of conventional matter inside the universe is $\Omega_{\mathrm{matter}} = 0.045$ [21]. The observed energy density of photons inside the universe is $\Omega_{\mathrm{photons}} = 0,000048$. The observed energy density of neutrinos inside the universe is $0.0009 < \Omega_{\mathrm{neutrinos}} < 0.048$.

The observed baryon–antibaryon asymmetry in the universe is $6.3 \cdot 10^{-10}$ [27]. Other numbers that characterize the universe are the density ratios of various nuclei. All these ratios are due to particle physics and are not discussed in the following.

The universe is observed to have *3 dimensions* also at galactic and at cosmological scales. At cosmological scales – within measurement errors – the universe is observed to be *flat*. Also, no singularities of any kind, no cosmic strings, and no different vacuum states have been observed.

All the observations just mentioned – about the horizon and about the interior of the universe – must be reproduced by the strand conjecture; otherwise, it is *falsified*. Obviously, some values depend on observational constraints, for example the present age of the universe. But taking these constraints as input, all other values must follow.

10 Strand cosmology: the horizon and its interior

The observed expansion of the universe can be derived from the field equations of general relativity. Given that the strand conjecture reproduces general relativity [4, 16], it also reproduces the expansion of the universe. However, the strand conjecture goes further. In fact, it suggests a simple cosmological model:

▷ The universe is *a single strand.*

For the present universe, this conjecture is illustrated in Figure 8. In the conjecture, this *universal strand* is woven into the cosmological particle horizon, leaves the horizon somewhere, continues into the interior, forms tangles and thus particles, and then continues again back to the horizon in another direction. There, the strand becomes again part of the weave that forms the cosmological particle horizon, until the strand again leaves into the interior at another location of the sky. In short,

▷ The cosmological horizon is a *one-sided weave* of strands all leaving towards the inside.

Such a one-sided weave is a true horizon. It implies:

Pr. 11 Nothing can be observed behind the cosmological horizon.

This prediction is made for any observer inside the universe – and only such observers are possible in the strand model. In short, the strand conjecture implies that all observers see a cosmological horizon. And they all see a different one. It is predicted that no observer can observe an effect whatsoever that requires an origin beyond the horizon. Any observation to the contrary would falsify the strand conjecture.

Strands divide the universe into two: the cosmological horizon and its interior. The cosmological horizon is a *weave* and provides a limit to the interior. The universal strand forms an interior network that contains *untangled* vacuum strands and *tangles of strands*, i.e., matter and radiation, as illustrated in Figure 8. In short,

Pr. 12 Strands imply the existence of space and particles inside the cosmological horizon.

This agrees with observation. In the strand conjecture, *woven*, *untangled* and *tangled* and strand segments are the three structures that make up the whole universe; they form, respectively, horizons, vacuum and matter.

Despite the limitations due to the circularity of definitions discussed in Appendix A, the strand conjecture for the universe implies numerous additional checks and testable predictions about cosmology. They are explored in the following.

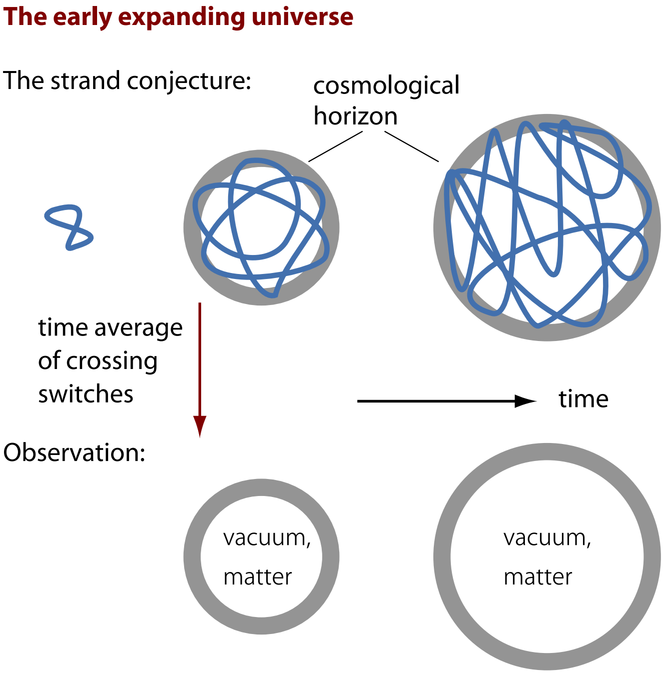


Figure 9: In the strand conjecture for the early universe, the universe increases in complexity over time and thereby forms a boundary: the cosmological horizon. When the universal strand increases in complexity, both the crossings on the horizon and the number of strands in the interior of the universe – the so-called *cosmological strands* – increase in number. Note the reckless use of background space in this illustration.

11 Strand predictions about uniqueness, time, expansion, and the horizon

Pr. 13 The strand conjecture implies that there is just *one* universe. First of all, strands do not allow separating or distinguishing "different universes". In addition, given that strands allow calculating the fundamental constants of the standard model uniquely [4], strands again imply that there is just *one* universe. No "other universes" with different values of the coupling constants are predicted to exist: such alternatives are not consistent with the strand conjecture and thus impossible. This conclusion agrees with observation. Even if the universe were made of several strands, these arguments would remain valid.

Pr. 14 Strands predict the *lack of different vacua*, with different values of the coupling constants or different values of any other parameter of the standard model [4, 16]. The strand conjecture forbids this option. This also implies the *lack of domain walls* in the universe. This agrees with observations.

Pr. 15 The strand conjecture for the history of the early universe is shown schematically in Figure 9. The strand conjecture implies that *the universe expands*, including its horizon and its contents. The expansion is observed. Strands therefore explain why the sky is dark at night. When asking what ‘drives’ the expansion of the universe in the strand conjecture, an answer that is not too wrong would be the following: At cosmological scales, tangle fluctuations tend to form more complex tangles with a larger probability than simpler ones. Once the cosmological horizon formed, its strand fluctuations – which include evaporation from the horizon – continue to drive the cosmological horizon outwards. This answer has to be taken with a grain of salt though, because time itself and space itself also result from the tangling of the universal strand.

Strands thus imply that the expansion of the universe is tied to the existence of a cosmological horizon. In the strand conjecture there is no expansion *without* a cosmological horizon, and vice versa. This is consistent with observation.

Pr. 16 The tangling of the fluctuating universal strand defines *cosmological time.* Cosmological time is intrinsically related to cosmological expansion.

A fluctuating universal strand implies that its tangling on a cosmological scale *increases in complexity with time.* There is a degree of circularity in this statement, as it assumes the existence of time, and time itself arises from strands. However, as argued in Appendix A, observers cannot describe observations without time; a certain degree of circularity is unavoidable. The circularity implies that an axiomatic presentation is not possible, but a consistent and correct description is.

In particular, cosmological tangledness has all properties expected from a global time coordinate: it is one-dimensional, has a smallest interval – the corrected Planck time – and increases continuously. Strands imply that *the universe has a finite age.* Strands imply that all components of the universe have a finite age, and that the age is the same for all components. This is indeed observed.

The present age of the universe is due to our human fate; the age is not a fundamental parameter of the strand conjecture and is not predicted by it, as expected.

Pr. 17 Strands imply, as Figure 9 illustrates, that there is *no sharp beginning* of time. A time coordinate can only be defined when the universe has a size of a few Planck lengths or more. Equivalently, time can only be defined when a cosmological horizon has formed. These statements do not contradict observations.

Pr. 18 Given that the universe is made of one strand, and given that the tangling defines cosmological time, it is predicted that *no* effect in the universe is due to causes that precede the big bang. (This is true even if it were made of several strands.)

If the universe is made of a single strand, it is an unknot. If it is made of several strands, it would be a link.

Thus, the universe could in principle be *cyclic,* but there is *no way* to prove or test this. In particular, strands imply that in a cyclic universe, there is *no effect* from one cycle to the next.

Pr. 19 As illustrated schematically in Figure 8 and in Figure 9, the strand conjecture implies the existence of a cosmological horizon at a *finite distance*. The horizon *limits* observable distances. A limit to distances and a cosmological horizon are indeed observed. Again, the present distance to the horizon is due to our human fate; the distance is not a fundamental parameter of the strand conjecture and is not predicted by it.

Pr. 20 In the strand conjecture, all horizons are *one-sided weaves.* If they were not one-sided, they would not be horizons.

Strands thus predict the *lack* of matter, energy and space *behind* a horizon for the observer belonging to the horizon. This is predicted both for black hole horizons and for the cosmological particle horizon.

The prediction of an actual, true *void* behind all horizons arises from the specific combination of gravity and quantum effects that is provided by strands. The prediction is in full contrast with classical cosmology, as derived from pure general relativity. Despite this contradiction, the prediction is not in contrast with observation, as nothing behind the cosmological horizon can be detected. In fact, the prediction that *nothing* exists behind the cosmological horizon will be of importance in the discussion on inflation and on FLRW models.

In short, the strand conjecture confirms the basic cosmological observations about our universe.

12 Strand predictions about vacuum, topology, dimensionality, singularities and shape

Pr. 21 In the strand conjecture, an *infinite* flat vacuum is *impossible*, because the situation is undefined. An infinite flat vacuum is an idealization. This agrees with observation.

Pr. 22 Because strands form tangles, as illustrated in Figure 8 and in Figure 9, and because tangles represent quantum particles, the strand conjecture implies that the universe is *not empty,* but filled with particles and physical space. For example, strands imply that we live neither in a pure de Sitter space nor in a pure anti-de Sitter space. This agrees with observation.

Pr. 23 Crossings and tangling are not possible in other dimensions. The strand conjecture implies that the interior of the universe has *trivial topology with three spatial dimensions* also on galactic and cosmological scales.

The quantum of action does not allow determining dimensionality at scales below the Planck scale. In the strand conjecture, dimensionality is an *intrinsic* property. No other number of dimensions than three is possible. This agrees with observation.

Pr. 24 Strands imply that there has never been a situation in which the universe had an *infinite* density ϱ or an *infinite* temperature T. In the strand conjecture, these observables are limited by the respective Planck values:

$$\varrho \leq \frac{c^5}{4\hbar G} \quad \text{and} \quad kT \leq \sqrt{\frac{c^5 \hbar}{4G}} \,, \tag{13}$$

where k is the Boltzmann constant. Strand predict the *lack of an initial singularity,* and also the *lack of any other kind of singularity.* This also implies the lack of a "big rip". So far, these statements do not contradict observations.

Pr. 25 A cosmological horizon that *rotates* against the average matter inside it is a theoretical possibility. Also more complex relative motions can be imagined. Strands predict that for statistical reasons, these options do not arise. However, no data is yet available on this issue.

Pr. 26 Strands also predict that a *non-spherical* horizon is impossible: Again, for statistical reasons, a spherical horizon is vastly more probable than a horizon of any other shape. This agrees with data.

Pr. 27 Strands imply that the interior of the universe – all matter and radiation – is, on average, flat, homogeneous and isotropic. This is observed. This will be explored in detail in the next section.

In short, the strand conjecture confirms the general observations made about space in the universe.

13 Strand predictions about matter's origin, expansion speed and density limits

In contrast to first impression, quantum effects can play a role at galactic and cosmological scales, as several authors have pointed out [28–30]. The following sections explore this field of enquiry.

Pr. 28 In strand cosmology, matter, radiation and space *arise* at the cosmological horizon. The strands that make up matter, radiation and space *'start'* at the cosmological horizon: they leave the horizon and enter the interior of the universe. The final effect is that the cosmological horizon is predicted to *shine.*

> ▷ All particles in the universe were radiated from the horizon.

This statement cannot be checked directly, but conclusions from it can.

Pr. 29 Whenever a matter tangle arises at or near the horizon, the horizon departs further. For every strand switch that occurs *inside* the horizon, something similar will occur *on* the horizon. This process 'pushes' the horizon further away.

This motion of the distant horizon is not limited by c, because it is not measured locally.

Pr. 30 Due to the superluminal expansion of the horizon and of space, strands also imply that distant matter appears to depart *faster* than c. This superluminal motion of the horizon and of matter near it is observed [24]: redshifts above 1 for distant matter are routinely observed in astronomy.

Pr. 31 A cosmological horizon allows deducing a limit for matter density, if the surface through which matter appears is chosen carefully. General relativity limits the luminosity, or energy flow, by $P \leq c^5/4G = 0.91(1) \cdot 10^{52}\,\mathrm{W}$ – provided that the surface is *physical* and *closed*, i.e., that every point of the surface can be assigned to a physical observer.

In this way, strands appear to imply that also the luminosity of the Hubble radius is limited by $c^5/4G$. This luminosity limit has been frequently explored in the literature [9–13].

The luminosity limit leads to an energy density limit, when the age t_0 of the universe is included. The limit energy density is given, for flat space, by twice their product:

$$\rho_0 \leq \frac{3}{8\pi G\,(t_0)^2} \approx 8.6 \cdot 10^{-27}\,\mathrm{kg/m^3} \quad . \tag{14}$$

This is the so-called *critical density* [23]. The value corresponds to a few atoms per cubic metre. It arises directly from the power limit.

Pr. 32 The power or luminosity limit of the universe, or equivalently, its energy density limit, can be checked in various ways: for photons, for neutrinos, for baryons, and for their sum. For *photons*, the luminosity of the horizon can be estimated using the Stefan-Boltzmann law $P = \sigma A T^4$ – where $\sigma = 5.67 \cdot 10^{-8}\,\mathrm{W/m^2K^4}$. In the universe, almost all photons are in the cosmic background radiation [21]. Even using the observed temperature $T_0 = 2.7\,\mathrm{K}$ of the cosmic background radiation as horizon temperature – a overestimate by 30 orders of magnitude, as we will see shortly – the resulting luminosity is about a thousand times smaller than nature's power limit. In other words, despite the considerable size of the cosmological horizon, at most one crossing switch every thousand Planck times on it is due to photons. Photons thus obey the power limit.

For *neutrinos*, the resulting value for the energy density is again small compared to the corrected Planck limit. With an estimated temperature $T_0 = 1.9\,\mathrm{K}$ of the neutrino background [21], the resulting neutrino luminosity is orders of magnitude smaller than the power limit.

For *baryons*, the number and the density of baryons in nature, for flat space, is also predicted to be limited by the critical density. The baryon number limit is

$$N_{0,\mathrm{baryons}} \leq \frac{c^3\,t_0}{2G\,m_{\mathrm{baryon}}} \quad . \tag{15}$$

Using $t_0 = 13\,800$ million years, the expression yields a predicted numerical value of $N_{0,\mathrm{baryons}} \leq 5.4 \cdot 10^{79}$. The limit clearly larger than the usual estimates from observations [21, 23].

More precisely, in ΛCDM cosmology, the measured baryon density value is $\Omega_b h^2 = 0.022 = 0.049\,\Omega_{\mathrm{crit}} \approx 4.2 \cdot 10^{-28}\,\mathrm{kg/m^3}$. The observed baryon density is indeed below the density limit naively predicted by the strand conjecture. Observations thus respect the baryon density limit.

As a further check, also the sum of all particle luminosities is observed to be below the critical density. They do not exceed maximum power. On the other hand, known particles alone do not reach the critical density.

As a note, one could argue that the radius of the surface for the power limit should be the distance to the cosmological horizon, not the Hubble radius. In ΛCDM cosmology, the cosmological horizon is 3.3 times further away than the Hubble radius $(4.4 \cdot 10^{26}\,\mathrm{m}$ instead of $1.3 \cdot 10^{26}\,\mathrm{m})$. If the power limit is applied to the cosmological horizon radius, there is a *contradiction* between the limit and the observed density. However, the surface defined by the cosmological horizon is not *physical*, in contrast to the Hubble radius. Therefore, the power limit does *not* apply to the cosmological horizon and *no* useful limit density can be deduced.

Pr. 33 With the changes of temperature over time that are deduced below, strands imply that the expansion rate of the universe first decreased because of the increase of matter content. Afterwards, when temperature was so low that matter was not generated any more, the expansion rate increased because of dark energy. This agrees with conventional cosmology.

Pr. 34 Strands appear to predict that in the present, dark-energy-dominated era, the horizon has a low temperature. Therefore, the matter density of the universe decreases roughly as $\rho \sim 1/R^3$. This prediction agrees with the ΛCDM model.

In short, the predictions of the strand conjecture about the basic properties of the universe either agree or at least do not contradict observations. However, *no prediction* for the value of the expansion speed has been deduced yet. No compelling way to approach the issue has been derived yet. In short, the understanding of the present *speed* of cosmic expansion, $3.3c$, requires an additional idea. This is still subject of research. It remains an open challenge to simulate the history of the universe, in particular of the early universe, using numerical or analytical methods. Such simulations will also allow checking whether strands fluctuations lead to a sufficient amplitude for the density fluctuations required to start galaxy formation.

14 Strand predictions for the cosmological horizon structure

Pr. 35 In the strand conjecture, the cosmological horizon is a *maximally tight* weave. A horizon made from a *loose* weave makes no operational sense: it is *indistinguishable* by measurements from a horizon that is closer and tighter.

Pr. 36 Observations show that in Planck units, the present cosmological horizon has a radius of about $4.3 \cdot 10^{26}\,\mathrm{m}$, or about $1.3 \cdot 10^{61}$ smallest lengths.

In the strand conjecture, the cosmological horizon therefore contains about $1.7 \cdot 10^{122}$ crossings. As a result, the cosmological horizon has *entropy* and *temperature*. This deduction cannot be tested directly, but indirect tests are possible.

Two tacit assumptions need to be verified. First: is the cosmological horizon tight, like a black hole horizon? As mentioned, there is no physical difference between a tight horizon and a loose horizon with lower strand density. Second: is there really one tether per smallest area? Again there is no physical difference between a large horizon with few tethers and smaller horizon with the maximum tether density.

Pr. 37 In other terms, strands imply that the number of tethers per horizon area that leave towards the interior is *constant over time.* The horizon tether density does *not* decay over time. For the same reason, the horizon tethers density does *not* increase over time.

Pr. 38 *Inside* the universe, the most numerous strand *crossings* are those due to empty space. These crossings are due to the *tethers* leaving the horizon. As mentioned, the cosmological horizon has one tether per minimum area.

The number of tethers of the cosmological horizon yields an upper limit for the crossing number *inside* the universe. The upper limit is based on the assumption that two random strands cross only once on the horizon and only cross at most once more in its interior. Figure 9 also makes the point.

Pr. 39 If all *matter* and *radiation* arises at the horizon, the number of strand segments making up the horizon must be compatible with the number of strand segments forming the particles in its interior. In the strand conjecture, it is assumed that, on average, a strand is only rarely part of two particle tangles.

Therefore, the *maximum number of particles made of one strand* (such as photons) that can be observed *inside* the horizon must be smaller than the number of tethers from the horizon. In the present universe, the observed number of photons is estimated to be around 10^{90} [21]. This number is indeed much smaller than the maximum number of tethers inside the universe, which is about 10^{122}.

Also (twice) the *maximum number of particles made of two strands* (such as quarks or gravitons) that can be observed *inside* the horizon, must be smaller that the number of tethers from the horizon. For quarks, this is the case; for gravitons there is no good observational estimate. In his work, Page [31] estimated that more than 10^{113} gravitons exist. Another estimate starts from the fact that gravitons carry, together, much less energy than photons, and have, on average, a 10^{25} times larger wavelength [32]. This kind of estimate yields a number of over 10^{110} gravitons in the universe. The number of gravitons does not exceed the maximum possible value.

Finally, (thrice) the *maximum number of particles made of three strands* (such as neutrinos or charged leptons) that can be observed *inside* the horizon must also be less that the number of tethers from the horizon. The higher number of these, the number of neutrinos in the universe, is expected to be very similar to the number of photons [31]. The total number of observed particles is thus indeed smaller than the (maximum) number of tethers from the cosmological horizon.

Pr. 40 In any case, the strand conjecture implies that *most* particles inside the universe are gravitons, because they arise naturally during expansion. This agrees with expectations [32]. On the other hand, the topic is not simple, as the discussion on the nature of dark energy will show below.

Pr. 41 The above predictions of the strand conjecture imply

> ▷ *Cosmic vacuum* arises from the horizon and is criss-crossed by strands.

This is the central property of the cosmic vacuum in the strand conjecture.

The structure of the cosmic vacuum merits a detailed investigation. The first task is a comparison with Minkowski vacuum.

In the strand conjecture for *infinite flat* space with a vanishing vacuum energy, the vacuum strand density *vanishes*. Also entropy and temperature *vanish*. For example, in an infinite and flat universe, there are *no* effects analogous to the Tolman temperature. The lack of vacuum strands confirms that flat infinite space, the Lorentz vacuum, is not an actual model of nature, but an idealized limit case. In such an infinite and flat space – with or without vacuum energy – strands imply that gravitational and inertial mass are *equal*, because both mass effects arise from the same strand process, namely the belt trick. In this limit, the equivalence principle is thus valid *exactly.*

The next task is to explore how the statements on Minkowski space are modified for the case of a universe with an expanding cosmological particle horizon. This will be done in the following.

Pr. 42 In the strand conjecture, energy is a quantum effect. In addition, in the strand conjecture, every *energy density* is a crossing switch density per time. The strand model of the universe implies that the vacuum, even if flat, is criss-crossed by tethers coming from the cosmological horizon. These strands *cross* inside the universe. As a result,

> ▷ *Dark energy* is a natural consequence of the strand model: it is due to the crossing switches in the cosmic vacuum. Because the density of crossing switches is low, dark energy has a small positive value.

This central result of the strand conjecture is already implied in Figure 9. The next step is to deduce quantitative predictions from this qualitative statement.

In short, in the strand conjecture, vacuum arises at the horizon. The expansion increases the number of tethers in the interior and leads to a small dark energy density. Also matter arises in the interior of the universe, when tethers tangle up. This occurs rarely; therefore, the matter density is small compared to the maximum imaginable value. The strand conjecture suggests that matter is only created when the universe is small and hot. All this is not in contrast with usual cosmology.

15 Strand predictions about the temperature of the horizon and the vacuum

Pr. 43 Figure 9 also implies that during expansion of the cosmological horizon, new strands and new crossings *continuously appear* in the cosmic vacuum. In addition, due to the expansion, the strands continuously *rearrange*. In other words, together with vacuum energy,

> ▷ Strands imply a non-vanishing *entropy* and *temperature* of the cosmic vacuum.

The cosmic vacuum is thus a *bath.* So far however, these properties have not been observed.

Pr. 44 Strands imply that the naive (absolute value of the) present temperature of the cosmological horizon, for de Sitter space, is given by

$$|T_{c0}| = \frac{\hbar c}{2\pi k} \frac{1}{R} \tag{16}$$

where R is the radius of the horizon. It is natural to assume that de Sitter space is an approximation for the present universe. Inserting the radius of the present cosmological horizon, $R_{c0} = 4.4 \cdot 10^{26}\,\mathrm{m}$, or $3.3c$ times 13 800 million years, yields

$$|T_{c0}| = 8.5 \cdot 10^{-31}\,\mathrm{K} \quad , \tag{17}$$

The resulting temperature of roughly $10^{-30}\,\mathrm{K}$ corresponds to a kinetic energy of around $10^{-34}\,\mathrm{eV}$. This vanishingly small horizon temperature confirms older research [33–35]; it is not in contradiction with observations. Strands thus imply that the temperature of the horizon is so low that it will *probably never* be checked by direct, local experiments. The temperature value is so low that is will also *not* be detected by cosmological measurements. The temperature of the present cosmic background radiation, $2.7\,\mathrm{K}$, is much larger and will mask the horizon temperature. This masking also occurs for neutrinos. Even if the situation were different for gravitons, the fact would be purely academic, as single gravitons cannot be detected [36, 37].

Pr. 45 The horizon temperature, properly speaking, is negative, as argued by Klemm [38] and by Cvetič et al. [39]. The entropy flow that results points towards the horizon. The sign and small value of the horizon temperature does not appear to influence the matter far inside the horizon.

Pr. 46 Strands imply and predict that the (absolute value of the) vacuum temperature of the *interior* of the universe is and has always been equal or *larger* than the (absolute value of the) temperature of the horizon. The (absolute value of the) horizon temperature is predicted to be an effective *lower bound* for vacuum temperature. The two temperatures are equal if there are in equilibrium. In general, they are not. This agrees with expectations.

Pr. 47 It might be worth noting that the 'black hole' *luminosity* of the horizon, despite its size, is much lower than the limit $c^5/4G$. The reason is that the temperature of the horizon decreases with the radius as $1/R$; as a result, the total emitted power, which depends on T^4, is extremely low and continues to decrease. The present luminosity of the horizon is negligible. In the past, however, the luminosity was much higher.

Pr. 48 Both the negative horizon temperature and the strand model appear to imply that there is *no measurable horizon relic radiation* from the times when the horizon was much smaller and hotter. However, this prediction cannot be checked by experiments, as the involved numbers are too small.

In short, strands imply that the vacuum temperature and the horizon temperature can *differ*, due to non-equilibrium effects. Strands also imply that the horizon temperature decreases with the expansion of the universe, and so does the vacuum temperature. The precise effects of the continuous addition of colder vacuum to the already existing warmer vacuum must still be worked out. The corresponding effect for photons could be a guide.

16 Qualitative strand predictions about horizon and interior entropy

Pr. 49 Strands imply that the cosmological horizon is a *tight* weave. As a consequence of weave tightness, the expression for *entropy* of a black hole, which is also a tight weave, can be used for the entropy S_{CH} of the cosmological horizon. Using $A = 4\pi R^2$ and $R = ...\,\text{m}$, one gets

$$S_{\text{CH}} = \frac{A\,kc^3}{4G\hbar} \;\approx\; 10^{122}k \ . \tag{18}$$

This entropy value agrees with other published estimates [25]. The concavity of the cosmological horizon, which contrasts with the convex shape a black hole horizon, should not change the result.

Pr. 50 Strands imply that the entropy of the cosmological horizon has the *same value* as or *more* than the entropy of the interior. This yields

$$S_{\text{interior}} \leq O(1)\frac{A\,kc^3}{4G\hbar} \;\approx\; 10^{122}k \ . \tag{19}$$

Since the universe is mainly empty, the interior entropy is predicted to be mainly the *entropy of the cosmic vacuum.* In other words, strands predict the existence of *entropy of dark energy*.

Pr. 51 Once the interior entropy is known, one can deduce an entropy density of the cosmic vacuum, or of dark energy, given by

$$s_{\text{interior}} = \frac{S_{\text{interior}}}{V} \leq 10^{44}k/\text{m}^3 \ . \tag{20}$$

This is an astonishingly high value, because a gas at standard conditions has an entropy of about $10^{26}k/\text{m}^3$.

In short, strands imply that the entropy density of the cosmic vacuum is positive and considerable. The next step is to deduce quantitative statements.

17 Accelerated expansion: observations and usual description

Observations of the distance-velocity relation of distant galaxies, using type Ia supernovae, lead to the conclusion that the expansion rate of the universe is *accelerating*. The acceleration of the expansion is a small effect that requires elaborate precise astronomical measurements at cosmological scales. At sub-galactic scales, including the solar system and all laboratories, *no* effects of the acceleration of the expansion are observed.

The most common description for the acceleration is that of ΛCDM cosmology: acceleration is due to *dark energy*, which is described by a non-vanishing, positive cosmological constant Λ that is a fundamental constant of nature. The cosmological constant Λ is part of the field equations (12) and of the Lagrangian of general relativity. Measurements yield the value [23]

$$\Lambda = 1.1 \cdot 10^{-52}\,\mathrm{m}^{-2} \quad . \tag{21}$$

This value is small: it is close to the inverse square of the Hubble radius.

Cosmological measurements about dark energy also find an energy per pressure value of

$$E/p = w = -1 \tag{22}$$

within measurement errors [23], consistent with a *constant* value of dark energy over time. The constancy of Λ has been measured over an age span that covers most of the age of the universe.

Apart form the distance-velocity relation, most other effects of the cosmological constant Λ are seen in simulations of the history of the universe. It might be that Λ is related to the rotation curves of galaxies; however, this issue is not settled. In any case, no effect of Λ is predicted to be observable at sub-galactic distances or in everyday life.

In short, in the ΛCDM model, the value of Λ is an unexplained, fundamental constant of nature.

18 Proposed explanations of expansion acceleration

Over the years, researchers regularly have tried to understand the *value* of Λ. In the concordance model of cosmology, the value is constant over time. The explanation attempt by Verlinde is based on the thermodynamics of space-time [28]. Other examples are the proposals by Hossenfelder [41] and by Padmanabhan [29]. None of these explanation proposals has gained general acceptance.

Several alternative and more radical explanations for the acceleration of the expansion of the universe also exist, triggered by the closeness between Λ and the inverse squared Hubble radius. In a first alternative, the measured acceleration is seen as a consequence of a peculiar (accelerating) observer status of the Earth [42]. This alternative explanation is not yet ruled out by observations [43,44].

In a second, similar alternative, the acceleration is seen as an artefact of the inhomogeneities of the universe. If the Earth is located in a so-called (partial) void, an apparent acceleration can arise, as explained by Wiltshire [45,46].

In a third alternative, certain researchers question the measurement data that led to the conclusion of acceleration [47, 48]. They argue that the conclusion is due to a bias in the selected set of type Ia supernovae that is used to determine distance values.

In a general fourth alternative, various authors have proposed *changes* to general relativity itself. These approaches have been compared and discussed in detail elsewhere [49]. Most of these proposals are not discussed in the present work, because the agreement with data is questionable. Many proposals disagree with general relativity – because they contradict maximum force, as discussed in reference [15] – or disagree with the standard model of particle physics. Therefore these proposals also disagree with the strand conjecture.

A fifth alternative is quintessence and the models related to it [50]. These approaches are less radical and just assume an evolution of Λ with time. A slow evolution of Λ might be compatible with data.

In short, so far, all observations failed to determine the *origin* of Λ. Equally, many models failed to calculate the *value* of Λ. If the strand conjecture is correct, it must be able to distinguish between ΛCDM cosmology, the proposed explanations of Λ, and the mentioned five alternatives. The strand conjecture must make specific, testable predictions, and must explain the origin, value and time-dependency of Λ. It is useful to start with some general considerations.

19 Strand predictions about the vacuum: energy, entropy, temperature and their time dependency

Pr. 52 Figure 9, showing the history of the universe, illustrates that the density of strands in the cosmic vacuum is *determined by the horizon*. The cosmic vacuum can be seen as a result of the tethers starting at the horizon. In particular, this implies that the strand density in the cosmic vacuum is *non-vanishing*, *flat*, *homogeneous* and *isotropic* across the interior of the universe. All this is observed. Therefore, both vacuum entropy density s_{vac} and vacuum energy density u_{vac} are predicted to be homogeneous and isotropic – as long as matter and radiation do not disturb it. There is no contradiction between this prediction and observations.

Pr. 53 The vacuum entropy density s_{vac} and the vacuum energy density u_{vac} are related.

$$u_{\text{vac}} = O(1) \cdot s_{\text{vac}}\, T_{\text{vac}} \quad . \tag{23}$$

This relation predicts that also the vacuum temperature is homogeneous and isotropic. So far, measurement did not allow any check of this statement – because the vacuum temperature is around 10^{-30} K – but the result is expected.

The next task is to deduce the *time dependency* and the *values* of all the vacuum quantities from strands. The following arguments are based, as before, on the number of tethers leaving from the cosmological horizon: it is assumed that one tether starts at each smallest horizon area $A_{\text{cPl}} = 4G\hbar/c^3$.

Pr. 54 First of all, the change in *strand density* in the cosmic vacuum can be estimated from the average number and the average length of strands, both of which depend on cosmological horizon surface and volume. Given a horizon radius R, the average strand length grows as $3R/4$, and the strand number grows as R^2. The total strand length thus grows as R^3. As a result, the strand density – i.e., strand length per volume – is thus independent of the size R of the horizon. In other words, *the strand density is constant over time.*

Pr. 55 The value of the strand density can be estimated from the geometry of the situation. The result is surprising: there is, within a factor $\mathcal{O}(1)$, a strand in every Planck volume. In other terms, strands make the astonishing prediction that vacuum is essentially *full of strands.* in contrast to the low density illustrated in Figure 8 and Figure 9.

Pr. 56 Secondly, the *crossing density* N_C/V has to be explored, and in particular, its change with radius R. The *close-by* crossing density, of strands passing each other *close by*, does not depend on R: it is constant. The *total* crossing density, of strands passing each other at *any* distance, will grow as $R^4/R^3 = R$. In short, even though strand density is constant, the crossing density *either is constant or increases;* its change depends on the distance range of the crossings that is under consideration.

Pr. 57 Again, the value of the close-by strand crossing density can be estimated from the geometry of the situation. Again, there is, within a factor $\mathcal{O}(1)$, a crossing in every smallest possible volume. Strands thus predict that vacuum is essentially *full of strand crossings.*

Pr. 58 Strands allow estimating the change of the crossing *switch* density. To do this, the average time taken by a crossing switch needs to be determined. In particular, it must be understood how this average time depends on the horizon size R. Only crossings of close-by strands can switch with a finite probability. The probability depends on the speed and amplitude of the average strand motion. In a highly dense strand vacuum, these two quantities will not depend on R. Thus the crossing switch density of close-by strands is expected to be *constant in time.*

In other terms, as long as close-by crossings dominate, the energy density Λ of the vacuum is *constant* in time. The same is predicted for the entropy density and the temperature of the vacuum. It needs to be noted that this result also assumes that vacuum is the dominant structure in the universe, i.e., that matter and radiation play a negligible role. This is indeed the case at the present age of the universe.

In short, strands imply that

> ▷ The cosmological constant Λ is a large-scale quantum effect.

Pr. 59 Strands should also allow estimating the *present value* of the vacuum energy density Λ. Close-by crossings will switch only rarely, due to the fully packed vacuum. The process is rare; as a result, Λ *is very small.* This is as expected, and is compatible with observations [23].

However, at present, there does not appear to be a good way to estimate the probability of a crossing switch in a fully packed vacuum. Nevertheless, a few results arise.

Pr. 60 Strands imply that dark energy is due to the horizon. But strands imply more:

> ▷ Strands imply that dark energy is not different from cosmic vacuum.

Strands imply that there is no cosmic vacuum without dark energy.

Pr. 61 Strands also imply something about the nature of dark energy:

> ▷ Dark energy resembles extremely cold particles – gravitons, photons, neutrinos, etc. – with cosmological-sized wavelength.

This result is a genuine strand prediction. It is not in contradiction with observation. However, there is no research literature on this option.

Pr. 62 In other terms, *strands imply that dark energy is not fundamentally different from known matter and radiation.* However, strands predict that at cosmological scales, different types of particles *cannot be distinguished* from each other. Strands thus predict

> ▷ Dark energy has no specific particle aspect that allows determining the involved particle types.

All this does not disagree with observations, but it challenges habits of thought.

Pr. 63 In conventional physical language, the *cosmological horizon* has a temperature and radiates; the radiated particles are *real*, as in every thermal radiation. However, the strand conjecture *modifies* this conclusion:

> ▷ There is *no difference* between *real* and *virtual* particle at cosmological scales.

The cosmic expansion can thus be said to generate a mixture of real and virtual particles. At cosmological scales, the two cases are *indistinguishable* in the strand model of dark energy.

Pr. 64 Summing up the previous three predictions, strands imply that at cosmological wavelengths, there is *no difference* between vacuum and particles. This unexpected result in the infrared limit complements the older, similar strand result in the ultraviolet: at Planck scales, strands do not allow distinguishing vacuum and matter. The equivalence in the ultraviolet is known since a long time [40], but the equivalence in the infrared appears to be new. It might be surprising that neither limit contradicts observations.

Pr. 65 Dark energy, like any other type of cosmic energy density, is characterized by the dimensionless factor w in the *equation of state* $p = w\varrho$ that relates the pressure p and the energy density ϱ. The factor w describes the type of behaviour of the energy density.

For photons and for relativistic matter, w has the value $1/3$; for slow (non-relativistic) matter, such as baryons, w has the value 0. For a fundamental constant, w has the value -1.

In the strand conjecture, the number w in the equation of state $p = w\varrho$ is ... negative because the vacuum effectively pushes the horizon away.

In the ΛCDM model, the equation of state for dark energy is $w = -1$, i.e., the cosmological constant is indeed constant. Observations confirm this w value for dark energy to within 10% [23].

Pr. 66 The value of the vacuum energy density cannot be estimated – yet. It is not clear whether strands imply a measurable or a negligible value of Λ. Also the time dependence is not clear. A comparison with the literature [51–53] is not yet possible.

In short, strands naturally allow to model dark energy in its properties.

20 Strand predictions about inflation

Pr. 67 Because in the strand model, flatness, homogeneity and isotropy arise naturally, strands have *no necessity for inflation* to explain these observations. The same holds for density fluctuations.

Given that in the strand conjecture, matter arises at the horizon, strands imply that there is *no re-entry* into the cosmological horizon. Strands predict that no stars, no galaxies, no matter – nothing – *ever* re-enters the horizon. Again, this is an argument against inflation.

Because strands do not allow fields outside those of the standard model, strands also exclude every field mechanism proposed for inflation in the past [54].

In short, for three reasons, strands imply that *inflation did not occur.* This conclusion does not contradict observations, but is hard to test in detail.

21 Discussion

The exploration has led to a number of conclusions.

- ▷ Strands are compatible with a very small or with a vanishing vacuum energy.
- ▷ Strands imply that vacuum energy density *remains constant* with increasing cosmological radius.
- ▷ Strands predict that $w = p/\varrho$ is negative and compatible with the value -1.
- ▷ Strands imply that the value of the vacuum energy density Λ is hard to estimate.

These results are disappointing. Still, it could well be that there are errors in the discussion so far. A check makes sense.

First of all, the universe could be made of several strands. Exploring the option, one finds that it has no effect on the numerical predictions.

Secondly, the number of horizon strands leaving towards the interior of the universe need not match those of a Schwarzschild black hole. The number of tethers could be less than one per minimal area. In this cases, the entropy would still be proportional to area, but the proportionality constant could be different from that of black holes. Again, this would have no effect.

Thirdly, the ratio between tether number and horizon area could be smaller, or even change over time – increasing or decreasing. In this case, the entropy of the horizon could depend on area in a different way. This would also change temperature end energy. But the option seems impossible: every strand in the horizon has a tether.

A fourth option needs to be explored. Could the time-dependence of Λ depend on the setting? Could it be that Λ is constant when a measurement is *local*, such as in quantum theory, but that it decays when a measurement is *global*? In other terms, can Λ be, at the same time, both constant and decaying? Or is Λ zero throughout? So far, the strand conjecture has difficulties to make hard statements. Strands appear to allow a constant, an increasing and a decaying cosmological constant Λ. Strands also suggest that if Λ is constant, strands do not allow a simple way of calculating it. If Λ decays, strands allow calculating it and suggest $\Lambda = 1/R^2$, as observed.

22 Outlook

From one fundamental principle the strand conjecture deduces the standard model of particle physics, without any modification [4]. From the same fundamental principle, at sub-galactic scales, the strand conjecture deduces general relativity, the lack of singularities, as well as the existence of a maximum force value $c^4/4G$ and a maximum power value $c^5/4G$ [16]. The agreement of these consequences with all observations so far encourages the exploration of larger scales.

In the domain of cosmology, fluctuating strands predict an expanding cosmological (particle) horizon. Strands starting at the cosmological horizon and transversing the interior form *matter*, *radiation* and *dark energy*. Matter and radiation are due to tangled strands, dark energy is conjectured to consist of crossing switches of untangled vacuum strands.

The strand conjecture implies that the cosmological horizon has entropy and temperature. The expansion of the universe results from the ever increasing tangling of the interior strand segments. The expansion rate first decreased because of the increase of matter content, and then increased because of dark energy.

Applying the power limit $c^5/4G$ to a surface inside the Hubble radius leads to an upper limit of energy density for the universe. All observed energy densities, taken together, are lower than the limit. Strands imply that, at present, the matter density decreases with time, as observed, because no new matter is being created any more.

In the strand conjecture, *dark energy* can be seen as being made of real and virtual photons, gravitons or neutrinos with cosmological wavelength. At those wavelengths, strands also predict that these particle types cannot be distinguished from each other. Strands predict that dark energy has a *low density value*, determined by the size, entropy and temperature of the cosmological horizon. Also a vanishing Λ is possible. Strands further suggest that, at present, *the density of dark energy is constant.* Future investigations should check these conclusions in all possible ways.

The strand conjecture can be tested in the domain of cosmology with simulations of the collective dynamics of large numbers of strand segments. This exploration will allow calculating the time-dependence and spatial variation of matter and radiation densities. Also the topic of dark matter and galaxy rotation curves must be explored. This will be done in an subsequent article. Further comparisons with cosmological measurements will then be possible.

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Appendix A On the circularity of the fundamental principle

Any strand crossing switch is assumed to take place in space. On the other hand, space, distances and all physical observables are assumed to arise from strands. The circularity can greatly be reduced – but not fully eliminated – with a more precise formulation.

In detail, crossing switches take place in a local *background space*, a space defined by the observer in order to allow descriptions of observations. In contrast, *physical space*, including physical distances and physical observables, arises from strand fluctuations and their crossing switches. When space is flat, background space and physical space are identical. When space is curved, background space is (usually) the local tangent space of physical space.

The strand conjecture stresses that a description of nature *without* a local background space and time is impossible. Every observation and every measurement of an object or a system requires the use of a local background space introduced by the observer. (There are many possible choices for the background – as many as there are possible observers.)

The need for a local background space is due to a fundamental contrast between *nature* and its *precise description*. The properties of nature and the properties required for a precise description *differ*; in fact, they *contradict* each other. A precise *description* of nature requires axioms, sets, elements, functions and points in (local background) space and time. In contrast, due to the uncertainty relation, at the Planck scale, *nature* itself does not provide the possibility to define points in (physical) space or time; they are emergent. Due to the uncertainty relation, neither sets nor elements exist in nature at the most fundamental level. As a result, an axiomatic description of nature is *impossible*. Axiomatic descriptions only exist for *parts* of physics, when the description and nature itself are approximated to have the *same* properties. Axiomatic descriptions are possible in quantum theory, in electrodynamics, in special relativity, or in general relativity. Axiomatic descriptions are impossible when the uncertainty relation is applied to space itself, thus not when describing *all* of nature at the same time. In particular, axiomatic descriptions are impossible for quantum gravity.

Because of the impossibility of an axiomatic description of nature and of quantum gravity, any complete description of nature must include a limited degree of *circularity*, in particular in its definition of time and space. By using background space, the strand conjecture indeed introduces such a limited amount of circularity: physical space is defined with the help of particles – e.g., with *rulers* made of matter or light – and particles are defined with the help of physical space – for example, with energy and spin that are *localized* in three spatial dimensions. Both physical space and particles arise in background space. At first sight, this eliminates the circularity. But background space itself arises as an approximation of local physical space. Therefore, a small amount of circularity remains. Despite this residual circularity, strands do allow a description of nature.

The main example for the difference between an axiomatic description and a complete, consistent and partially circular description is the dimensionality of space. The number of spatial dimensions is not a consequence of the fundamental principle or of some axiom; the number of spatial dimensions is included in the fundamental principle right from the start. The number of dimensions is included at the start because it is the only consistent choice: Only three dimensions allow crossing switches, allow particle tangles, allow spin $1/2$, allow Dirac's equation, allow deducing U(1), broken SU(2) and SU(3), and allow deducing Einstein's field equations. *Only* a background space with *three* dimensions allows the existence of tangles and allows a precise description of nature.

In short, due to the use of a local background space, the strand conjecture *cannot* be tested by asking whether it is an *axiomatic* description of nature. No unified description of nature can be axiomatic. Every unified description and every model of quantum gravity must be somewhat *circular*. Nevertheless, like any unified description, the strand conjecture *can* be tested by asking whether it is a correct, complete and consistent description of nature. So far, it passes all experimental and theoretical tests [4, 5, 16, 20].

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