Testing a microscopic model for black holes deduced from maximum force

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Maximum force $c^4/4G$ and maximum speed c imply Einstein's field equations of general relativity. Combining the two limits with the quantum of action and an old idea by Dirac leads to a Planck-scale model for the microscopic degrees of freedom of space, particles and horizons. The model is based on fluctuating, tangled, one-dimensional strands. The model reproduces black hole mass, energy, temperature and entropy. Strands confirm that the mass and charge of a black hole are distributed over its horizon. The model implies black hole radiation, the hoop conjecture, the theorems of horizon mechanics, the no-hair theorem, Bekenstein's entropy bound, the lack of remnants, as well as both the magnetic moment limit and the g-factor of black holes. Comparisons with fuzzballs and firewalls are possible. Important predictions are numbered; so far, all agree with observations or expectations. In addition, the standard model of particle physics arises automatically from fluctuating strands.

Keywords: general relativity; black holes; quantum gravity; strand conjecture.

I The search for the microscopic degrees of freedom of black holes

What are the microscopic degrees of freedom of black holes? Whatever they are, they must be related to the microscopic degrees of freedom of vacuum, and to gravitons. Over the past decades, many candidates have been proposed [1–10], though with limited success. Any proposed microscopic degrees of freedom of black holes must *reproduce* space, curvature, mass and gravitation in all their macroscopic and microscopic aspects. But in addition, any proposal must also provide *new* results that go beyond the usual description of space as a continuous manifold made of points. In the following, it will be shown how an idea on quantum theory due to Dirac, combined with a recent result on general relativity, leads to a microscopic model of black holes that provides such new results. The other quantum gravity aspects of the model have been explored in detail elsewhere [11, 12].

II The first origin of the strand conjecture: tethers

Bohr had the habit to present quantum theory as a consequence of the smallest action value $\hbar$ [13]. In 1925, Dirac included the speed limit c into quantum theory. From 1929 on, Dirac presented the *string trick* or *belt trick* in his lectures. The belt trick is illustrated in Figure 1. It describes the basic properties of spin $1/2$ – namely the return to the original situation ofter a rotation by 4π – as the consequence of *tethering*. As Dirac told Gardner ([14], page 47) the belt trick also shows that a spin value below $\hbar/2$ is impossible. This confirms Bohr's statement: a smallest angular momentum value $\hbar/2$ also limits the smallest observable action to the value $\hbar$.

A less known companion to the belt trick, the fermion trick, is illustrated in Figure 2. The fermion trick describes the basic properties of fermions – the return to the original situation ofter double particle exchange – as the consequence of tethering. Here, *tethers* are connections of a structure to spatial infinity with unobservable, uncuttable, flexible, fluctuating, one-dimensional strands.

In nature, matter particles are found to have spin $1/2$ and to behave as fermions. However, particle tethers are not observed at all. In retrospect, Dirac's tether trick was the first hint that matter particles could

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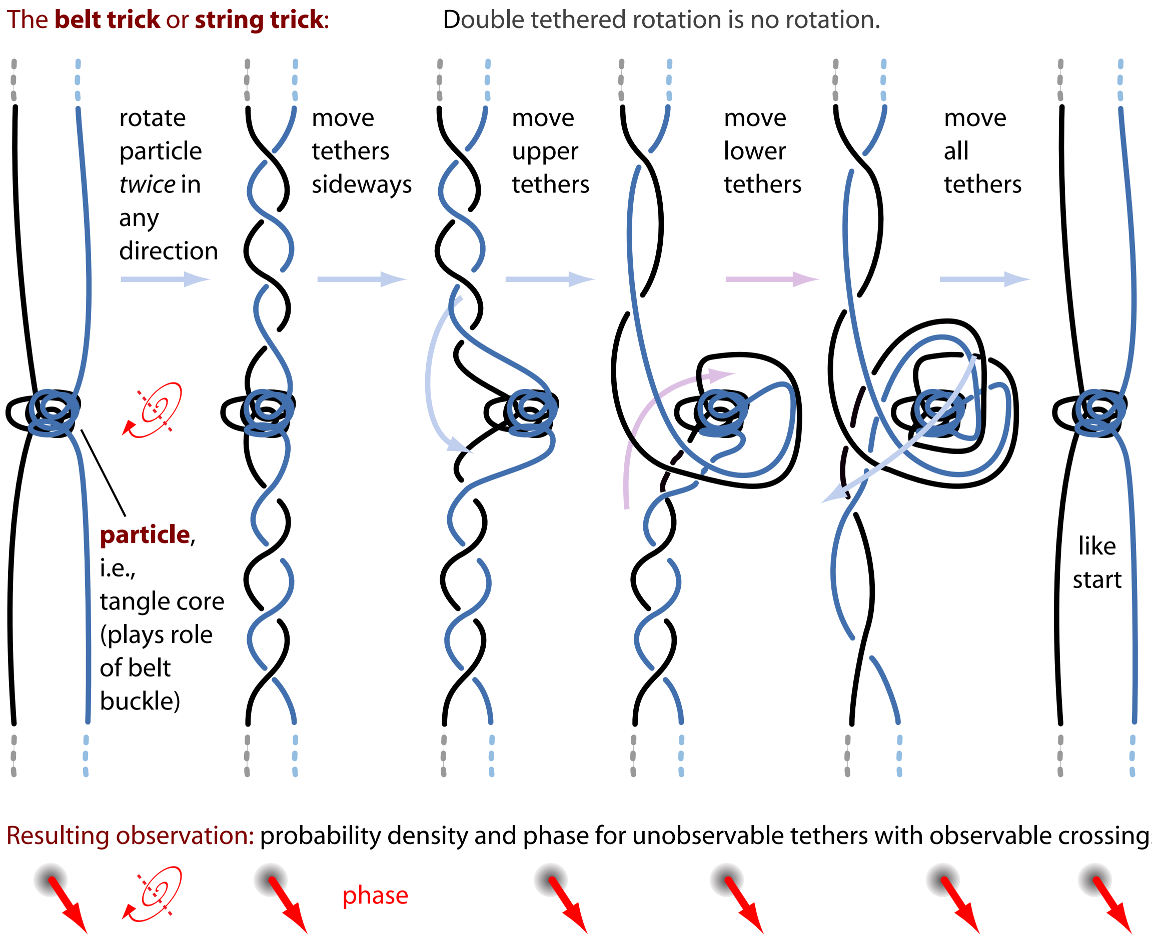

FIG. 1: The *belt trick* or *string trick*: a double rotation, by 4π, of a tethered structure, such as a tangle core, is equivalent to no rotation – if the tethers are allowed to fluctuate and untangle as shown. Untangling is impossible after a particle rotation by just 2π: tangle cores with 4 or more tethers thus show the properties of spin $1/2$. As a consequence, a tethered core is able to rotate continuously. In the strand conjecture, the trick also couples rotation and *displacement* of the tangle core. The coupling of core rotation frequency and core displacement determines particle mass. (The figure is modified from reference [15].)

be described using *unobservable* tethers with *observable* crossing switches. (The concept of 'crossing switch' is defined below.) Fifty years later, in 1980, Battey-Pratt and Racey showed that tethers also imply the full free Dirac equation [16]. In short, Battey-Pratt and Racey proved that Dirac's trick implies Dirac's equation. (For example, strand crossings have the same geometrical properties as wave functions: position, density, and phase. Wave functions turn out to be time-averages of tether crossings [11, 15].) In other terms, *every quantum effect* can be seen as result of observable crossing switches of unobservable tethers.

III The second origin of the strand conjecture: maximum force

The invariant limit speed c realized by massless radiation is a fundamental principle of nature. Indeed, special relativity contains and implies a maximum speed; furthermore, the maximum speed implies and contains special relativity. In a similar way, the invariant maximum force

$$F_{\mathrm{max}} = \frac{c^4}{4G} \approx 3.0 \cdot 10^{43}\,\mathrm{N} \quad , \tag{1}$$

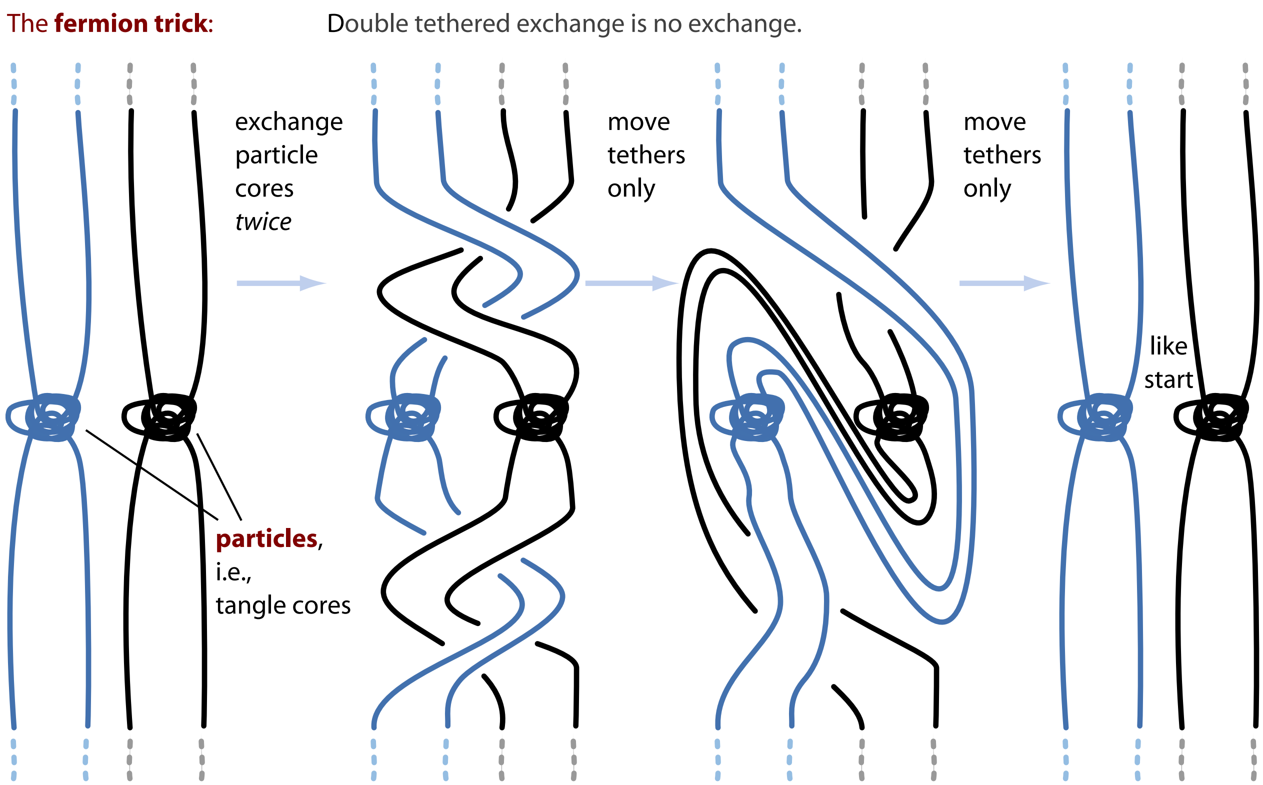

FIG. 2: The *fermion trick*: a double particle exchange of two tethered particles is equivalent to no exchange – if the tethers are allowed to fluctuate and untangle as shown. Untangling is impossible after single exchange. Tangle cores with 4 or more tethers thus show the defining properties of fermions.

that is realized by gravitational horizons is a principle of nature. Here, G is the gravitational constant. The force limit is a principle of nature for two analogous reasons.

It is known since at least 1973 that *general relativity contains and implies a maximum force* [17–37]. The gravitational force between two black holes never exceeds $c^4/4G$, as shown by Gibbons [23]. The force limit is also given by the maximum energy per distance possible in general relativity: the energy Mc^2 of a Schwarzschild black hole divided by its diameter $D = 4GM/c^2$ defines the maximum possible force value. The ratio is independent of the black hole size and mass. Even charged or rotating black holes do not yield larger ratios. Finally, the force on a test mass being lowered towards a gravitational horizon with a rope never exceeds the upper bound, – if the size of the test mass is considered. In fact, no physical system allows exceeding the upper force limit. All proposed counter-examples to the force limit disappear under closer scrutiny [38–42]. In fact, the force bound agrees with all known observations and passes all theoretical tests [42].

It is known since 2003 that the force bound $c^4/4G$ *implies and contains* Einstein's field equations. Two proofs are known [24, 25, 42, 43]. One proof starts by deducing the first law of horizon mechanics from the force bound. The first law can be used to deduce the field equations. The other proof starts by showing that the force bound implies a limit on the deformation of space. The deformation limit in turn implies a relation between curvature and energy, which leads to the field equations.

In short, because the field equations *follow* from maximum force $c^4/4G$ and from maximum speed c, both limits are *principles* of nature. It should be mentioned that alternative and *equivalent* formulations of the principle of maximum force exist. There is the principle of maximum power or maximum luminosity $c^5/4G$, the principle of maximum mass flow rate $c^3/4G$, and others [42]. Each of these principles is equivalent to general relativity, and equivalent to the others.

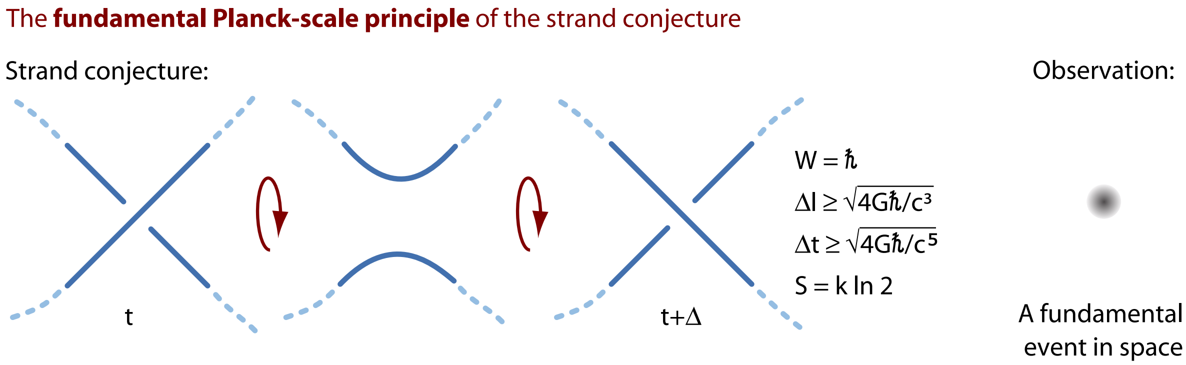

FIG. 3: The fundamental principle of the strand conjecture specifies the simplest observation – a simplified version of Dirac's trick, shown in Figure 1, taking place at the Planck scale – that is possible in nature: the almost point-like *fundamental event* results from a *skew strand switch*, or *crossing switch*, at a position in three-dimensional space. The strands themselves are *not observable*. They are impenetrable and are best imagined as having Planck size radius. The observable switch defines the action unit $\hbar$. The double Planck length limit and the double Planck time limit arise, respectively, from the smallest and from the fastest crossing switch possible. The paper plane represents background space, i.e., the local tangent Euclidean space defined by the observer. (The figure is modified from reference [15].)

IV Combining tethers and maximum force

Taken together, the three limits – special relativity's c, quantum theory's $\hbar$, and general relativity's $c^4/4G$ – imply that there are no trans-Planckian effects in nature of any kind (as long as G is substituted by $4G$ in all Planck quantities). These (corrected) Planck limits and all their consequences agree with all observations. Therefore, the (corrected) Planck limits must also hold in a theory of quantum gravity.

In fact, there is a simple way to ensure the lack of trans-Planckian effects. One requires that the smallest quantum of action $\hbar$, the smallest length $\sqrt{4G\hbar/c^3}$ and the smallest time $\sqrt{4G\hbar/c^5}$ hold everywhere and at every instant of time.

Can unobservable strands also explain gravitation and quantum gravity? It turns out that this is the case, provided that the fluctuating strands have Planck radius. With this property, the validity of the maximum force principle is ensured, and thus the validity of general relativity. Also quantum gravity follows, including the finite value and the surface dependence of black hole entropy, as shown below. It thus appears that also *every gravitational effect* can be seen as result of observable crossing switches of unobservable tethers.

V The fundamental principle of the strand conjecture

In the strand conjecture, all physical systems found in nature – matter, radiation, space and horizons – are made of unobservable strands that fluctuate at the Planck scale.

▷ A *strand* is defined as smooth curved line – a one-dimensional, open, continuous, everywhere infinitely differentiable subset of $\mathbb{R}^3$ or of a curved 3-dimensional Riemannian space, with trivial topology and without endpoints – that is surrounded by a perpendicular disk of Planck radius $\sqrt{\hbar G/c^3}$ at each point of the line, whose shape is not self-intersecting, and that is randomly fluctuating over time.

(It is most practical to define and visualize strands as having Planck-size radius. A few issues about this choice are not discussed here.) The definition of strands leads to the following statements:

▷ Strands are unobservable. However, crossing switches of skew strands – exchanges of over- and underpasses – are observable. Crossing switches determine the Planck units G, c and $\hbar$; this fundamental principle is illustrated in Figure 3.

The defining Figure 3 therefore combines the essence of Dirac's trick with the Planck limits, and thus combines quantum theory with maximum force and general relativity.

The strand conjecture claims that the fundamental principle contains and implies all observations, all equations of motion, and all Lagrangians. In particular, the fundamental principle implies:

- ▷ Physical space is a (three-dimensional) *network* of fluctuating strands – i.e., of strands that are neither woven nor tangled nor knotted, as illustrated in Figure 4).
- ▷ Horizons are (two-dimensional) *weaves* of fluctuating strands – i.e., similar to a fabric made of woven threads, and illustrated in Figure 6.
- ▷ Particles are (localized) *rational tangles* of fluctuating strands – using the term from topological knot theory, defined and illustrated in Figure 10.
- ▷ Physical motion *minimizes* the number of observable crossing switches of fluctuating unobservable strands.

The fundamental principle thus contains all physical systems, and also every physical process. This includes particle motion, gauge interactions, particle scattering, and gravitational waves. Classifying tangles of strands leads to the particle spectrum, classifying tangle deformations with Reidemeister moves leads to the three gauge groups and couplings, and exploring tether exchanges leads to particle mixings. The belt trick leads to particle mass. The full Lagrangian of the standard model follows from strands, including massive neutrino with PMNS mixing [11, 15]. In the following, however, only gravitation is explored, and in particular, only black holes.

By definition, the fundamental principle of the strand conjecture of Figure 3 states that action, length, time and entropy are *limited from below*:

$$W \geq \hbar \ , \ \ \Delta l \geq \sqrt{4G\hbar/c^3} \ , \ \ \Delta t \geq \sqrt{4G\hbar/c^5} \ , \ \ S \geq k \ln 2 \ . \tag{2}$$

Above all, strands visualize these inequalities. Together with Figure 3, these inequalities contain quantum gravity and allow understanding black holes. The number $\ln 2$ in the minimum entropy is due to the 2 possible strand configurations.

Strands themselves have *no* observable properties: no colour, no tension, no mass, and no energy. Due to the impossibility of observing them, strands have *no* meaningful equation of motion. Indeed, all results in the following are *independent* of the detailed fluctuating motion one might imagine for strands. (This could be called the *congruence principle* of the strand conjecture.) The independence of fluctuations eliminates any imaginable arbitrariness of the description of space, horizons and particles with strands.

Strands *cannot be cut*; they are not made of parts. Strands cannot interpenetrate; they *never* form an actual crossing. When the term 'crossing' is used in the present context, only the two-dimensional projection shows a crossing. Strands thus have a *crossing* in space when a strand segment passes over another. In three dimensions, strands are *always at a distance*. Like in Dirac's trick, a *crossing switch* – the change from an overpass to an underpass – cannot arise through strand interpenetration, but only via strand deformation.

In the strand conjecture, *all physical observables* – such as energy, mass, action, momentum, length, velocity, surface, volume, entropy, field intensities or quantum numbers – arise from combinations of crossing switches. No physical observable is a property of strands themselves; all physical observables arise from *shape configurations* of *several* strands. In other terms: every physical observable *emerges* from strand crossings.

VI Flat and curved physical space from strands

The *only* observable aspect of strands – as in Dirac's trick – are their crossing switches, and thus, for example, the distribution of crossing switches. To relate strands to observations, it is important to deduce the behaviour of crossing switches from the fundamental principle for the system under consideration. The simplest case is empty physical space.

A strand network of untangled strands describing physical space is illustrated in Figure 4. The illustration uses background space to define physical space. *Background* space is what is needed to *talk* about nature.

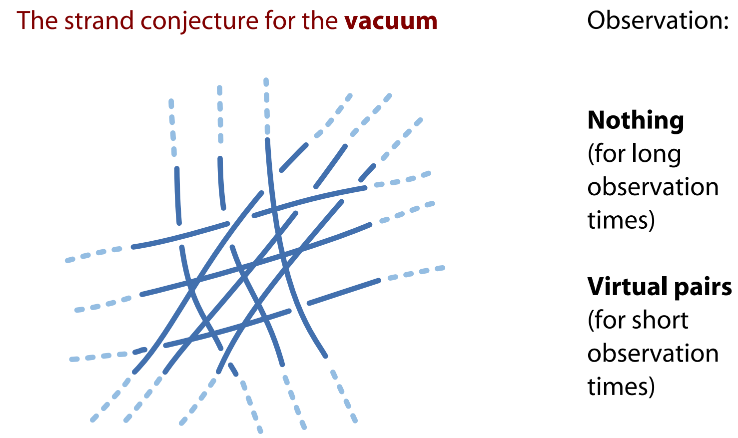

FIG. 4: A simplified and idealized illustration of the strand conjecture for a flat vacuum, i.e., for flat physical space. The space of the picture is *background* space. *Physical* space is generated by strand crossing and their switches. Strands fluctuate in all directions: their crossings are washed out. (Typical strand distances can be many orders of magnitude larger than their diameters.) For sufficiently long time scales (longer than a few Planck times), the lack of crossing switches leads to a vanishing energy density; for short time scales, particle–antiparticle pairs, i.e., rational tangle–antitangle pairs, arise in the vacuum due to the shape fluctuations of the strands. (The figure is modified from reference [12]; the reference also discusses the relation between background space and physical space.)

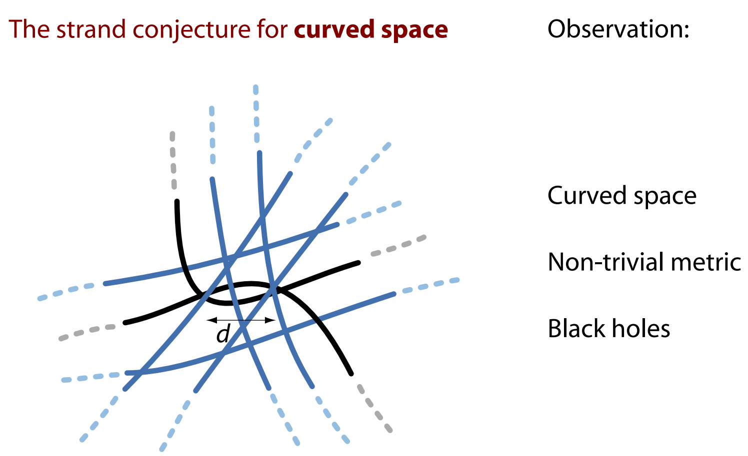

FIG. 5: An illustration of the strand conjecture for a *curved* vacuum. The strand and crossing configuration is *not* homogeneous and is midway between that of a flat vacuum and that of a horizon. Strands in black differ in their configuration from those in a flat vacuum. The value of the curvature is inversely proportional to the distance d. (The figure is modified from reference [12].)

Physical space is what can be *measured* about space: curvature, vacuum energy, entropy, temperature etc. The circularity issues that arise are discussed and solved in reference [12].

In other terms, a network of *untangled*, *unwoven* and *unknotted* strands represents *empty and flat* physical space. The time-average of the fluctuations, taken on a scale of a few Planck times or more, yields three-dimensional physical flat space, including its continuity, homogeneity, isotropy and Lorentz-invariance. On time scales longer than a few Planck times, there are (on average) *no* crossing switches, and thus neither matter nor energy – just empty space. *Physical space results from washing out the crossing switches of the strand network.* Strands thus imply that *no deviation* from the continuity, homogeneity, isotropy, dimensionality and Lorentz-invariance of (physical) flat space can be observed. This is the case at any practical energy (except for the corrected Planck energy itself) and is valid *despite* the existence of a smallest length $l_{\min} = \sqrt{4G\hbar/c^3}$.

Strands not only visualize flat space; strands also visualize curved space. Spatial curvature is illustrated in Figure 5. In simple terms, the fundamental principle of the strand conjecture implies:

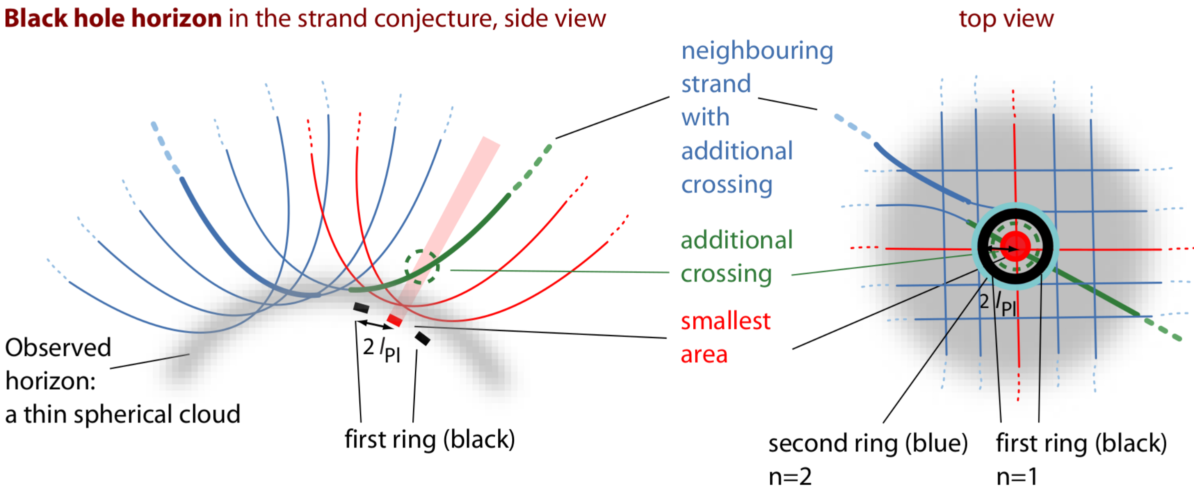

FIG. 6: The strand conjecture is illustrated for a Schwarzschild black hole, as seen by a distant observer: the horizon is a cloudy or fuzzy surface produced by crossing switches of the strands woven *tightly* into it. Due to the additional crossings *above* the horizon, the number of microstates per smallest area is larger than 2, and given by the base e of the natural logarithms (see text). This horizon model yields the entropy of black holes. (The figure is modified from reference [12].)

▷ *Flat space* is a homogeneous network of crossing fluctuating strands.

▷ *Curvature* is an inhomogeneous crossing (switch) density in the vacuum network.

The strand configuration for curved space differs from that of flat space: certain strands break the isotropy and homogeneity. The main curvature value depends on the configuration of the strands leading to the inhomogeneity. The curvature can evolve over time. This strand model for curved space implies that curved space-time is, locally, a Minkowski space. Thus, strands lead to a *pseudo-Riemannian space-time*.

VII Horizons and black holes from strands

In the strand conjecture,

▷ Horizons are one-sided, tight *weaves* of strands.

In this statement, *one-sided* means that all strands leave the horizon on one side, the side of the observer. One-sidedness means that there is ‘nothing’, not even an unobservable strand, on the other side of the horizon – except for rare fluctuations. For a distant observer *at rest*, a one-sided weave also implies that no space and no events are observable behind it. The weave thus acts as a *limit* to observation. Figure 6 gives a schematic illustration of a Schwarzschild black hole, both as a cross section and as a top view. For any black hole horizon, all strands come in from far away, are *woven* into the horizon, and leave again to far away. Due to their Planck radius, the weave of strands forming a horizon is as *tight* as possible: seen from above the horizon, there is one crossing for each smallest area.

For a *falling* observer, the strands do not form a weave, but continue on the other side and form a (distorted) network, i.e., curved vacuum. Such an observer does not notice anything special when approaching the horizon, or when crossing it. A tangle weave thus shows all the qualitative properties that characterize a horizon.

The weave model for horizons allows determining the *energy* and thus the *mass* of a Schwarzschild black hole. Energy E has the dimension action per time. Because every crossing switch is associated with an action $\hbar$, the horizon energy is found by determining the number N_{cs} of crossing switches, multiplied by $\hbar$, that occur per unit time. Because the number will depend on the surface area of the horizon, the energy will be proportional to horizon area. In a horizon weave, the crossing switches *propagate* from one crossing to the next, over the surface of the whole (tight) weave. Because the horizon weave is *tight*, each crossings has the size of the minimum length squared, given by the corrected Planck area $A_{\mathrm{cPl}} = 4\,G\hbar/c^3$. Because the horizon weave is *tight*, the propagation speed is one smallest length per shortest switch time: switch propagation thus occurs at the speed of light c. In the time T needed to circumnavigate a *spherical*, non-rotating horizon of area $A = 4\pi R^2$ at the speed of light, *all* crossings of the horizon switch. This yields:

$$E = \frac{N_{\mathrm{cs}}\hbar}{T} = \frac{A/(4G/c^3)}{2\pi R/c} = \frac{c^4}{2G}\,R \quad . \tag{3}$$

The weave model of a horizon thus reproduces the relation between the energy – or mass – and the radius of a Schwarzschild black hole.

The weave model also fixes the number of microstates per horizon area. Figure 6 shows that for a given smallest area containing just one strand crossing, the effective number N of microstates *above* that smallest area is *larger* than 2. A number larger than 2 occurs because sometimes, fluctuating neighbouring strands also *cross above* the smallest area.

The probability for a neighbouring strand to cross above the smallest area will depend on the distance at which the neighbouring strand leaves the lowest woven layer of the horizon. To calculate the probability, one imagines the central crossing surrounded by an infinite series of rings, each with a smallest area value $A_{\mathrm{cPl}} = 4\,G\hbar/c^3$. The rings can be numbered with a number n, as illustrated in Figure 6. The central crossing corresponds to $n = 0$. Ring number n thus *encloses* an area given by the smallest area A_{cPl} times n. Now, the probability p_1 that a strand from ring 1 crosses above the centre is

$$p_1 = \frac{1}{2} = \frac{1}{2!} \quad . \tag{4}$$

Likewise, the probability that a strand from ring n crosses above the centre is

$$p_n = \frac{1}{n+1}\,p_{n-1} = \frac{1}{(n+1)!} \quad , \tag{5}$$

because the strand has to continue in the correct direction above every ring on its way to the centre. The last expression is a result of the *extension* of strands; it would not arise if the fundamental constituents of horizons were not extended. The expression yields an effective number N of microstates above the central crossing given by

$$N = 2\, +\, \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ... + \frac{1}{n!} + ... = \mathrm{e} = 2.718281... \tag{6}$$

In this expression, the term 2 is due to the two options at the central point; the term $1/2!$ arises from the first ring around it, as illustrated in Figure 6; the subsequent terms are due to the subsequent rings. Expression (6) implies that the average number N of strand microstates for each smallest area, i.e., for each *corrected Planck area* $A_{\mathrm{cPl}} = 4\,G\hbar/c^3$ on the black hole horizon, is given by $N = \mathrm{e} > 2$. In the weave model, every corrected Planck area therefore contains somewhat *more* than 1 bit of information.

The calculation of the entropy of the complete black hole horizon is straightforward. It starts with the definition of entropy

$$S = k\;\ln N_{\mathrm{total}} \quad , \tag{7}$$

where k is the Boltzmann constant and N_{total} is total number of microstates of the complete horizon. The full horizon area A can be seen as composed of *a large number* of corrected Planck areas. The total number of microstates is thus the product of the number N of microstates for every corrected Planck area:

$$N_{\mathrm{total}} = N^{A/(4\,G\hbar/c^3)} \quad . \tag{8}$$

This is standard thermodynamics. The next step is to insert the result (6) due to strands. This yields

$$N_{\mathrm{total}} = \mathrm{e}^{A/(4\,G\hbar/c^3)} \quad . \tag{9}$$

The total number of horizon microstates, inserted into expression (7), yields the horizon entropy S of a black hole with surface A:

$$\frac{S}{k} = \frac{A}{4\,G\hbar/c^3} \quad . \tag{10}$$

This is the expression discovered by Bekenstein [44].

To summarize: in the strand conjecture, black hole entropy is *finite* because the microscopic degrees of freedom are discrete. Black hole entropy is due to one quarter of surface area A because the microscopic degrees of freedom are extended. The factor $1/4$ is due to the same factor appearing in the maximum force value $c^4/4G$. As Figure 6 illustrates, strands imply that horizon entropy is located *at and slightly around* the horizon. This agrees with expectations. The strand conjecture for black holes also confirms and visualizes a result by Zurek and Thorne from 1985: the entropy of a black hole is the logarithm of the number of ways in which it could have been made [45].

In the strand conjecture, the above calculation of the black hole entropy counts certain states more than once. Because strands can bend, reorienting the complete horizon sphere does not produce a different micro-state. The possible orientations of a sphere are given by the possible orientations of the poles and by the possible orientations of the sphere around the pole axis. The poles of the sphere can point to any of the $A/A_{\rm cPl}$ minimal surfaces that make up the horizon; in addition, the sphere can be rotated around the axis in $\sqrt{A/A_{\rm cPl}} \cdot \mathcal{O}(1)$ ways. The corrected value for the number of microstates of a spherical horizon is therefore

$$N_{\rm total} = \frac{N^{A/A_{\rm cPl}}}{(A/A_{\rm cPl})^{3/2} \cdot \mathcal{O}(1)} \quad . \tag{11}$$

This value yields the corrected black hole entropy

$$\frac{S}{k} = \frac{A}{4\,G\hbar/c^3} - \frac{3}{2}\,\ln\frac{A\,c^3}{4G\hbar} - \ln\mathcal{O}(1) \quad . \tag{12}$$

The last term is negligibly small. The second term shows that the strand conjecture makes a specific prediction for the logarithmic correction to the entropy of a Schwarzschild black hole. The value of the correction is much too small to ever be tested in experiments; but it agrees with previous calculations [46].

In other terms, woven strands imply both the energy E and the entropy S of Schwarzschild black holes. As usual, the ratio $E/2S$ determines the black hole *temperature* [47]:

$$T_{\rm BH} = \frac{\hbar c}{4\pi k}\,\frac{1}{R} = \frac{\hbar}{2\pi kc}\,a \quad . \tag{13}$$

In the last equality, the surface gravitational acceleration $a = GM/R^2 = c^2/2R$ was introduced, using expression (3).

In short, black holes are also warm. Their finite temperature implies that *black holes radiate*. Strands thus reproduce black hole *evaporation*. Radiation and evaporation are due to strands detaching from the horizon. If a single strand detaches, a photon is emitted. If a tangle of two or three strands detaches, a graviton or a fermion is emitted. When all strands have detached, the full black hole has evaporated.

VIII Deducing general relativity from thermodynamics and strands

In a path-breaking paper, Jacobson showed in 1995 that the thermodynamic properties of the microscopic degrees of freedom of space and of black holes imply Einstein's field equations of general relativity [48]. He deduced the result without requiring any details about the nature and properties of these degrees of freedom.

Jacobson started with three thermodynamic properties:

- the entropy–area relation $S = A\,kc^3/4G\hbar$,
- the temperature–acceleration relation $T = a\,\hbar/2\pi kc$,

- the relation between heat and entropy $\delta Q = T\delta S$.

As shown above, these three relations are valid also for strands: strands reproduce the entropy relation (7) of black holes, the temperature (13) of black holes, and their heat–entropy relation from (3).

Using the three thermodynamic properties, the thermodynamic relation

$$\delta E = \delta Q \quad , \tag{14}$$

which is valid *only* in case of a horizon, yields

$$\delta E = \frac{c^2}{8\pi G} a\, \delta A \quad . \tag{15}$$

This is the first law of horizon mechanics. This law can be rewritten, using the energy–momentum tensor T_{ab}, as

$$\int T_{ab}\, k^a \mathrm{d}\Sigma^b = \frac{c^2}{8\pi G} a\, \delta A \quad , \tag{16}$$

where $\mathrm{d}\Sigma^b$ is the general surface element and k is the Killing vector that generates the horizon. The Landau–Raychaudhuri equation [49] – a purely geometric relation – allows rewriting the right-hand side as

$$\int T_{ab}\, k^a \mathrm{d}\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab}\, k^a \mathrm{d}\Sigma^b \quad , \tag{17}$$

where R_{ab} is the Ricci tensor that describes space-time curvature. The equality between integrals implies a relation between the integrands:

$$T_{ab} = \frac{c^4}{8\pi G} \left(R_{ab} - (\frac{R}{2} + \Lambda)\, g_{ab} \right) \quad , \tag{18}$$

where R is the Ricci scalar and Λ is a constant of integration. These are Einstein's field equations of general relativity, without any modification. The thermodynamics of horizons thus do not determine the value of the cosmological constant Λ, as expected.

Jacobson stresses that the field equations are valid everywhere and for all times – not only near horizons – because a suitable general coordinate transformation can position a horizon at any point in space-time. If horizons and black holes are thermodynamic systems, so is curved space itself. It is thus correct to say that the field equations result from *thermodynamics of space*. Jacobson's argument thus shows that space, gravity and horizons are made of the same microscopic degrees of freedom.

Given that strands realize the premises of Jacobson's argument, it is possible to formulate a first, basic prediction:

Pr. 1 *No deviations* between general relativity and the strand conjecture arise.

This prediction also implies the full validity of the Hilbert Lagrangian, without any modification. As a result, all processes described by general relativity are reproduced by strands. For example, this includes universal $1/r^2$ gravity; it is illustrated in Figure 7.

So far, the *cosmological horizon* and its effects were not considered. Strands thus imply that general relativity holds without any measurable deviation and with full certainty only for *sub-galactic* distances, where the horizon has *no* influence. The validity of the strand model for galactic and cosmological distances will be explored in the future.

The equivalence of strands and general relativity is not a surprise: strands and their properties were deduced from maximum force, which already implies general relativity. In fact, the equivalence is not a strong statement for a further reason. Jacobson's deduction of the field equations is *independent* of the microscopic model of space. After Jacobson's result, various kinds of microscopic degrees of freedom for space have been conjectured and explored [1–10]. All these proposals recover the field equations. In other terms, these explorations have shown that extracting the *correct* microscopic degrees of freedom of space from all the proposals in the literature is *not possible* using arguments from quantum gravity alone.

The correct microscopic degrees of freedom of space and gravitation must also reproduce the standard model of particle physics. In addition, they must explain particle masses coupling constants and mixing angles. These seem *the only criteria* that differentiate between the various microscopic models of gravitation. Interestingly, strands are one of the few approaches to quantum gravity that do reproduce the Lagrangian of the standard model, as shown in detail elsewhere [11, 15].

IX Strand predictions about physical space

Pr. 2 In the network that defines empty space, the tangling of strands is not possible in other dimensions or in fundamentally different ways. Therefore, strands predict that flat physical space is *three-dimensional*, *unique* and *well-behaved* at all scales. Flat physical space is a three-dimensional *continuum* that is *homogenous* and *isotropic*, without observable deviations. Curved space is *Riemannian*. These predictions agree both with expectations [50] and with the most recent observations [51, 52].

Any evidence for lower or higher dimensions, other spatial topologies, quantum foam, different vacuum states, domain walls, cosmic strings, regions of negative energy, 'space-time noise', 'particle diffusion', 'space viscosity', crystal behaviour of space, or any other deviation from a well-behaved pseudo-Riemannian space-time manifold would directly *falsify* the strand conjecture.

Pr. 3 As a consequence of the fundamental principle the maximum local energy speed in nature is c. Strands predict this at all energy scales, in all directions, at all times, at all positions, for every physical observer. Strand predict the lack of *observable* violations of Lorentz-invariance, for all energies and all physical systems. It predicts no variable speed of light, no time-dependent speed of light, no time-dependent energy of light, i.e., no 'tired' light, no energy-dependent speed of light, no helicity-dependent speed of light, no polarization-dependent speed of light, no 'double relativity' and no 'deformed special relativity'. Strands thus predict the lack of dispersion, birefringence and opacity of the vacuum. All this agrees with observations [53].

Pr. 4 Strand crossings resemble fermionic or anti-commuting coordinates as used in supergravity, resemble non-commutative space [54, 55], resemble Clifford algebras, and even resemble the internal spaces of the *aikyon* approach based on octonions [56]. A strand crossing can also be seen as a four-dimensional subspace, spanned by the four angles describing the crossing geometry, specific to a point in background space; a crossing thus resembles twistor space [57]. Though strands resemble all these types of internal spaces, they do so only at *certain* points in space and at *certain* instants in time, as a result of the fluctuations. Figure 4 and Figure 5 thus imply that strands do *not* produce fixed internal spaces.

Pr. 5 The strand conjecture for the vacuum predicts the *lack of trans-Planckian effects.* If *any effect* due to space intervals smaller than the minimal length can be observed – for example in electric dipole moments [58], in higher order effects in quantum field theory, or in the discreteness of space – the strand conjecture is falsified. The same holds for time intervals shorter than the corrected Planck time.

As a consequence, strands also imply a *limit* to curvature κ:

$$\kappa \leq \frac{1}{l_{\mathrm{min}}} = \sqrt{\frac{c^3}{4G\hbar}} \ . \tag{19}$$

In particular, this limit implies the *lack of singularities* in nature, of any kind. Physical space is Riemannian in all observations.

X Strand predictions about black holes

For black holes, the fundamental principle of the strand conjecture leads to numerous consequences.

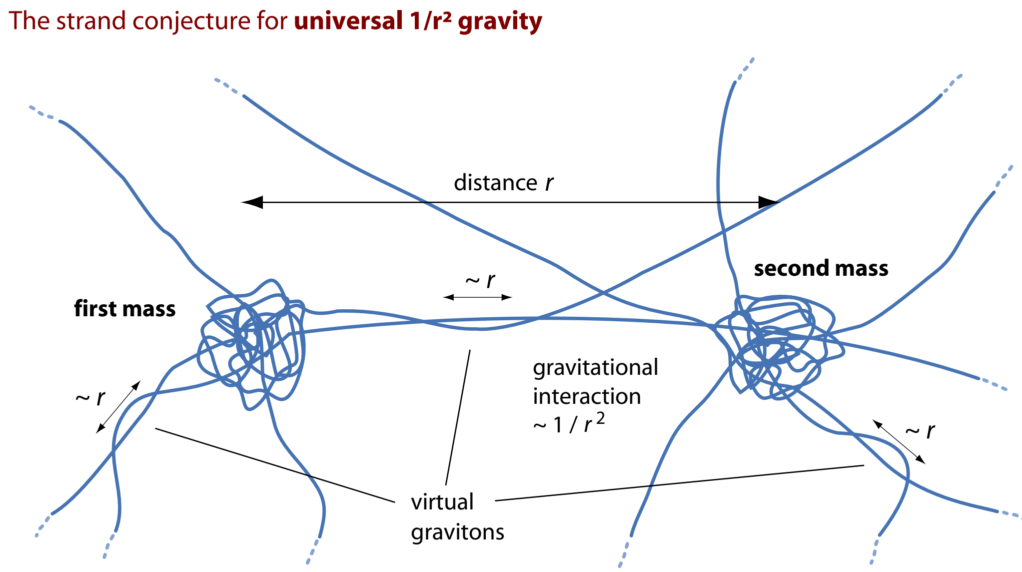

FIG. 7: Gravitational attraction results from strands. More precisely, everyday *gravitation* is due to tether pair twists and their influence on tether fluctuations. When speeds are low and spatial curvature is negligible, as illustrated here, twisted tether pairs – i.e., virtual gravitons – from any mass lead to a universal $1/r^2$ attraction of other masses. The average length of twisted pairs of tethers scales with r. As a consequence, the curvature around such a mass scales as $1/r^3$. These results are valid for infinite, approximately flat space. The strand structure of gravitons is illustrated in Figure 9. (The figure is taken from reference [12].)

Pr. 6 No black hole merger can exceed maximum power $c^5/4G$. Indeed, the highest luminosity values observed so far are those observed in black hole mergers by LIGO and VIRGO [31]. At present, the highest peak powers were observed for the events GW170729 and GW190521. They showed values of $4.2(1.5) \cdot 10^{49}$ W or 230 ± 80 solar masses per second [59] and of $3.7(9) \cdot 10^{49}$ W or 207 ± 50 solar masses per second [60]. All these values are well below the (corrected) Planck limit of $c^5/4G = 50\,756(12)$ solar masses per second.

Pr. 7 The strand conjecture implies that a *rotating* black hole realizes a belt trick that involves a *vast* number of tethers. Figure 8 shows such a configuration during rotation. Animations illustrating the process were available on the internet before the strand conjecture formulated this equivalence, programmed by Jason Hise. In the figure, the *ergosphere* is the spatial region in which the crossing switches take place during rotation. The figure does not show that the horizon of a rotating black hole is somewhat flattened at the poles.

Pr. 8 The strand conjecture for black holes illustrated in Figure 6 implies that the horizon entropy, the horizon energy and the horizon temperature are *limit values* for all physical systems of the same size. These limits arise directly from the corrected Planck limits of expressions (2) that define the strand conjecture. So far, they agree with observations.

In particular, because strands cannot be tighter – closer to each other – than in a horizon, the limit

$$\frac{m}{L} \leq \frac{c^2}{4G} \tag{20}$$

arises for every physical system of size L. The limit has a value of $3.3666(1) \cdot 10^{26}\mathrm{kg/m}$ or about $1/6$ of a solar mass per km. Equality is predicted to hold *only for black holes.* The strand conjecture thus naturally implies that, for a given mass value, black holes are the densest objects in nature. Strands thus illustrate and imply both the *hoop conjecture* and the *Penrose conjecture:* for a given mass, because of the minimum size of crossings, a spherical horizon – a tight weave – has the smallest possible diameter. Other possible weave shapes have larger size. This agrees with expectations.

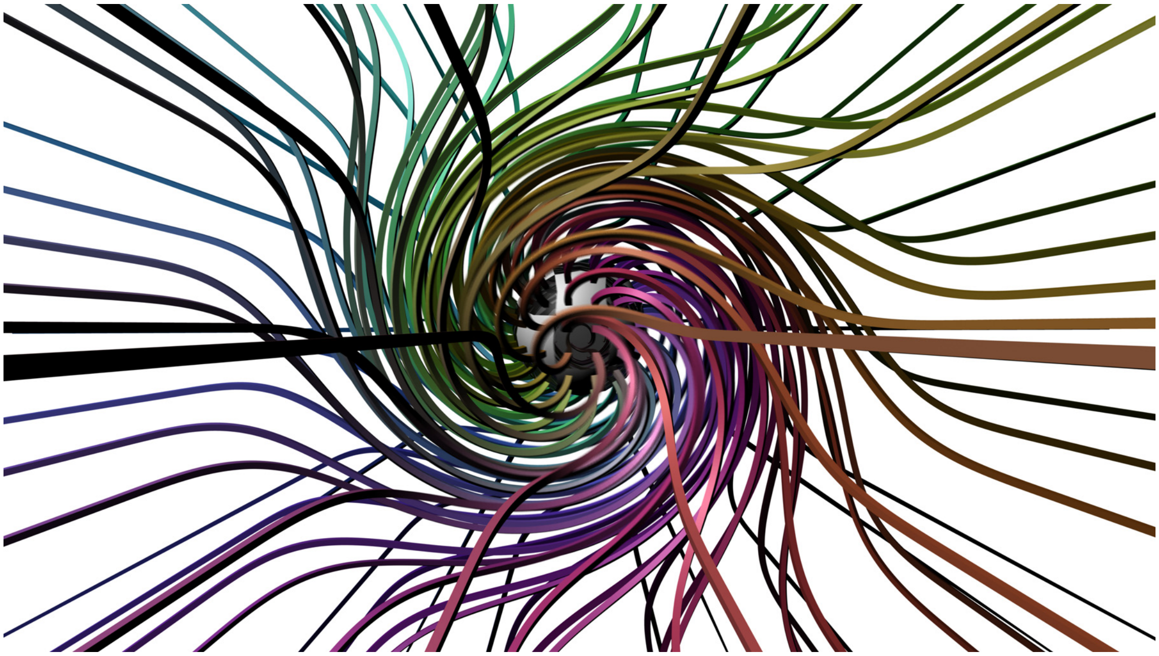

FIG. 8: The strand conjecture leads to an unusual model of a black hole rotating about the vertical axis (© Jason Hise). The flattening at the poles is not shown. For a complete animation, see the online videos at youtube.com/watch?v=LLw3BaliDUQ and youtube.com/watch?v=eR9ZCwYPhhU. The figure also confirms the moment of inertia of Schwarzschild black holes and the g-factor of rotating charged black holes (see text).

Pr. 9 The strand conjecture illustrated in Figure 6 implies that black holes *evaporate*. Through fluctuations, single strands or tangles of strands can detach from the horizon weave. The strand conjecture allows deducing several predictions about evaporation.

- First of all, the emission of particles will depend on the size of the black hole and on the tangling of the particle tangles, i.e., on particle mass values.
- For large black holes, the evaporation is a low probability process, and the evaporation rate of such a black hole is small. This agrees with expectations.

  To calculate evaporation rates for different particles, probabilities for corresponding untangling processes must be calculated. At present, no mathematical tools to do this appear to exist. However, it is expected that particles made of one strand (photons) are emitted more frequently than particles made of two or three strands (gravitons and fermions). This agrees with all calculations [61].
- The smaller the black hole, the higher the total luminosity, because strands detach with higher probability from a horizon with higher curvature.
- For small black holes, the curvature of the black hole facilitates the emission of massive particle tangles. The relative probability for the emission of massive particles in black hole radiation is predicted to *increase* for smaller black holes, because for small black holes, the curvature helps emission of massive tangles.
- Just before the completion of the evaporation process, black holes still radiate with a luminosity near but *below* the maximum possible value, the Planck power $c^5/4G$.

All these predictions agree with predictions made in the research literature decades ago [61].

Pr. 10 Black holes evaporate until the horizon weave has completely *dissolved* into separate strands or tangles. Strands predict *the lack of black hole remnants* that differ from usual elementary particles.

Pr. 11 Together with the strand description of black hole evaporation, strands predict and illustrate the lack of black holes with microscopic mass values. The corrected Planck limits for energy density, size, temperature and luminosity deduced above imply that all black holes obey

$$m > \sqrt{\frac{\hbar c}{4G}} \ , \tag{21}$$

thus have a mass that is *larger than the corrected Planck mass.* This agrees with observations and expectations.

Pr. 12 The weave model of horizons also implies that elementary particles, which are tangles – not weaves – are *not* black holes. This agrees with expectations and with observations.

Pr. 13 The strand conjecture automatically implies that the horizon area of a small black hole is *quantized* in multiples of the smallest area $4G\hbar/c^3$. This result was already deduced by Bekenstein [62]. However, strands also imply that area quantization of black holes is *not observable*, because in principle, no apparatus can have the sensitivity to detect this smallest area value. Such an apparatus would have to be able to count and thus to observe strands. This is impossible.

Pr. 14 The strand conjecture for black holes of Figure 6 and the statistical properties of their fluctuations also imply that white holes do *not exist.* For reasons of probability, evaporation cannot take place backwards. This agrees with observations.

Pr. 15 Because black hole horizons are weaves in the strand conjecture, black holes are predicted to have *no hair*, i.e., no nuclear charges, no baryon number, no lepton number or other quantum numbers. In previous papers [11, 15] it became clear that all these quantum numbers are topological properties of tangles. In the strand conjecture, these quantum numbers are not defined for horizons. All quantum numbers except electric charge – which is defined with the help of crossing or tangle chirality and is explored below – do not make sense for weaves. The *no-hair theorem* is thus natural in the strand conjecture.

It is ironic that the strand conjecture can also be seen as a way to describe particles and horizons *only* with the help of “hair”, if one uses “hair” as a synonym for “strand” or “tether”. Using this terminology, one could say that the “hair conjecture” implies the no-hair theorem.

Pr. 16 In the strand conjecture, horizons are tight, one-sided weaves. Any matter tangle that falls towards a horizon and is near it for a distant observer is flattened. As a result, at most one Planck mass can arrive at a horizon during a Planck time. Expressions (2) then yield the mass rate limit

$$\frac{\mathrm{d}m}{\mathrm{d}t} \leq \frac{c^3}{4G} \ . \tag{22}$$

This limit – again valid for any point in space – is also valid in general relativity – and in nature in general. So far, the limit, given by $1.00928(3) \cdot 10^{35}\,\mathrm{kg/s}$ or $50\,756(12)$ solar masses per second, is not violated by any observation – including black hole mergers. Also, numerical simulations of general relativity did not exceed the limit [37].

Pr. 17 In any physical system, strand crossings can be more or less tight, and switch more or less frequently. The limit case for a system of size R and energy E is the one with the *tightest* possible strands, as defined by the smallest length in expressions (2). This directly yields

$$\frac{2\pi}{\hbar c}\, ER \geq \frac{S}{k} \ . \tag{23}$$

This is *Bekenstein’s entropy bound.* The strand conjecture implies that equality is realized by horizons – and only by horizons – because horizons are the strand configurations that are as tight as possible and whose crossings switch as rapidly as possible. This agrees with expectations.

Pr. 18 The strand conjecture *limits* energy density (and pressure) to the (corrected) Planck value:

$$\frac{E}{V} \leq \frac{c^7}{16\, G^2 \hbar} = 2.8958(1) \cdot 10^{112}\,\mathrm{J/m^3} \ . \tag{24}$$

The energy density limit implies a lower limit for black hole size, for particle size and for the size of any localized system. Therefore, strands do *not allow mass singularities* in nature, neither dressed nor naked. Cosmic censorship is automatically realized in the strand conjecture. So far, both the density limit and the lack of mass singularities agree with observations.

Pr. 19 As explained above, the strand conjecture for black holes of Figure 6 allows a further conclusion. For observers at rest outside the black hole, the weave model of horizons implies that *nothing* can be observed behind the horizon. In simple terms, nothing is 'inside' a black hole horizon. In particular, strands imply the *lack of a tightly concentrated mass* inside a black hole.

Pr. 20 Strands imply that the mass of black holes is distributed over their horizon. Therefore, all black holes, including Schwarzschild black holes, have a finite *moment of inertia* I. Since strands reproduce general relativity, the moment of inertia of Schwarzschild black holes is given, as in general relativity, by the limit deduced for slowly rotating Kerr black holes, as determined by Raine [63] (page 68), by Ha [64, 65] and by Thorne et al. [66] (page 39):

$$I = MR^2 \ . \tag{25}$$

This result again disagrees with the idea that black hole mass is concentrated in a putative central singularity. Falsifying this value for the moment of inertia would falsify the strand conjecture.

The value of the moment of inertia is *larger* than that of a spherical mass shell, for which $I = 2MR^2/3$. The strand model visualizes the difference between a black hole and a mass shell in the following manner: Figure 8, showing the belt trick with a large number of belts, implies that every smallest surface on the horizon contributes the same number of crossing switches. Every smallest surface on the horizon thus contributes equally to the angular momentum, independently of its distance from the axis of rotation. Because mass is evenly distributed over the horizon, the total moment of inertia is $I = MR^2$.

Pr. 21 In the strand conjecture, *electric charge* is a result of the *chiral* linking of strands [11, 15]. Because horizons are weaves of strands, the electric charge Q of black hole horizons is *limited*. Strands visualize the limit directly.

A simple way to deduce the charge limit is to use the force limit $c^4/4G$. The electric force between two charged black holes must be smaller than the maximum force:

$$\frac{Q^2}{4\pi\epsilon_0 \, R^2} \leq \frac{c^4}{4G} \ . \tag{26}$$

Using the black hole relation $M = Rc^2/2G$ of equation (3), this can be rewritten as

$$\frac{Q^2}{4\pi\epsilon_0} \leq GM^2 \ , \tag{27}$$

which is the established limit for a Reissner-Nordström black hole. Finding an exception to the charge limit would falsify the strand conjecture. However, such an exception would also falsify maximum force and general relativity. Unfortunately, no observations that allow testing the region near the limit are available so far. In fact, it is expected that virtual pair production prevents such observations.

Pr. 22 Strands model black holes as weaves. Because strands model electric charge with crossing chirality, this implies that the electric charge of a black hole is distributed *over its surface*. The predicted charge distribution is consistent with the distribution of black hole *mass* mentioned above. Indeed, the strand conjecture implies that electric charge exists only for massive objects, and that charge and mass cannot be separated.

Pr. 23 Being weaves, black holes can be either non-rotating or rotating. For rotating black holes, the strands in the weave provide a limit to the angular momentum of the black hole. Angular momentum, like spin, is a result of strand crossing switches [11, 15]. In a rotating black hole, the weave rotates. Because the equatorial speed is limited by c, a maximal rotation frequency ω arises, with the value $\omega \leq c/R$. Using the limit $J \leq E/\omega$, this yields

$$J \leq \frac{2G}{c} M^2 \ . \tag{28}$$

As expected, a rotating weave behaves like a Kerr black hole [67]. A higher angular momentum would contradict the fundamental principle, and in particular the minimum time for crossing switches. So far, the angular momentum limit for extremal black holes agrees with observations [68].

Pr. 24 The *irreducible mass* of a rotating black hole is determined by the number of strands N_s that make it up. Strands thus predict that the *total* mass of a rotating black hole is a monotonous function of the irreducible mass and of its rotational energy, up to the angular momentum limit. This agrees with expectations and with observations.

Pr. 25 The description of rotating black holes or masses with strands also suggests that moving tethers describe what is usually called *frame dragging*. In the strand conjecture and in general relativity, frame dragging occurs around all rotating masses, at all distances, and independently of whether the mass is a black hole or not. Like all other observable effects, also frame dragging results from crossing switches.

Pr. 26 Strands also allow exploring black holes that are both charged and rotating – the Kerr-Newman case. In the strand conjecture, electric charge is due to the chirality of tangles [11, 15].

Strands imply that when a charged *black* hole rotates, the tethers move as shown in Figure 8. This motion implies and predicts that the g-factor for such black holes is

$$g = 2 \quad . \tag{29}$$

Strands make this prediction (at tree level in the case of elementary particle) for all rotating systems for which the crossings that generate mass and those that generate charge rotate together. In all these cases, the g-factor is 2 because of the belt trick [11, 15]. The value 2 for the g-factor of black holes agrees with the usual predictions [69–71]. The question whether the g-factor is exactly 2 or whether it shows corrections that depend on the fine structure constant α – especially in the case of maximally charged black holes – is still open. So far, however, no way to test these predictions appears to be possible.

Pr. 27 Strands allow expressing the results on rotating charged black holes with an additional limit. As deduced above, strands imply a charge limit for any black hole given by equation (27). The definition of the g-factor

$$\mu = g \frac{Q}{2M} J \tag{30}$$

implies, for $g = 2$, that

$$\frac{\mu}{J} = \frac{Q}{M} J \quad . \tag{31}$$

This means that

$$|\frac{\mu}{J}| \leq \sqrt{G}\sqrt{4\pi\epsilon_0} \quad . \tag{32}$$

Strands thus confirm the limit conjectured by Barrow and Gibbons [33]. So far, all observations and thought experiments agree with the limit.

In summary, the Barrow–Gibbons limit was derived from three strand properties: the horizon is a rotating weave; secondly, the electric charge, being due to chiral crossings, rotates with the mass; and thirdly, the crossings cannot rotate faster than the speed of light.

Pr. 28 Strands predict that black hole horizons fulfil the four laws of horizon mechanics [72]. The zeroth law, the constancy of surface gravity, is intrinsically valid in the weave model of horizons. The first law was deduced in Section VIII. The second law is realized by the detachment of strands during black hole evaporation. The third and last law is again automatic in the strand conjecture, as a result of the impossibility to reach the Planck limits. In fact, all four laws are built into the fundamental principle of the strand conjecture of Figure 3.

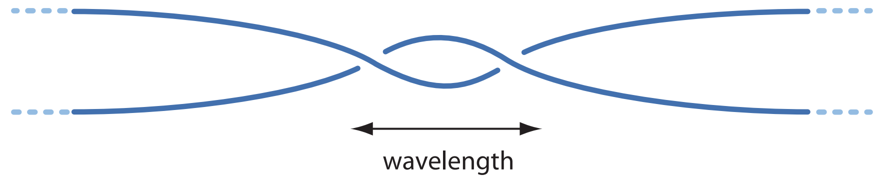

FIG. 9: The strand conjecture for the graviton is a twisted pair of strands. The configuration has spin 2, boson behaviour, and zero mass. A macroscopic gravitational wave is an ensemble of a large number of gravitons. (The figure is modified from reference [12].)

Pr. 29 Strands predict that black holes have *vanishing* magnetic charge, because strands, due to their extension, do not allow magnetic charge to exist [11]. This agrees with expectations and observations so far.

Pr. 30 Strands confirm that every horizon is a physical system that on the one hand can be seen as an extreme form of (curved) space, and on the other hand can be described as an extreme form of (falling) matter. Both points of view on horizons lead to tight, one-sided weaves as models for horizons. Horizons are thus systems at the border between space and matter. Alternatively, in the strand conjecture, *horizons are a mixture of space and matter.* This agrees with expectations.

Pr. 31 The thermodynamic properties of strand fluctuations in black holes have implications for the shape oscillations of horizons. Shape oscillations of black hole horizons increase (and decrease) the local curvature. This increases (and decreases) the local evaporation through radiation, i.e., through strand detachment. As a result, horizon shape oscillations are *damped* and disappear over time. This agrees with theoretical expectations.

Pr. 32 The strand conjecture for black holes illustrated in Figure 6 implies that for a distant observer at rest, horizons are not surfaces, but *thin cloudy volumes.* Strands thus imply that black hole horizons resemble stretched horizons. In contrast to an observer at rest, an observer *falling towards* and into the black hole experiences a three-dimensional strand network instead of an (almost) two-dimensional strand weave. The two descriptions can be transformed into each other with suitable deformations of the involved strands. The strand conjecture thus provides a model of a black hole that resembles a *'firewall'* [73] and a *'fuzzball'* [74, 75].

Pr. 33 The strand conjecture implies that black holes (with all their quantum properties) are *impossible in higher dimensions*, because higher dimensions *do not allow* forming stable weaves. Strands thus imply that black holes can be imagined in higher dimensions only if quantum effects are (at least partially) neglected. However, this statement is hard or even impossible to verify.

Pr. 34 The strand conjecture for black holes illustrated in Figure 6 suggests that black holes can *reflect* an incoming quantum particle, instead of swallowing it, but that the probability is *extremely* low: the incoming particle must have an energy so low that its wavelength is comparable to the size of the black hole. For such a low energy, the particle strands are similar in shape to vacuum strands, and the motion of the scattered particle around the black hole resembles the motion of vacuum strands around a traveling black hole. This low probability agrees with expectations [76].

XI Strand predictions about quantum gravity and gravitons

Strands predict the results of many quantum gravity experiments.

Pr. 35 Gravity is due to the exchange of virtual gravitons. The tangle model of the graviton, a twisted pair of strands that moves and rotates, is illustrated in Figure 9. Gravitons do surround masses: every Dirac trick generates twisted strand pairs. Strands also predict that gravitons have spin 2, because gravitons return to their original state after a rotation of the tangle core by π. Gravitons are predicted to be massless, because their core is not localized. Because cores can swap positions along the strands, gravitons are predicted to be bosons. As a result, coherent gravitons are predicted to yield gravitational waves with spin 2 and velocity c, as observed. Graviton exchange is also at the basis of universal $1/r^2$ gravity, as illustrated in Figure 7.

Pr. 36 In the strand conjecture, *single gravitons cannot be detected*, for two reasons. First, strands imply the indistinguishability between graviton observation from any other quantum fluctuation of or at a detector. Equivalently, in the strand conjecture, the absorption of a single graviton does not lead to a detectable particle emission. Secondly, even if gravitons were detectable, in the strand conjecture, they have an extremely small cross section, of the order of the square of the Planck length. The low cross section is due to the graviton tangle shown in Figure 9. The tangle implies a low detection probability, as expected [77, 78].

Pr. 37 Strands – and expressions (2) – imply that the gravitational constant G *does not run* with energy. In the language of perturbative quantum field theory, strands imply that G is *not renormalized*. This prediction agrees with expectations [79, 80] and with observations, though the available data is sparse and many opposite views exist.

Pr. 38 Strands imply that quantum superposition effects for gravitational systems are unobservable, because the exchange of graviton destroys entanglement. This is as expected [81].

In particular, strands imply that in a *double-slit experiment* with quantum particles, the particles pass both slits at the same time; the particle core splits in two pieces during passage – though in different fractions at every passage. Therefore strands predict that the gravitational field of a quantum particle arises at both slits, at every passage, though each time in different fractions.

Pr. 39 The impossibility to detect single gravitons implies the *lack* of unknown, observable *quantum corrections* to general relativity. Equivalently, strands predict the lack of observable quantum effects in *semiclassical gravity.*

So far, these predictions are not contradicted by any observation.

XII Strand predictions about elementary particle masses

The strand conjecture reproduces known physics. In addition, new results are possible [11, 15]. This section just mentions a few results about elementary particle masses.

Given that black holes are made of large numbers of woven strands, it is natural to assume that elementary particles are made of *a few* woven strands. Indeed, in the strand conjecture, all elementary particles are *rational tangles* – i.e., woven, unknotted tangles – made of one, two or three strands [11, 15]. Tangles made of four or more strands are composed, not elementary. An example of an elementary rational tangle is shown in Figure 10.

Among tangles made of a few strands, those made of one strand are bosons; more precisely, they are photons. Massive elementary particles tangles are made of two or three strands. Every fermion tangle, being a tethered structure that is tangled in the region of its tangle core, has non-vanishing mass. Every fermion tangle reproduces spin 1/2 behaviour under rotations – using Dirac's belt trick – and fermion behaviour under the exchange of positions of tangle cores. All tangles reproduce the gauge groups U(1), SU(2) and SU(3) as the result of Reidemeister moves on their tangled cores.

Only *rational* tangles – i.e., tangles that arise through the motion or braiding of their tethers – allow reproducing the transformation of particles observed in experiments. And only rational tangles allow a classification into a finite number of families that correspond to the observed elementary particles. These arguments are worked out in detail in references [11] and [15].

Mass is the property of tangles that creates virtual gravitons around them. Mass is also given by action per time, divided by c^2. This implies:

▷ The *particle mass* (in corrected Planck units) is the probability of strand crossing switches occurring, per Planck time, in spontaneous belt tricks of the particle tangle.

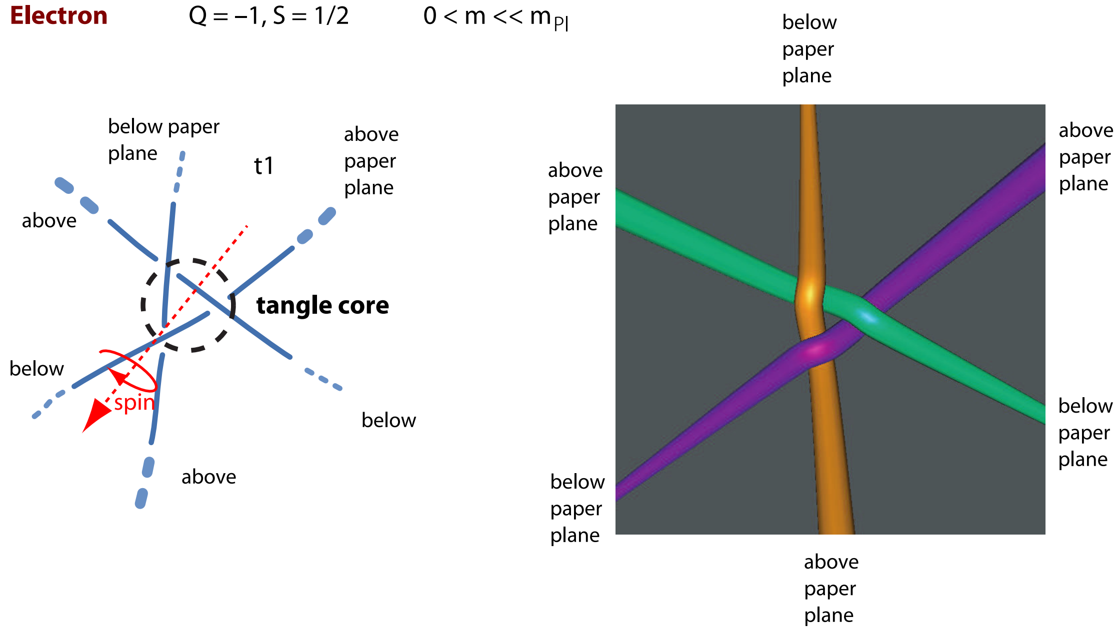

FIG. 10: In the strand conjecture, elementary particles are modelled as rational tangles of strands. Tangles are called *rational* when they are formed by switching tethers. Tethers and strand segments are unobservable; only crossing switches are observable. Due to their fluctuations, tangles lead to the observation of particles that are localized in the region of the tangle core. Only rational tangles model the observed behaviour of elementary particles. Antiparticles are mirror tangles. The chirality determines the electric charge. Fermion tangles, such as the electron tangle shown in the figure, automatically have spin 1/2. Spin is core rotation. Wave functions are washed out tangle crossings; probability densities are washed out crossing switches. (The figure is modified from reference [12].)

Rational tangles directly allow deducing a number of predictions about mass values of elementary particles.

Pr. 40 Strands promise, through the analogy between thermodynamic effects and gravitational attraction, to allow calculating the gravitational mass of quantum particles. The value of gravitational mass is predicted to depend on the geometric *tangle shape* of the particle – and on nothing else.

Since particle mass is due to their (tight) tangle shape, the mass values of all elementary particles are predicted to be zero or positive, discrete but not quantized, equal to that of their antiparticles, fixed, unique, calculable and constant in time and space. This agrees with data. *If particle masses would be found to vary over space or time, the strand conjecture would be falsified.*

In the strand conjecture, as shown in references [11, 15], only particles with positive mass can have electric and weak charge. In addition, it was shown that all mass values are due to Yukwawa coupling to the Higgs. Only those particles that couple to the Higgs are observed to be massive. All this agrees with observation.

Pr. 41 The tangle model of elementary particles implies that both the gravitational and the inertial mass of elementary particles are due to tether fluctuations. *Gravitational* mass describes the virtual gravitons around a mass: they arise in the tethers due to the belt trick. *Inertial* mass describes how a rotating mass advances through the vacuum with the belt trick, as described by Battey-Pratt and Racey [16] and many others after them. In the strand conjecture, these two processes are exactly the same: both involve tether fluctuations around the core, and in particular, both involve the belt trick. Therefore, inertial and gravitational mass are equal – for infinite, flat space. Strands thus imply that the *equivalence principle* holds, in its weak and strong forms – at least for sub-galactic scales, when there is no effect of the cosmological horizon. This agrees with observations [82].

Pr. 42 Strands imply that elementary particle mass values *run* with four-momentum. The reason is that the tangles completely reproduce quantum field theory, and that elementary particles are surrounded by virtual particle pairs; thus their mass values run with four-momentum. This agrees with observations – e.g., [83] – and expectations.

Pr. 43 Because spontaneous tangle fluctuations leading to the belt trick are *rare*, the gravitational mass m of elementary particles is to be much smaller than the corrected Planck mass:

$$0 < m \ll \sqrt{\hbar c/4G} \ . \tag{33}$$

This inequality agrees with observations and agrees with old arguments [84]. Strands thus provide a general answer to the *mass hierarchy* problem.

Pr. 44 Strands imply that falling particles are fluctuating and diffusing tangles. This implies that *more complex* particle tangles have *higher* gravitational mass (for equal number of tethers). The same connection has already been deduced for inertial mass in a different way [11]. The connection yields the correct mass sequences for all hadrons and predicts normal mass ordering for neutrinos. *If neutrino masses would* not *obey normal ordering, the strand conjecture would be falsified.*

The tangle model also explains that neutrinos mix and that their mass values are stable under renormalization, as shown in references [11, 15]. Strands thus allow non-vanishing neutrino mass in the standard model of particle physics. More estimates about fermion masses using the tangle model are found in reference [12].

More precise strand estimates of particle masses require the development of better approximations and of suitable computer simulation programs. *The failure to reproduce the correct mass value of a single particle, at any single energy value, would falsify the strand conjecture.*

XIII Conclusions

Thinking about nature as made of strands is unusual. It is also unusual to describe every physical process as a combination of fundamental events due to strand crossing switches. This unusual model for black hole horizons provides no new or unexpected results about black holes: strands predict all their classical and quantum properties. Due to this agreement, the strand model of black holes is interesting only for one reason.

Strands also allow deducing quantum field theory, the spectrum of forces, and the spectrum of elementary particles. In fact, strands predict the lack of any deviation from the standard model of particle physics, with massive neutrinos with PMNS mixing, as argued in detail elsewhere [11, 15]. As a new result,

Pr. 45 Strands predict that the mass values of elementary particles, their coupling constants and their mixing angles, including their running with energy, can be calculated ab initio from tangle geometry.

Only a few predictions on particle masses have been given here; more predictions about mass and quantum gravity are found elsewhere [12]. As long as the predictions due to the strand conjecture are not falsified, the conjecture remains a candidate for a complete description of nature.

Acknowledgments and declarations

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