Testing a conjecture on the origin of space, gravitation and mass

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Abstract

A Planck-scale model for the microscopic degrees of freedom of space and gravitation is explored in detail. The conjectured model is based on a single fundamental principle that involves fluctuating one-dimensional strands. First, the fundamental principle is used to deduce classical and quantum properties of space and gravitation, from the field equations of general relativity to gravitons. Secondly, the fundamental principle is used to deduce a detailed list of predictions. They include the lack of any change to general relativity at all sub-galactic scales, the validity of black hole thermodynamics, the lack of singularities, the lack of quantum foam, the lack of unknown observable quantum gravity effects, the maximum local momentum flow or force $c^4/4G$, and the maximum local luminosity or power $c^5/4G$. All predictions are tested against experiments and are found to agree with observations so far. Finally, it is shown that the fundamental principle implies a model for elementary particles that allows deducing ab-initio upper and lower limits for their gravitational mass values.

Keywords: general relativity; quantum gravity; particle mass; strand conjecture.

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1 The quest to uncover the microscopic aspects of space and gravitation

The nature of physical space and gravitation remains a matter of intense research. What are the microscopic degrees of freedom of black holes, the microscopic nature of the vacuum, and the microscopic details of curvature? Many possibilities have been proposed and explored [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The so-called *strand conjecture* proposes a microscopic model for black holes, particles, space and gravity that is based on one-dimensional fluctuating constituents that are called *strands*. The model is based on a single fundamental principle that describes nature at the Planck scale.

In order to show that any proposed microscopic degrees of freedom are candidates for a description of nature, it is necessary to show that they *reproduce* space, curvature, mass and gravitation in all their macroscopic and microscopic aspects. But this requirement is not sufficient. It is also necessary to show that strands provide *additional* results about gravitation that go beyond the usual description of space as a continuous manifold made of points. As many predictions as possible should be derived, and the proposed tests should be as strict as possible. This is the aim of the present work.

It will appear that the strand conjecture agrees with all observations about general relativity and quantum gravity at all sub-galactic scales. Strands provide a model for elementary particles and their gauge interactions, and suggest a way to estimate their mass values.

2 The origin of the strand conjecture

When Max Planck discovered the quantum of action $\hbar$ in 1899, he found the underlying quantity that explains the observation of all quantum effects in nature [13]. Bohr described quantum theory as consequence of the minimum observable action value $\hbar$ [14]. Heisenberg introduced the canonical commutation relation and Schrödinger introduced the wave function. Pauli included spin and Dirac the maximum energy speed c. From around 1929 onwards, Dirac regularly made use of the so-called *string trick* or *belt trick* in his lectures. The trick, illustrated below in Figure 11, assumes that fermions are connected to spatial infinity by tethers that are unobservable, but whose crossings are observable. With help of the trick, Dirac used to describe spin $1/2$ behaviour as result of tethered rotation. Nevertheless, he never published anything about this connection. Answering a letter from Gardner, Dirac wrote that the trick demonstrates that angular momenta below $\hbar/2$ are not possible in nature [15]. A smallest angular momentum $\hbar/2$ still implies a smallest observable action value $\hbar$.

Historically, tethers were the first hint that nature might be built from unobservable extended constituents. It took several decades to understand that also the complete Dirac equation could be deduced from unobservable tethers. This was first achieved by Battey-Pratt and Racey in 1980 [16]. Independently, in 1987, Kauffman conjectured a direct relation between the canonical commutation relation – and thus Planck's constant $\hbar$ – and a crossing switch of tethers [17]. Again, without stating so explicitly, the assumption was that tethers are unobservable, whereas their crossings are. In the early twenty-first century, independently of the work by Battey-Pratt and Racey and of that by Kauffman, Dirac's trick again led to the discovery of the relation between crossing switches of unobservable tethers, $\hbar$, wave functions, and the Dirac equation [18]. In short, Dirac's trick implies Dirac's equation. It thus appeared that *every quantum effect* can be thought as being due to unobservable extended constituents whose crossings are observable. Because the term 'string' had acquired a different meaning in the meantime, and because the term 'tether' does not describe the full scope of the involved extended constituents, the alternative term *strand* appeared more appropriate.

A question arises naturally: can unobservable strands also explain gravity? The finite value of black hole entropy and its surface dependence provided first hints [19, 20]. Indeed, it turns out that both the properties of black holes and Einstein's field equations can be deduced from crossing switches of unobservable strands. This deduction is repeated below. It thus appeared that *every gravitational effect* can be thought as being due to unobservable extended constituents whose crossings are observable.

The strand conjecture for fundamental physics appears promising also from another perspective. The central parts of quantum field theory can be summarized by the statements that all observable action values W obey

$$W \geq \hbar$$

and that all observable energy speeds v obey

$$v \leq c \quad .$$

General relativity can be summarized by the statement that all observable power values P obey

$$P \leq c^5/4G \quad ,$$

as discussed in various publications [21, 22, 23, 24]. These three limit statements based on Planck units imply several consequences. First, all Planck units are invariant and universal *limits* that encode fundamental aspects of nature. Second, all equations of motion, starting with Dirac's equation and Einstein's field equations, *follow* from the Planck units. Third, at Plank scales, physics is fundamentally *simple*, being described by limit statements. The three inequalities imply and contain a large part of modern physics. Finally, at Plank scales, a description of nature that makes only use of algebra and combinatorics appears possible. In other words,

▷ Planck units suggest the possibility of a complete and unified description of motion based on inequalities.

Planck units suggest that nature is fundamentally simple. All these ideas are realized by the strand description of nature.

The strand conjecture also provides a way to describe nature's processes as built of fundamental events. Such an approach is in line with the conviction of Veltman [25] that Feynman diagrams are more fundamental than the Lagrangian. Building observations from fundamental events has

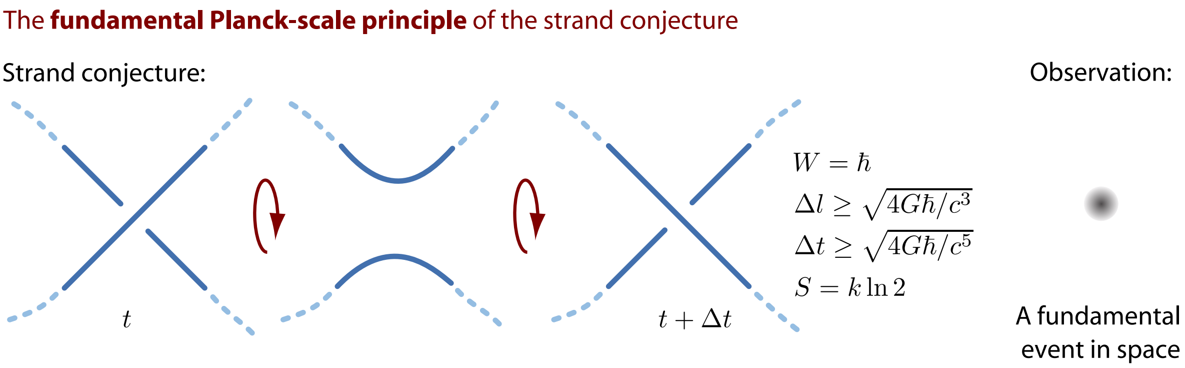

Figure 1: The fundamental principle of the strand conjecture – a simplified version of Dirac's trick of Figure 11 at the Planck scale – specifies the simplest observation possible in nature: the almost point-like *fundamental event* results from a *skew strand switch*, or *crossing switch*, at a position in three-dimensional space. The strands themselves are *not observable*. They are impenetrable and are best imagined as having Planck size radius. The observable switch defines the action unit $\hbar$. The double Planck length limit and the double Planck time limit arise, respectively, from the smallest and from the fastest crossing switch possible. The paper plane represents background space, i.e., the local tangent Euclidean space defined by the observer.

also been explored by Krugly [26]. This view of nature is also related to the extensive work based on causal sets [27].

Finally, the strand conjecture is also related to the growing interest in qubits: a skew crossing of two strands provides a simple implementation of a qubit. Describing all of nature with strands is thus one specific way of describing all of nature with qubits [28, 29, 30].

3 The strand conjecture and its fundamental principle

The strand conjecture states: all physical systems found in nature – matter, radiation, space and horizons – are made of strands that fluctuate at the Planck scale but remain unobservable.

▷ A *strand* is defined as smooth curved line – a one-dimensional, open, continuous, everywhere infinitely differentiable subset of $\mathbb{R}^3$ or of a curved 3-dimensional Riemannian space, with trivial topology and without endpoints – that is surrounded by a perpendicular disk of Planck radius $\sqrt{\hbar G/c^3}$ at each point of the line, and whose shape is randomly fluctuating over time.

The strand conjecture is then formulated in the following way:

▷ Strands are unobservable. However, crossing switches of skew strands – exchanges of over- and underpass – are observable. Crossing switches determine the Planck units

G, c and $\hbar$; this fundamental principle is illustrated in Figure 1.

The defining Figure 1 thus combines the central point of Dirac's trick with the Planck limits. This fundamental principle implies:

▷ Physical space is a (three-dimensional) *network* of fluctuating strands – i.e., of strands that are neither woven nor tangled nor knotted, as illustrated in Figure 2).

▷ Horizons are (two-dimensional) *weaves* of fluctuating strands – i.e., similar to a fabric made of woven threads, and illustrated in Figure 3.

▷ Particles are (localized) *rational tangles* of fluctuating strands – using the term from topological knot theory, defined and illustrated in Figure 10.

▷ Physical motion *minimizes* the number of observable crossing switches of fluctuating unobservable strands.

Using Figure 1, the strand conjecture appears to imply general relativity, fermions, bosons and the gauge interactions – with all their observed properties. The figure also illustrates the most fundamental event and the most fundamental process in nature, from which all other processes are built, including all motion in nature. The following sections check these claims in detail for gravitation at sub-galactic scales. After all the checks are passed successfully, a number of new results are presented.

The implications of the strand conjecture for particle physics, gauge interactions, and the standard model have been explored elsewhere [18, 31]. Tangles of strands imply the particle spectrum, Reidemeister moves imply the three gauge groups and couplings, and tether exchanges imply particle mixings. As a result, the complete Lagrangian of the modern standard model follows from strands. In the following, however, only gravitation is explored.

The fundamental principle of the strand conjecture states that action, length, time and entropy are *limited from below*:

$$W \geq \hbar \ , \ \ \Delta l \geq \sqrt{4G\hbar/c^3} \ , \ \ \Delta t \geq \sqrt{4G\hbar/c^5} \ , \ \ S \geq k \ln 2 \ . \tag{1}$$

Strands visualize these inequalities. In fact, these inequalities – describing the minimum possible action, the minimum length, the minimum time, and the minimum entropy – together with Figure 1, contain everything that is needed to deduce the rest of the present work. One notes that the minimum length and time are given by *twice* the Planck values. The number $\ln 2$ in the minimum entropy is due to the 2 strand configurations, which resemble the two orientations of a qubit.

Apart from their crossings in space – a strand segment passing over another – strands have *no* observable properties. Strands have no colour, no tension, no mass, and no energy. Due to the impossibility of observing strands, strands have no meaningful equation of motion. Indeed, all results in the following are *independent* of the detailed fluctuating motion one might imagine for strands – as long as crossing switches reproduce observations. This important aspect eliminates any apparent arbitrariness of the description of space, horizons and matter with fluctuating strands.

Strands cannot be cut; they are not made of parts. Strands cannot interpenetrate; they *never* form an actual crossing. When the term 'crossing' is used in the present context, only the two-dimensional projection shows a crossing. In three dimensions, strands are *always at a distance*. In particular, a crossing *switch* – the change from an overpass to an underpass – cannot arise through strand interpenetration, but only via strand deformation. These aspects are also implied in Dirac's trick.

In the strand conjecture, *all physical observables* – action, momentum, energy, mass, velocity, length, surface, volume, tension, entropy, field intensities, quantum numbers, etc. – arise from combinations of crossing switches. No physical observable is a property of strands themselves; all physical observables arise from *shape configurations* of *several* strands. In more fashionable wording: all physical observables *emerge* from strand crossings. In order to visualize the minimum length in nature, it is easiest to visualize strands as having Planck-size radius.

4 A structured exploration of gravitation from strands

The present work explores gravity at sub-galactic scales. It first derives *space* from the fundamental principle of strands, then continues with the derivation of *horizons* and the derivation of the *field equations* and of *gravitons*. Then the consequences and testable *predictions* of crossing switches in the domains of space and gravitation are explored. This is done comprehensively: all the tests of general relativity and of quantum gravity found in the research literature are covered. Special emphasis is put on the properties of black holes, because they include the most extreme field configurations and curvature values. As a result, black holes allow the most compelling tests of any description of gravitation, and thus of the strand conjecture. Because all predictions from the strand conjecture follow from the fundamental principle, falsifying *just one* specific prediction automatically falsifies the *whole* conjecture.

Appendix A discusses the circularity of the fundamental principle. Appendix B briefly summarizes how crossing switches of fluctuating strands lead to quantum theory and elementary particles.

Strand *cosmology* is not explored in the present work, with small exceptions. In cosmology, the strand conjecture is completed by the statement that nature consists of a *single strand.* A typical strand segment comes from the cosmological horizon, is tangled into some particle, and returns to the horizon; there it follows the horizon surface, enters the interior again at another position, is tangled into some other particle in the interior, and returns back to the horizon; this occurs a huge number of times. Strand cosmology implies numerous testable predictions about cosmological observables, dark energy and dark matter. The deduced predictions and their tests will be explored in a subsequent paper.

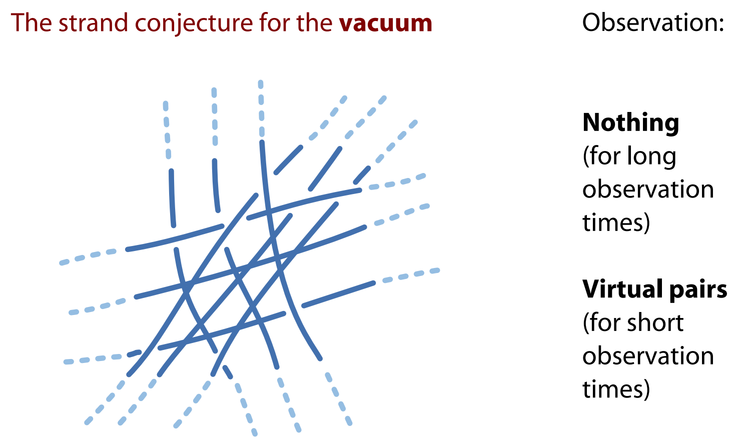

Figure 2: A simplified and idealized illustration of the strand conjecture for a flat vacuum, i.e. for flat physical space. The space of the picture is *background* space. *Physical* space is generated by strand crossings. Strands fluctuate in all directions. (Typical strand distances are many orders of magnitude larger than their diameters.) For sufficiently long time scales, the lack of crossing switches leads to a vanishing energy density; for short time scales, particle–antiparticle pairs, i.e., rational tangle–antitangle pairs, arise in the vacuum due to the shape fluctuations of the strands, as illustrated in Figure 14 below. The difference between background space and physical space is discussed in Appendix A.

5 Deducing physical space from strands

Because strands are unobservable, it is *not* possible to describe them with differential equations. Because strands are unobservable, it is *not* possible to speak about their motion or their dynamics. The *only* observable aspect of strands – as in Dirac's trick – are their crossing switches, and thus, for example, the distribution of crossing switches. To relate strands to physics, it is important to deduce the behaviour of crossing switches from the fundamental principle. This is done now, starting with physical space.

In the strand conjecture, a *network* of fluctuating strands is conjectured to yield *physical* flat empty space. A strand network is illustrated in Figure 2. The picture uses background space to define physical space. *Background* space is what is needed to *talk* about nature. *Physical* space is everything that can be *measured* about space: curvature, vacuum energy, entropy, temperature etc. The circularity issues that arise are discussed in Appendix A.

A network of *untangled*, *unwoven* and *unknotted* strands models *empty and flat* physical space. The time-average of the fluctuations, on a scale of a few Planck times or more, yields three-dimensional physical flat space, including its continuity, homogeneity, isotropy and Lorentz-invariance. On sufficiently long time scales, there are (on average) *no* crossing switches, and thus neither matter nor energy – just empty space. Strands thus imply that *no deviation* from the continuity, homogeneity, isotropy, dimensionality and Lorentz-invariance of (physical) flat space can be observed – at any energy – *despite* the existence of a smallest length $l_{\rm min} = \sqrt{4G\hbar/c^3}$.

It is important to note that the connection between physical space and strand networks is not arbitrary, but *inescapable*. Only strand *networks* have the properties that space has: extension, three dimensions and lack of matter. Below, in Section 11, a number of falsifiable predictions are deduced from this connection.

Strands imply that in contrast to the highest speed, to the smallest action and to the highest force, *the smallest length cannot be observed.* In nature, light realizes c, atomic processes realize $\hbar$, and black holes realize $c^4/4G$. Strands imply, that in contrast, *no* physical system or process can realize the smallest length. To measure the smallest length, a single strand would have to be observed; this is impossible. For the same reason, also the smallest time interval $t_{\rm min} = \sqrt{4G\hbar/c^5}$ cannot be observed.

In the strand conjecture, rational, i.e. unknotted *tangles* of fluctuating strands define elementary particles and explain their quantum behaviour, as summarized in Appendix B. Fluctuations of the vacuum strands sometimes lead to the formation of short-lived rational tangle–antitangle pairs, as illustrated in Figure 14. These tangle pairs model virtual particle–antiparticle pairs. They arise in vacuum at short time scales.

In summary, vacuum is conjectured to be a consequence of fluctuating strands. The fundamental principle suggests that, microscopically, vacuum is a three-dimensional *network* of distant and fluctuating strands. In contrast to other proposals, the strand conjecture implies that space has the *same* number of dimensions and the *same* topology both at macroscopic scales and at Planck scales. The apparent circularity of the fundamental principle is discussed in Appendix A. The strand conjecture thus implies that in the absence of gravity, (local) Minkowski space, i.e., (local) flat space-time, arises at *all* measurable scales and energies, down to Planck scales. Strands thus predict that up to Planck energy, physical space does *not* change.

6 Deducing horizons and black holes from strands

This section shows that the fundamental principle for strands explains both horizons and black holes, and that strands allow deducing black hole thermodynamics in a straightforward way.

In the strand conjecture, *woven* fluctuating strands define horizons. More precisely, the strand conjecture implies that

▷ Horizons are one-sided, tight *weaves*.

In this statement, *one-sided* means that all strands leave the horizon on one side, the side of the observer. One-sidedness means that there is ‘nothing’, not even an unobservable strand, on the other side of the horizon. A schematic illustration of a Schwarzschild black hole, shown both as a cross section and as a top view for a distant observer at rest, is given in Figure 3. For a black hole, and for any other horizon, all strands come in from far away, are *woven* into the horizon, and leave

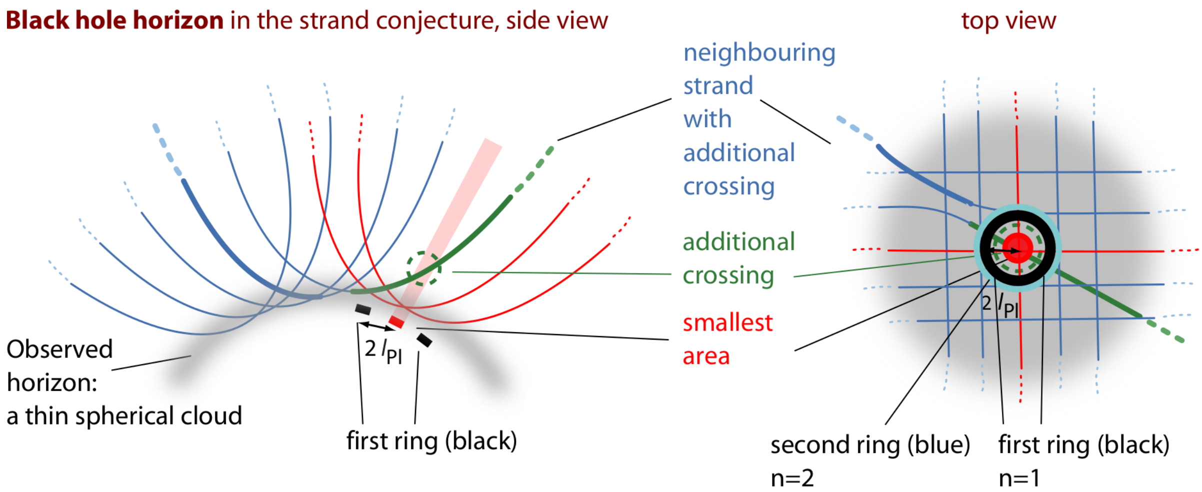

Figure 3: The strand conjecture illustrated for a Schwarzschild black hole, as seen by a distant observer at rest: the horizon is a cloudy or fuzzy surface produced by the crossing switches of the strands woven *tightly* into it. Due to the additional crossings *above* the horizon, the number of microstates per smallest area is larger than 2, and given by the base e of the natural logarithms (see text). This yields the entropy of black holes.

again to far away. If strands are imagined as having Planck radius, the weave of strands forming a horizon is as *tight* as possible: seen from above, there is one crossing for each smallest area.

At a larger scale, a weave becomes a two-dimensional surface. For a distant observer at rest, a one-sided weave also implies that no space and no events are observable behind it. The weave thus acts as a *limit* to observation. For a falling observer, the strands do not form a weave, but continue on the other side and form a (distorted) network, i.e., curved vacuum. Such an observer does not notice anything special when approaching the horizon, as seen by an observer at spatial infinity, or when crossing it, in its own reference frame. A one-sided weave thus shows the qualitative properties that characterize a horizon.

The strand conjecture for horizons allows determining the *energy* and thus the *mass* of a spherical, non-rotating horizon. Energy E has the dimension action per time. Because every crossing switch is associated with an action $\hbar$, the horizon energy is found by determining the number N_{cs} of crossing switches, multiplied by $\hbar$, that occur per unit time. This number will depend on the surface area of the horizon. In a horizon, crossing switches *propagate* from one crossing to the next, over the surface of the whole (tight) weave. Since the horizon weave is *tight*, the propagation speed is one smallest crossing per shortest switch time: switch propagation thus occurs at the speed of light c. Since the horizon weave is *tight*, each crossings has the size of the minimum length squared, given by $A_{cPl} = 4G\hbar/c^3$. In the time T needed to circumnavigate a *spherical*, non-rotating horizon of area $A = 4\pi R^2$ at the speed of light, *all* crossings of the horizon switch.

This yields:

$$E = \frac{N_{\mathrm{cs}}\hbar}{T} = \frac{A/(4G/c^3)}{2\pi R/c} = \frac{c^4}{2G}\, R \ . \tag{2}$$

The woven strand model of a horizon thus reproduces the relation between the energy – or mass – and the radius of a Schwarzschild black hole.

Strands also determine the number of microstates per horizon area. Figure 3 shows that for a smallest area on the horizon, i.e., for an area that contains just one strand crossing, the effective number N of microstates *above* that smallest area is *larger* than 2. The number would be two if each smallest area would contain just one crossing, with its 2 possible signs. However, a number larger than 2 occurs because also fluctuating neighbouring strands sometimes *cross above* that smallest area. It will be shown now that these additional crossings lead to an average number of microstates for each smallest area given by $\mathrm{e} = 2.718281...$

The probability for a neighbouring strand to cross above a given (central) smallest area will depend on the distance at which the neighbouring strand leaves the lowest woven layer of the horizon. To calculate the probability, one imagines the central crossing surrounded by an infinite series of rings, each with a smallest area value $A_{\mathrm{cPl}} = 4\,G\hbar/c^3$. As illustrated in Figure 3, the rings are numbered with a number n. The central crossing corresponds to $n = 0$. Ring number n therefore *encloses* n times the smallest area A_{cPl}. The probability that a strand from ring 1 reaches the centre, forming an additional crossing above it, is

$$p_1 = \frac{1}{2} = \frac{1}{2!} \ . \tag{3}$$

The probability that a strand from ring n reaches the centre and forms an additional crossing is

$$p_n = \frac{1}{n+1}\, p_{n-1} = \frac{1}{(n+1)!} \ , \tag{4}$$

because the strand has to continue in the correct direction above every ring on its way to the centre. This expression is a result of the *extension* of strands; it would not arise if the fundamental constituents of the horizon would not be extended – in short, if they would not be strands. The expression yields an effective number N of microstates above the central crossing given by

$$N = 2 \ + \ \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ... + \frac{1}{n!} + ... = \mathrm{e} = 2.718281... \tag{5}$$

In this expression, the term 2 is due to the two options at the central point; the term $1/2!$ arises from the first ring around it, as shown in Figure 3; the following terms are due to the subsequent rings. Expression (5) implies that the average number N of strand microstates for each smallest area, i.e., for each *corrected Planck area* $A_{\mathrm{cPl}} = 4\,G\hbar/c^3$ on the black hole horizon, is given by $N = \mathrm{e}$. In the strand conjecture, every corrected Planck area therefore contains *more* than 1 bit of information (which would correspond to $N = 2$).

The calculation of the entropy of the complete black hole horizon starts with the usual definition

$$S = k \ \ln N_{\mathrm{total}} \ , \tag{6}$$

where k is the Boltzmann constant and N_{total} is total number of microstates of the complete horizon. Because the full horizon area A can be seen as composed of *many* corrected Planck areas, the total number of microstates is given by the product of the number of states for every corrected Planck area:

$$N_{\text{total}} = N^{A/(4\,G\hbar/c^3)} \quad . \tag{7}$$

So far, only standard thermodynamics was used. The next step is to insert the result (5) due to strands. This yields

$$N_{\text{total}} = \mathrm{e}^{A/(4\,G\hbar/c^3)} \quad . \tag{8}$$

This total number of horizon microstates can then be inserted into expression (6) for the entropy. The horizon entropy S of a black hole with surface A is then given by

$$\frac{S}{k} = \frac{A}{4\,G\hbar/c^3} \quad . \tag{9}$$

This is the expression discovered by Bekenstein [19]. Strands provide a number of insights into the expression of black hole entropy.

In the strand conjecture, the *finiteness* of the entropy is thus due to the *discreteness* of the microscopic degrees of freedom. Strands also imply that both the *surface dependence* of the entropy and the factor $1/4$ – including the lack of factors like $\ln 2$ or a Barbero-Immirzi parameter – are due to the *extension* of the microscopic degrees of freedom.

As Figure 3 illustrates, strands further imply that horizon entropy is located *at and slightly above* the horizon. This agrees with expectations.

The strand conjecture for black holes also confirms and visualizes a result by Zurek and Thorne from 1985: the entropy of a black hole is the logarithm of the number of ways in which it could have been made [32].

In the strand conjecture, the above calculation of the black hole entropy counts certain states more than once. Because strands can bend, reorienting the complete horizon sphere does not produce a different micro-state. The possible orientations of a sphere are given by the possible orientations of the poles and by the possible orientations of the sphere around the pole axis. The poles of the sphere can point to any of the A/A_{cPl} minimal surfaces that make up the horizon; in addition, the sphere can be rotated around the axis in $\sqrt{A/A_{\text{cPl}}} \cdot \mathcal{O}(1)$ ways. The corrected value for the number of microstates of a spherical horizon is therefore

$$N_{\text{total}} = \frac{N^{A/A_{\text{cPl}}}}{(A/A_{\text{cPl}})^{3/2} \cdot \mathcal{O}(1)} \quad . \tag{10}$$

This value yields the corrected black hole entropy

$$\frac{S}{k} = \frac{A}{4\,G\hbar/c^3} - \frac{3}{2}\,\ln\frac{A\,c^3}{4G\hbar} - \ln\mathcal{O}(1) \quad . \tag{11}$$

The last term is negligibly small. The second term shows that the strand conjecture makes a specific prediction for the logarithmic correction to the entropy of a Schwarzschild black hole.

The value of the correction is much too small to ever be tested in experiments; but it agrees with previous calculations [33].

In short, strands appear to imply the energy E and the entropy S of Schwarzschild black holes. As usual, the ratio $E/2S$ determines the *temperature* of such black holes [34]:

$$T_{\mathrm{BH}} = \frac{\hbar c}{4\pi k}\,\frac{1}{R} = \frac{\hbar}{2\pi k c}\,a \quad . \tag{12}$$

In the last equality, the surface gravitational acceleration $a = GM/R^2 = c^2/2R$ was introduced, using expression (2). In short, black holes are warm.

The finite temperature value implies that *black holes radiate*. As a consequence, strands reproduce black hole *evaporation.* Radiation and evaporation are due to strands detaching from the horizon. If a single strand detaches, a photon is emitted. If a tangle of two or three strands detaches, a graviton or a fermion is emitted. (These last two statements are only possible after the strand models for every elementary particle have been deduced; this is done in references [18] and [31].) When all strands have detached, the complete black hole has evaporated.

The expressions (1) and the fundamental principle of Figure 1 contain a further result of interest. The gravitational acceleration on the surface of a black hole is $a = GM/R^2 = c^2/2R$; this is the maximum value possible. The value of black hole energy (2) implies that the black hole mass is given by $M = Rc^2/2G$. Taken together, this yields a limit on force $F = Ma$ given by

$$F \leq \frac{c^4}{4G} = 3.0 \cdot 10^{43}\,\mathrm{N} \quad . \tag{13}$$

This is the *maximum force* that can be observed at any point in nature. The existence of a maximum force is inextricably tied and equivalent to the minimum size of masses in nature. All derivations of its value make use of this connection; for example, $c^4/4G$ is also the maximum possible gravitational force between two black holes [21, 22, 23, 24].

It is important to note that the connection between black hole horizons and strand weaves is not arbitrary, but, again, *inescapable*. Only strand weaves imply that horizons have extension, two dimensions, entropy, energy, and temperature. Below, in Section 13, numerous testable predictions are deduced from the connection between strand weaves and horizons.

In summary, strands reproduce the known thermodynamic properties of black holes. These properties can all be deduced from the fundamental principle, i.e., from the expressions (1) and from Figure 1, which includes the extension of strands. As will become clear next, these results on black holes are sufficient to derive general relativity.

7 Deducing general relativity from thermodynamics

In 1995, in a path-breaking paper, Jacobson showed that the thermodynamic properties of the microscopic degrees of freedom of space and of black holes imply Einstein's field equations of general relativity [35]. He started with three thermodynamic properties:

- the entropy–area relation $S = A\, kc^3/4G\hbar$,
- the temperature–acceleration relation $T = a\, \hbar/2\pi kc$,
- the relation between heat and entropy $\delta Q = T\delta S$.

Using these three properties, the basic thermodynamic relation

$$\delta E = \delta Q \quad , \tag{14}$$

which is valid *only* in case of a horizon, yields the first principle of horizon mechanics

$$\delta E = \frac{c^2}{8\pi G} a\, \delta A \quad . \tag{15}$$

This expression can be rewritten, using the energy–momentum tensor T_{ab}, as

$$\int T_{ab}\, k^a \mathrm{d}\Sigma^b = \frac{c^2}{8\pi G} a\, \delta A \quad , \tag{16}$$

where $\mathrm{d}\Sigma^b$ is the general surface element and k is the Killing vector that generates the horizon. The Raychaudhuri equation [36] – a purely geometric relation – allows rewriting the right-hand side as

$$\int T_{ab}\, k^a \mathrm{d}\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab}\, k^a \mathrm{d}\Sigma^b \quad , \tag{17}$$

where R_{ab} is the Ricci tensor that describes space-time curvature. This equality between integrals implies that the integrands obey

$$T_{ab} = \frac{c^4}{8\pi G} \left(R_{ab} - (\frac{R}{2} + \Lambda)\, g_{ab} \right) \quad , \tag{18}$$

where R is the Ricci scalar and Λ is an undetermined constant of integration. These are Einstein's field equations of general relativity. The value of the cosmological constant Λ is thus *not determined* by the thermodynamic properties of horizons.

As Jacobson explained, the field equations are valid everywhere and for all times, because a suitable coordinate transformation can position a horizon at any point in space and at any instant of time. Achieving this just requires a change to a suitable accelerating frame of reference.

Given that horizons and black holes are thermodynamic systems, so is curved space. In other words, the field equations result from *thermodynamics of space*. Jacobson's argument thus shows that space is made of microscopic degrees of freedom, and that gravity is due to the same microscopic degrees of freedom.

Jacobson's argument also implies that space is a thermodynamic system in *three* spatial dimensions. The argument assumes three dimensions from the start; the argument does not work for other numbers of dimensions.

8 Deducing general relativity from strands

As explained in Section 6 above, strands imply the existence of horizons and of black holes. Above all, strands imply their thermodynamic properties: strands reproduce the entropy relation (6) of black holes, the temperature (12) of black holes, and their heat–entropy relation from (2). These are the three conditions for using Jacobson's argument to derive general relativity. Strands thus fully reproduce the argument. Therefore,

▷ Fluctuating strands lead to general relativity.

However, the result must be taken with caution. Jacobson's deduction of the field equations is *independent* of the details of the fluctuations and *independent* of the microscopic model of space, as long as the three thermodynamic properties given at the start are valid. After Jacobson's result, various kinds of microscopic degrees of freedom for space have been conjectured, including those found in references [2, 3, 4, 5, 6, 7, 9]. These explorations have effectively shown that finding the *correct* microscopic degrees of freedom of physical space among all the proposals in the literature is *not possible* using arguments from gravity or quantum gravity alone.

In addition, only a few independent investigations have concluded that vacuum is made of fluctuating lines [10, 11]. (There is a loose relation also with those quantum gravity explorations which deduce that space has less than three dimensions at Planck scales, such as [37] and the references cited therein.) However, all these investigations do *not* subscribe to the fundamental principle of the strand conjecture.

Among all the proposed microscopic models of space, strands might be seen as the simplest one. However, the simplicity of the strand conjecture is *not* a sufficient argument in its favour.

Any promising candidate for the microscopic degrees of freedom of space and gravitation must also reproduce the standard model of particle physics and, above all, explain the fundamental constants. This seems *the only way* to differentiate between the various microscopic models of gravitation. (This point is also made, independently, by others, such as Eichhorn [38].) Given that strands appear to reproduce the Lagrangian of the standard model – as argued in references [18] and [31] – it is worth exploring them also in the domain of gravitation. *The inability to reproduce the standard model or any experiment finding a deviation from the standard model would falsify the strand conjecture.* So far, no issues and no deviations have been observed. Despite all the caveats, the strand conjecture remains of interest.

In summary, in the strand conjecture, the field equations – and thus the Hilbert action – appear as consequences of fluctuations of impenetrable, featureless, unobservable strands. The first prediction of strands in the domain of gravitation is:

Pr. 1 *No deviations* between general relativity and the strand conjecture arise.

As a result, all processes described by general relativity are reproduced by strands. Therefore, *the smallest deviation between general relativity and observations would falsify the strand conjecture.* Below, in Section 14, the lack of deviations is made more precise: the prediction is limited, for

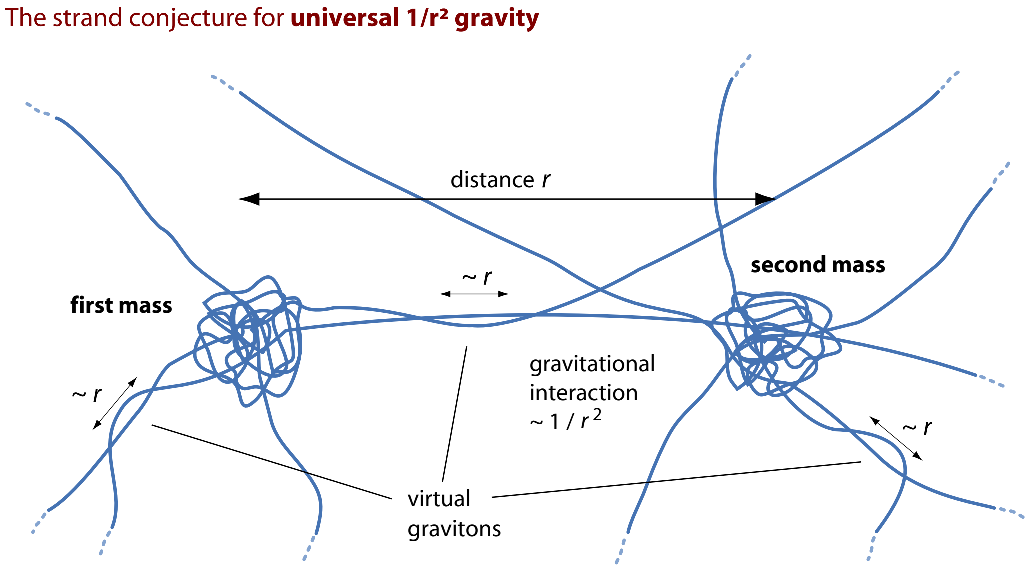

Figure 4: Gravitational attraction results from strands. When speeds are low and spatial curvature is negligible, as illustrated here, twisted tether pairs – i.e., virtual gravitons – from any mass lead to a $1/r^2$ attraction of other masses. The average length of twisted pairs of tethers scales with r. As a consequence, the curvature around such a mass scales as $1/r^3$. These results are valid for infinite, approximately flat space.

the time being, to sub-galactic distances. The equivalence for galactic and cosmological distances will be explored in a separate paper.

It must be stressed again that the ability to reproduce general relativity is *not unique* to the strand conjecture. Reproducing general relativity is not a sufficient argument in favour of any conjecture about the microscopic aspects of space. However, the *new* results given in Section 16 do provide arguments in favour of the strand conjecture. But before these new results are discussed, is is useful to derive more details about gravitation and space and to test the strand conjecture in detail.

9 Deducing gravity at low curvature and speeds from strands

In everyday situations, the effects of tethers can be simplified. In such situations, relative speeds are much lower than the speed of light c and spatial curvature can be neglected. This simplifies gravity.

In the strand conjecture, every mass, i.e., every system of tangles, is connected to the border of space by tethers. In particular, in the strand conjecture, every space-time effect, including gravity, is due to the behaviour of tangle tethers. Indeed, the nearer a mass is to a second mass, the more frequently the tethers from the two masses cross. Figure 4 illustrates the situation. The strand conjecture states:

▷ Everyday *gravitation* is due to tether pair twists and their influence on tether fluctuations.

Around every mass, the tethers crossings fluctuate; averaged over time, the fluctuations lead to a crossing switch density. This density of tether pair twists corresponds to a density of virtual gravitons. The resulting crossing switch density leads to a local temperature of space, and to a local negative potential energy. There are several ways to show that the crossing switch density around a spherical mass leads to *universal* $1/r^2$ *gravity*. Each way can be seen a simplification of Jacobson's original argument for the case of flat space and low speeds.

A. Given a spherical surface A enclosing a gravitating mass M at its centre, the acceleration a of a test mass located somewhere on the surface is related to the local vacuum temperature T. This relation is due to and described by the Fulling–Davies–Unruh effect that is deduced from strands in Section 11:

$$a = \frac{2\pi\, kc}{\hbar} T \quad , \tag{19}$$

where k is the Boltzmann constant. The vacuum temperature T is found by dividing the energy E contained inside the sphere by *twice* the maximum possible entropy S for that sphere. The temperature T is thus given by

$$T = \frac{E}{2S} = \frac{2G\hbar}{kc} \frac{M}{A} \quad . \tag{20}$$

Using $A = 4\pi r^2$ yields a temperature at the enclosing sphere given by

$$T = \frac{G\hbar}{2\pi\, kc} \frac{M}{r^2} \quad . \tag{21}$$

Inserting this expression into the Fulling–Davies–Unruh acceleration a yields

$$a = G \frac{M}{r^2} \quad . \tag{22}$$

This is universal gravitation, as discovered by Hooke and popularized by Newton. Since spatial curvature was neglected, and since the central mass was assumed to be at rest, this expression is only valid for flat space and small speeds.

B. An alternative deduction of universal $1/r^2$ gravity from black hole entropy was given by Verlinde [39]. The gravitational force F on a test mass m is given by the vacuum temperature T created by the central mass M and by the change of entropy S with distance x that is induced by the motion of the test mass:

$$F = T \frac{\mathrm{d}S}{\mathrm{d}x} \quad . \tag{23}$$

In the strand conjecture, the change of entropy $\mathrm{d}S/\mathrm{d}x$ when a test particle m moves by a distance x can be determined in a simple manner. When the test particle m moves by a (reduced) Compton wavelength, its tangle core has rotated by a full turn. Therefore, the entropy change is $2\pi k$ per (reduced) Compton wavelength. Thus,

$$\frac{\mathrm{d}S}{\mathrm{d}x} = \frac{2\pi\, kc}{\hbar} m \ . \tag{24}$$

Using the vacuum temperature T found in expression (21) yields an expression for the gravitational force given by

$$F = G \frac{Mm}{r^2} \ . \tag{25}$$

In short, strands imply universal gravity in the same way as Verlinde's entropic gravity does. Strands can be seen as a specific model of entropic gravity.

C. A further analogy for the attraction of a test mass by a large mass is the process of *thermodiffusion*. Thermodiffusion is the motion of a molecule in a fluid solvent with a temperature gradient. Recent research has shown that the *thermodiffusion coefficient* – describing the speed of the motion – is determined by the entropy of solvation [40]. Translated to the strand conjecture, the motion of a test mass due to gravity can be seen as motion along the temperature gradient of the vacuum. In this (partial) analogy, the gravitational mass of a particle – describing the speed of the motion – is given by the entropy that arises when the particle tangle is added to the vacuum.

D. Figure 4 can also be taken as illustrating how virtual gravitons lead to universal $1/r^2$ gravity. In flat three-dimensional space, the density of tether pair twists around a mass decays automatically with $1/r^2$. The image thus visualizes how strands reproduce the classical limit of quantum gravity.

In summary, everyday gravity can be described with strands in various equivalent ways: as fluctuation hindrance via tether crossings, as a process lowering entropy, as thermodiffusion, or as exchange of virtual gravitons. All of them show that strands imply universal $1/r^2$ gravitation. Additional descriptions may even appear in future.

10 Deducing curvature from strands

Strands not only visualize flat space; strands also visualize curvature. The fundamental principle of the strand conjecture implies:

▷ *Flat space* is a homogeneous network of fluctuating strands.
▷ *Curvature* is an inhomogeneous crossing (switch) density in the vacuum network.

An illustration of spatial curvature is given in Figure 5. The strand configuration differs from that of flat space. Certain strands break the isotropy and homogeneity. The main curvature value

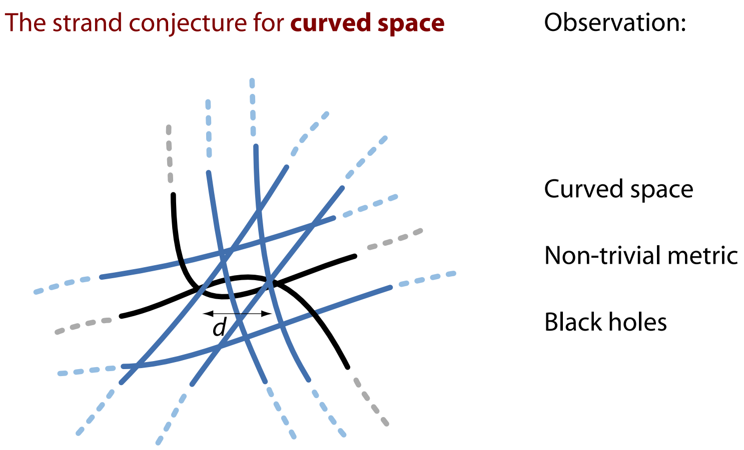

Figure 5: An illustration of the strand conjecture for a *curved* vacuum. The strand and crossing configuration is *not* homogeneous when averaged over time. Strands in black differ in their configuration from those in a flat vacuum. The value of the curvature is inversely proportional to the distance d. Also, the strand configuration is midway between that of a flat vacuum and that of a horizon.

depends on the configuration of the strands leading to the inhomogeneity. The curvature can evolve over time. This strand model for curved space implies that curved space-time is, locally, a Minkowski space. As a result, strands lead to a *pseudo-Riemannian space-time*.

In short, strands visualize space, black holes, gravity and curvature. It is now time to test the strand conjecture in detail.

11 Strand predictions about flat physical space

The fundamental principle of the strand conjecture – Figure 1 and expressions (1) – implies several testable predictions about flat physical space and the everyday vacuum. All predictions are deduced from the model for flat space given in Figure 2. *If any of the following predictions is refuted, the strand conjecture is falsified.*

Pr. 2 Because tangling of strands is not possible in other dimensions, strands predict that flat physical space is *three-dimensional*, *unique* and *well-behaved* at all scales. Flat physical space is a three-dimensional *continuum* that is *homogenous* and *isotropic*, without observable deviations.

So far, these predictions about physical space agree with expectations [41] and with the most recent observations (reference [42] claims Planck-scale sensitivity; see also reference [43]). Any evidence for other dimensions, other topologies, quantum foam, different vacuum states, different vacuum states, or crystal behaviour of space, or any other deviation from a well-behaved pseudo-Riemannian space-time manifold would directly falsify the strand conjecture.

Pr. 3 As a consequence of the fundamental principle in Figure 1 and of the expressions (1), the maximum energy speed in nature is c. This applies at all energy scales, in all directions, at all times, at all positions, for every physical observer. In short, the strand conjecture predicts no *observable* violation of Lorentz-invariance, for all energies and all physical systems. This agrees with observations so far – but not with other proposals in the literature, such as [44].

Pr. 4 The strand conjecture for space of Figure 2 and for the photon [18] imply, right from the start, that light moves with speed c. There is no variable speed of light, no time-dependent speed of light, no time-dependent energy of light, i.e., no 'tired' light, no energy-dependent speed of light and no helicity-dependent speed of light. Strands predict the lack of dispersion, birefringence and opacity of the vacuum. So far, this agrees with observations [45].

Pr. 5 The strand conjecture for the vacuum illustrated in Figure 2 predicts that there are *no* observable effects of the flat vacuum. Space is continuous for all observations, at all scales. For example, 'space-time noise', 'particle diffusion' or 'space viscosity' do not exist and will not be observed. Strands imply the lack of any degradation of distant star images. This agrees with observations [46].

Pr. 6 The strand conjecture for the vacuum illustrated in Figure 2 implies *no deviations from special relativity* for any measurable energy scale, as long as gravity plays no role. No 'double' or 'deformed special relativity' holds in nature, even though a maximum energy-momentum for elementary particles does exist. Strands predict that whenever special relativity is not valid, either general relativity needs to be used, or quantum field theory, or both together. This agrees with all observations so far.

Pr. 7 The strand conjecture for the vacuum illustrated in Figure 2 predicts the *lack of trans-Planckian effects.* For example, the existence of a *minimal length* given by

$$l_{\text{min}} = \sqrt{4G\hbar/c^3} \tag{26}$$

is predicted. If *any effect* due to space intervals smaller than the minimal value, the corrected Planck value, can be observed – for example in electric dipole moments [47], in higher order effects in quantum field theory, or in the discreteness of space – the strand conjecture is falsified. So far, observations agree with this and all other corrected Planck limits.

Pr. 8 Strands also predict the lack of observable effects of time intervals shorter than the corrected Planck time given by

$$t_{\text{min}} = \sqrt{4G\hbar/c^5} \ . \tag{27}$$

If *any effect* due to time intervals smaller than the corrected Planck time can be observed –

including a discreteness of time – the strand conjecture is falsified. So far, all observations agree with this limit.

Pr. 9 The strand conjecture for the vacuum illustrated in Figure 2 implies a *finite information amount* in any finite volume, be it empty or not, including the universe itself. Inside a corrected Planck volume, at most one strand crossing can be present. A strand crossing is the most fundamental two-state quantum system. (As such, a crossing resembles the 'Ur' introduced by Weizsäcker [28], which was later renamed 'qubit'.) So far, all observations and calculations confirm that both information and information density are always *finite*.

Pr. 10 Strands imply and predict that space's constituents are *discrete*. The discreteness of the constituents is not in contrast with the continuity of physical space. The discreteness is the reason for the finite entropy of horizons and for the limits to physical observables, and thus agrees with observations. The discreteness of the microscopic constituents agrees with all modern approaches to quantum gravity. Strands also imply that the discreteness cannot be confirmed directly, but only indirectly.

Pr. 11 The strand conjecture for the vacuum repeats that space's microscopic constituents are *extended*. Strands share this property with other microscopic models: loops, superstrings, super-membranes, spin networks, tensor networks, bands, knots, causal sets, triangulations, graphs, microscopic wormholes or exotic manifolds. Extension appears to be the simplest way to bring together the – at first sight contrasting – requirements of a minimal length, of space continuity, and of constituent discreteness, which all agree with observations.

Pr. 12 The strand conjecture for the vacuum states that its microscopic constituents have *no observable properties*. The constituents cannot have observable cross sections, cannot carry observable fields, cannot have mass, momentum or energy, and cannot carry quantum numbers. Strands require that observables and quantum numbers must be emergent.

Pr. 13 The strand conjecture predicts that a flat infinite space, as illustrated in Figure 2, would have a *vanishing vacuum energy* and thus a vanishing cosmological constant. The strand conjecture predicts the same result also for quantum field theory (see Appendix B). The often-cited discrepancy by a factor of 10^{120} between the value of the observed vacuum energy density and the value predicted from quantum field theory does *not* arise in the strand conjecture. The vacuum energy and the cosmological constant in the presence of a cosmological horizon are predicted to be small and positive, as detailed in the forthcoming paper on cosmology. All this agrees with observations.

Pr. 14 Figure 2 implies that strands do *not* produce fermionic or anti-commuting coordinates like in supergravity, or non-commutative space [48, 49], or a Clifford algebra, or a twistor space [50], or any other internal space at *every* point in physical space at every instant in time. Strands do approximate some of these structures, but only at *certain* points in space at *certain* times. The positions of these points and their internal spaces fluctuate. At the time being, it seems that this property of the strand conjecture is more in agreement with observations and provides more explanatory power; however, this impression could change in the future.

Pr. 15 Because of the strand configuration of Figure 2, space is unique, isotropic and homogeneous, as just mentioned. This implies that are *no different vacuum states*, and no boundaries or phase transitions between them. Together with the discussion of Section 15 below, this implies the lack of cosmic strings, domain walls and regions of negative energy. This agrees with observations.

Pr. 16 Because the fluctuating strand network generates space as in Figure 2, *the topology of space* – and of the universe – *is trivial*. This is predicted for all scales and all positions – and agrees with observations so far. In particular, strands predict that there is *no quantum foam*. This agrees with observations [43].

Pr. 17 The strand network of the vacuum and the weave model of black holes in Figure 3 imply that Minkowski space is *stable* against the spontaneous formation of black holes and against the spontaneous formation of singularities. This confirms the result found by Christodoulou [51].

Pr. 18 Both quantum theory and the strand conjecture imply that an accelerated observer in flat space observes a *temperature*. In the strand conjecture, the temperature is due to the strand bending that is induced on the vacuum strands by the acceleration. The local vacuum temperature T observed by an observer undergoing acceleration a appears after inserting the relativistic acceleration-length limit $L = c^2/a = 2R$ for accelerating systems (which can be deduced in special relativity from the invariance of c) into the temperature of black holes (12), derived above. The resulting temperature value is

$$T = \frac{\hbar}{2\pi\, kc}\, a \quad . \tag{28}$$

and is called the *Fulling–Davies–Unruh effect*. In the strand conjecture, the thermal particles detected by the accelerating observer arise from the vacuum strands that he encounters. The expression is thus equivalent to the expression for black hole temperature. It is not clear yet whether the Fulling–Davies–Unruh effect will ever be observed experimentally.

Pr. 19 In contrast to an accelerating observer, an inertial observer in infinite flat space measures a *vanishing* vacuum temperature. This result arises in quantum theory, in the strand conjecture, and in experiments. The two results about the vacuum temperature observed by inertial and accelerated observers will be of interest in cosmology.

Pr. 20 In the strand conjecture, it makes no sense to speak of a strand density, because strands are not observable. Only crossing switches are observable. All predictions deduced from the strand conjecture must depend on crossing switch density *only*. Predictions deduced from the strand conjecture must not depend on strand density itself. So far, this is the case.

Pr. 21 The strand conjecture for the vacuum is in contrast with several other microscopic descriptions of space for an additional reason: Strands do not *obey* equations of motion, but *fluctuate randomly*. The details of strand fluctuations are neither observable nor important. Predictions deduced from the strand conjecture must not depend on the motion of single strands. So far, this is the case.

The *randomly fluctuating* motion of the constituents leads to a homogenous crossing switch density. Such a homogeneous crossing switch density leads to a homogeneous, continuous and Lorentz-invariant space-time. Random fluctuations of strands thus lead to the properties of space found in observations.

12 Strand predictions about curved space

Pr. 22 A fluctuating, untangled strand network generates flat physical space. A weakly tangled network, illustrated in Figure 5, generates *curved physical space*. As a result, also curved space has *three* dimensions, at all distance and energy scales, in all directions, at all times, at all positions, for every physical observer. So far, this is observed.

Pr. 23 The value of spatial curvature κ around a mass is due to the tether crossing switch density induced by the mass. In the strand conjecture, the crossing switch density decreases with distance r from the mass. So does the strand inhomogeneity. As illustrated in Figure 4, this yields the proportionality

$$\kappa \sim \frac{1}{r^3} \quad . \tag{29}$$

This relation agrees with expectations: the result also arises from the field equations, which were derived from the fundamental field equations above. Strands thus propose a simple visualization of the result. In simple terms, a factor $1/r^2$ is due to Gauss' law, and a factor $1/r$ is due to the average size of twisted pairs of tethers – the virtual gravitons – around the mass. The third power in the decrease of the curvature around a mass is thus due to the three dimensions of space and to the extension of strands. Without extended constituents, an explanation of the $1/r^3$ dependence does not seem possible.

Pr. 24 Strands and expressions (1) imply a *limit* to curvature κ. The limit is given by the inverse of the smallest length:

$$\kappa \leq \frac{1}{l_{\text{min}}} = \sqrt{\frac{c^3}{4G\hbar}} \quad . \tag{30}$$

This limit again implies the lack of singularities in nature, of any kind. So far, this prediction is not in contrast with observations.

Pr. 25 Strands imply that the Ricci scalar R is non-zero in a region of space only if tangles – i.e., only if massive particles – are found in that region. Again, the behaviour is as expected.

The expressions (1) imply that the maximum value for the Ricci scalar R is given by inverse minimum area:

$$R \leq \frac{1}{l^2_{\text{min}}} = \frac{c^3}{4G\hbar} \quad . \tag{31}$$

So far, this prediction is not in contrast with observations.

13 Strand predictions about black holes

The fundamental principle of the strand conjecture – Figure 1 and expressions (1) – allows drawing numerous testable conclusions about black holes. Since black holes are the most extreme gravitational configurations, tests of back hole limits are the most telling. The following tests have been collected from the research literature on black holes. *If any of the following predictions is wrong, the strand conjecture is falsified.*

Pr. 26 The strand conjecture for black holes illustrated in Figure 3 implies that the horizon entropy, the horizon energy and the horizon temperature are *limit values* for all physical systems of the same size. These limits arise directly from the corrected Planck limits of expressions (1) that define the strand conjecture. So far, they agree with observations.

In particular, because strands cannot be tighter – closer to each other – than in a horizon, the limit

$$\frac{m}{L} \leq \frac{c^2}{4G} \tag{32}$$

arises for every physical system of size L. The limit has a value of $3.3666(1) \cdot 10^{26}\mathrm{kg/m}$ or about $1/6$ of a solar mass per km. Equality is predicted to hold *only for black holes.* The strand conjecture thus naturally implies that, for a given mass value, black holes are the densest objects in nature. Strands thus illustrate and imply both the *hoop conjecture* and the *Penrose conjecture:* for a given mass, because of the minimum size of crossings, a spherical horizon – a tight weave – has the smallest possible diameter. Other possible weave shapes have larger size. This agrees with expectations.

Pr. 27 The strand conjecture illustrated in Figure 3 implies that black holes *evaporate*. Through fluctuations, single strands or tangles of strands can detach from the horizon weave. The strand conjecture allows deducing several predictions about evaporation.

- First of all, the emission of particles will depend on the size of the black hole and on the tangling of the particle tangles, i.e., on particle mass values.
- For large black holes, the evaporation is a low probability process, and the evaporation rate of such a black hole is small. All this agrees with expectations.

  To calculate evaporation rates for different particles, probabilities for corresponding untangling processes must be calculated. At present, no mathematical tools to do this appear to exist.
- For small black holes, the curvature of the black hole allows the emission of massive particle tangles. The relative probability for the emission of massive particles in black hole radiation is predicted to *increase* for smaller black holes.

- The smaller the black hole, the higher the total luminosity, because strands detach with higher probability from a horizon with higher curvature.
- Just before the completion of the evaporation process, black holes still radiate with a luminosity near but *below* the maximum possible value, the Planck power $c^5/4G$.

All these predictions agree with predictions made in the research literature [52]. Whether they will ever be confirmed by observations remains to be seen.

Pr. 28 Black holes evaporate until the horizon weave has completely *dissolved* into separate strands or tangles. Strands predict *the lack of black hole remnants* that differ from usual elementary particles.

Pr. 29 Together with the strand description of black hole evaporation, strands predict and illustrate the lack of black holes with microscopic mass values. The corrected Planck limits for energy density, size, temperature and luminosity deduced above imply that all black holes obey

$$m > \sqrt{\frac{\hbar c}{4G}} \ , \tag{33}$$

thus have a mass that is *larger than the corrected Planck mass.* This agrees with observations and expectations.

Pr. 30 The weave model of horizons also implies that elementary particles, which are tangles – not weaves – are *not* black holes. This agrees with expectations and with observations.

Pr. 31 The strand conjecture automatically implies that the horizon area of a small black hole is *quantized* in multiples of the smallest area $4G\hbar/c^3$. This implication has been already deduced in the past [53]. However, strands also imply that area quantization of black holes is *not observable*, because in principle, no apparatus can have the sensitivity to detect this smallest area value. Such an apparatus would have to be able to count and thus to observe strands.

Pr. 32 The strand conjecture for black holes of Figure 3 and the statistical properties of their fluctuations also imply that white holes do *not exist.* For reasons of probability, evaporation cannot take place backwards. This agrees with observations.

Pr. 33 Because black hole horizons are weaves in the strand conjecture, black holes are predicted to have *no hair*, i.e., no nuclear charges, no baryon number, no lepton number or other quantum numbers. In a previous paper [18] it became clear that all these quantum numbers are topological properties of tangles. In the strand conjecture, these quantum numbers are not defined for horizons. All quantum numbers except electric charge – which is defined with the help of crossing or tangle chirality and is explored below – do not make sense for weaves. The *no-hair theorem* is thus natural in the strand conjecture. It is ironic that the strand conjecture can also be seen as a way to describe particles and horizons *only* with the help of “hair”, if one uses “hair” as a synonym for “strand” or “tether”. Using this terminology, one could say that the “hair conjecture” implies the no-hair theorem.

Pr. 34 The fundamental principle of the strand conjecture – in particular the expressions (1) – implies that in all processes, near or far from horizons, the power and luminosity limit

$$P \leq c^5/4G = 9.0709(3) \cdot 10^{51}\,\mathrm{W} \tag{34}$$

and the force and momentum flow limit

$$F \leq c^4/4G = 3.0257(2) \cdot 10^{43}\,\mathrm{N} \tag{35}$$

are always valid at any given point in space. These limits – one quarter Planck mass per Planck time, or $50\,756(12)$ solar masses per second, times c and times c^2 – are predicted to apply to every local process in nature [21, 22, 23, 24]. A solar mass of $1.9885(5) \cdot 10^{30}\,\mathrm{kg}$ is assumed here.

No Earth-bound process approaches the force and power limit, by far. Astrophysical observations are necessary to check the limits. Galaxies, quasars, galaxy clusters, and blazar jets all emit below 10^{-5} solar masses per second. In supernovae and hypernovae, both accretion and emission are below 10^{-2} solar masses per second. Gamma ray bursts emit at most 1 solar mass per second. The fastest observed and simulated accretion processes achieve 10 solar masses per second. The highest observed luminosities so far are those observed in black hole mergers by LIGO and VIRGO [54]. At present, the highest peak powers were observed for the events GW170729 and GW190521. They showed values of $4.2(1.5) \cdot 10^{49}\,\mathrm{W}$ or 230 ± 80 solar masses per second [55] and of $3.7(9) \cdot 10^{49}\,\mathrm{W}$ or 207 ± 50 solar masses per second [56]. All these values are well below the (corrected) Planck limit of $50\,756(12)$ solar masses per second.

Present data therefore does not yet allow distinguishing between the corrected Planck luminosity limit $P \leq c^5/4G$ and the conventional Planck limit $P \leq c^5/G$ that is four times larger. *Future discoveries might change this and allow a direct test of this aspect of general relativity and of the strand conjecture.*

Pr. 35 In the strand conjecture, horizons are tight, one-sided weaves. Any matter tangle that falls towards a horizon and is near it for a distant observer is flattened. As a result, at most one Planck mass can arrive at a horizon during a Planck time. Expressions (1) then yield the mass rate limit

$$\frac{\mathrm{d}m}{\mathrm{d}t} \leq \frac{c^3}{4G} \quad . \tag{36}$$

This limit – again valid for any point in space – is also valid in general relativity – and in nature in general. So far, the limit, given by $1.00928(3) \cdot 10^{35}\,\mathrm{kg/s}$ or $50\,756(12)$ solar masses per second, is not violated by any observation – including black hole mergers. As a future check, it could be interesting to check existing numerical simulation packages of general relativity against this limit.

Pr. 36 The strand conjecture and expressions (1), with their lower limit on crossing switch time and on other observables, *limit* energy density (and pressure) to the (corrected) Planck value:

$$\frac{E}{V} \leq \frac{c^7}{16\,G^2\hbar} = 2.8958(1)\cdot 10^{112}\,\mathrm{J/m^3} \quad . \tag{37}$$

The energy density limit implies a lower limit for black hole size, for particle size and for the size of any localized system. Therefore, strands do *not allow singularities* in nature, neither dressed nor naked. Cosmic censorship is automatically realized in the strand conjecture. So far, both the density limit and the lack of singularities agree with observations.

Pr. 37 In any physical system, strand crossings can be more or less tight, and switch more or less frequently. The limit case for a system of size R and energy E is the one with the *tightest* possible strands, as defined by the smallest length in expressions (1). This directly yields

$$\frac{2\pi}{\hbar c}\,ER \geq \frac{S}{k} \quad . \tag{38}$$

This is *Bekenstein's entropy bound*. The strand conjecture implies that equality is realized by horizons – and only by horizons – because horizons are the strand configurations that are as tight as possible and whose crossings switch as rapidly as possible. This agrees with expectations.

Pr. 38 In the strand conjecture, *electric charge* is a result of the *chiral* linking of strands [18]. Because horizons are weaves of strands, the electric charge Q of black hole horizons is *limited*. Strands visualize the limit directly.

A simple way to deduce the charge limit is to use the force limit (13) deduced above from strands. The electric force between two charged black holes must be smaller than the maximum force:

$$\frac{Q^2}{4\pi\epsilon_0\,R^2} \leq \frac{c^4}{4G} \quad . \tag{39}$$

Using the black hole relation $M = Rc^2/2G$ of equation (2), this can be rewritten as

$$\frac{Q^2}{4\pi\epsilon_0} \leq GM^2 \quad , \tag{40}$$

which is the established limit for a Reissner-Nordström black hole. Finding an exception to the charge limit would falsify the strand conjecture. However, such an exception would also falsify maximum force and general relativity. Unfortunately, no observations that allow testing the region near the limit are available so far. In fact, it is expected that virtual pair production prevents such observations.

Pr. 39 Strands model black holes as weaves. Because strands model electric charge with crossing chirality, this implies that the electric charge of a black hole is distributed *over its surface*. The predicted charge distribution is consistent with the distribution of black hole *mass*. Indeed, the strand conjecture implies that electric charge exists only for massive objects, and that charge and mass cannot be separated. This agrees with results from general relativity [57, 58, 59].

Pr. 40 Being weaves, black holes can be either non-rotating or rotating. For rotating black holes, the strands in the weave provide a limit to the angular momentum of the black hole. Angular momentum, like spin, is a result of strand crossing switches [18]. In a rotating black hole, the weave rotates. Because the equatorial speed is limited by c, a maximal rotation frequency ω arises, with the value $\omega \leq c/R$. Using the limit $J \leq E/\omega$, this yields

$$J \leq \frac{2G}{c} M^2 \quad . \tag{41}$$

As expected, a rotating weave behaves like a Kerr black hole [60]. A higher angular momentum would contradict the fundamental principle, and in particular the minimum time for crossing switches. So far, the angular momentum limit for extremal black holes agrees with observations [61].

Pr. 41 The strand conjecture implies that a *rotating* black hole realizes a belt trick that involves a *huge* number of tethers. Surprisingly, animations illustrating such a process were available on the internet [62] before the strand conjecture formulated this equivalence. They were programmed by Jason Hise. Figure 6 shows such a configuration during rotation. In this description, the *ergosphere* is the region in which the crossing switches take place during the belt trick. In contrast to the figure, the horizon of a rotating black hole is flattened at the poles, and so is its ergosphere.

Pr. 42 The *irreducible mass* of a rotating black hole is determined by the number of strands N_{s} that make it up. Strands thus predict that the *total* mass of a rotating black hole is a monotonous function of the irreducible mass and of its rotational energy, up to the angular momentum limit. This agrees with expectations and with observations.

Pr. 43 The description of rotating black holes or masses with strands also suggests that moving tethers describe what is usually called *frame dragging*. In the strand conjecture and in general relativity, frame dragging occurs around all rotating masses, at all distances, and independently of whether the mass is a black hole or not. Like all other observable effects, also frame dragging results from crossing switches.

Pr. 44 Strands also allow exploring black holes that are both charged and rotating – the Kerr-Newman case. In the strand conjecture, electric charge is due to chiral tangling [18, 31]. First of all, strands imply that when a black hole rotates, the tethers move as shown in Figure 6. This motion implies and predicts that the g-factor for such black holes is

$$g = 2 \quad . \tag{42}$$

Strands make this prediction (at tree level in the elementary particle case) for all rotating systems for which the crossings that generate mass and those that generate charge rotate

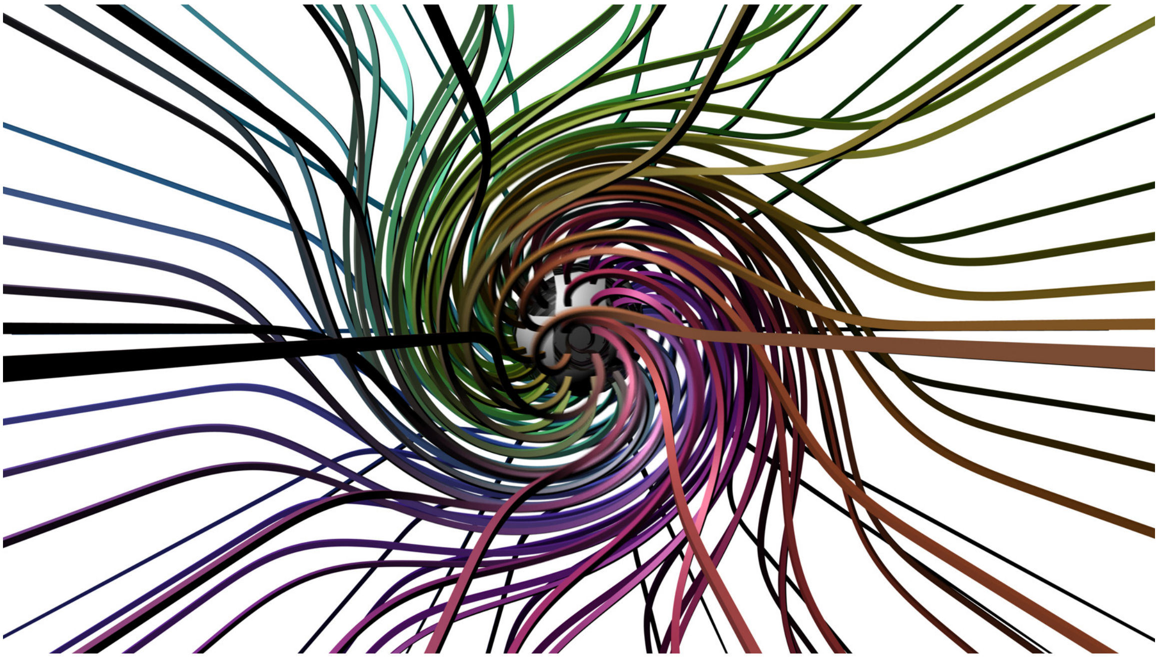

Figure 6: The strand conjecture for a rotating black hole rotating about the vertical axis (© Jason Hise). The flattening of the horizon at the poles is not shown. For a complete animation of the process, see the online video at reference [62].

at the same speed. In these cases, the g-factor is 2, because of the belt trick [18]. The complete animation depicting black hole rotation as a belt trick with a *large* number of tethers illustrates the origin of the g-factor even more clearly [62]. The value 2 for the g-factor of black holes agrees with the usual predictions [63, 64, 65]. The question whether the g-factor is exactly 2 or whether it shows corrections that depend on the fine structure constant α – especially in the case of maximally charged black holes – is still open. So far, however, no way to test these predictions appears to be possible.

Pr. 45 Strands allow expressing the results on rotating charged black holes with an additional limit. As deduced above, strands imply a charge limit for any black hole given by equation (40). The definition of the g-factor

$$\mu = g \frac{Q}{2M} J \tag{43}$$

implies, for $g = 2$, that

$$\frac{\mu}{J} = \frac{Q}{M} J \quad . \tag{44}$$

This means that

$$|\frac{\mu}{J}| \leq \sqrt{G}\sqrt{4\pi\epsilon_0} \quad . \tag{45}$$

Strands thus confirm the limit conjectured by Barrow and Gibbons [66]. So far, all observations and thought experiments agree with the limit. *If this inequality is violated, the strand conjecture is falsified.*

In summary, the Barrow–Gibbons limit was derived from three strand properties: the horizon is a rotating weave; secondly, the electric charge, being due to chiral crossings, rotates with the mass; and thirdly, the crossings cannot rotate faster than the speed of light.

Pr. 46 Strands predict that black holes have *vanishing* magnetic charge, because strands, due to their extension, do not allow magnetic charge to exist [18]. This agrees with expectations and observations so far. More checks might be possible in the future. *The observation of magnetic charge would falsify the strand conjecture.*

Pr. 47 Strands confirm that every horizon is a physical system that on the one hand can be seen as an extreme form of (curved) space, and on the other hand can be described as an extreme form of (falling) matter. Both points of view on horizons lead to tight, one-sided weaves as models for horizons. Horizons are thus systems at the border between space and matter. Alternatively, in the strand conjecture, *horizons are a mixture of space and matter.* This agrees with expectations.

Pr. 48 The thermodynamic properties of strand fluctuations in black holes have implications for the shape oscillations of horizons. Shape oscillations of black hole horizons increase (and decrease) the local curvature. This increases (and decreases) the local evaporation through radiation, i.e., through strand detachment. As a result, horizon shape oscillations are *damped* and disappear over time. This agrees with theoretical expectations.

Pr. 49 The strand conjecture for black holes of Figure 3 allows a further conclusion. For observers at rest outside the black hole, the weave model of horizons implies that *nothing* can be observed behind the horizon. In simple terms, nothing is 'inside' a black hole horizon. This corresponds to expectations. In particular, strands suggest the *lack of a tightly concentrated mass* and thus also the *lack of a singularity* inside a black hole.

In principle, a horizon could also be modelled by a tight ball, a tight clew, or a tight skein of strands. A black hole could thus be made of many strands in an involved three-dimensional tight tangle. At first sight, such a configuration would seem to be more dense than a tight weave. However, such a configuration is physically indistinguishable from a woven horizon, because only crossing switches at the surface of the ball would be possible and thus be observable.

Pr. 50 Because strands imply that the mass of black holes is distributed over their horizon, all black holes, including Schwarzschild black holes, have a finite *moment of inertia* I. Since strands reproduce general relativity, the moment of inertia of Schwarzschild black holes is given, as in general relativity, by the limit deduced for slowly rotating Kerr black holes [58, 59, 67]:

$$I = MR^2 \quad . \tag{46}$$

This result again disagrees with the idea that black hole mass is concentrated in a putative central singularity. Falsifying this value for the moment of inertia would falsify the strand conjecture.

The value of the moment of inertia is *larger* than that of a spherical mass shell, for which $I = 2MR^2/3$. The strand model visualizes the difference between a black hole and a mass shell in the following manner: Figure 6, showing the belt trick with a large number of belts, implies that every smallest surface on the horizon contributes the same number of crossing switches. Every smallest surface on the horizon thus contributes equally to the angular momentum, independently of its distance from the axis of rotation. Because mass is evenly distributed over the horizon, the total moment of inertia is $I = MR^2$.

Pr. 51 The strand conjecture for black holes illustrated in Figure 3 implies that for a distant observer at rest, horizons are not surfaces, but *thin cloudy volumes.* Strands thus imply that black hole horizons resemble stretched horizons. In contrast to an observer at rest, an observer *falling towards* and into the black hole experiences a three-dimensional strand network instead of an (almost) two-dimensional strand weave. The two descriptions can be transformed into each other with suitable deformations of the involved strands. The strand conjecture thus provides a model of a black hole that resembles a *'firewall'* [68] and a *'fuzzball'* [69, 70].

Pr. 52 The strand conjecture implies that black holes (with all their quantum properties) are *impossible in higher dimensions*, because higher dimensions *do not allow* forming stable weaves. Strands thus imply that black holes can be imagined in higher dimensions only if quantum effects are (at least partially) neglected. However, this statement is hard or even impossible to verify.

Pr. 53 The strand conjecture for black holes illustrated in Figure 3 suggests that black holes can *reflect* an incoming quantum particle, instead of swallowing it, but that the probability is *extremely* low: the incoming particle must have an energy so low that its wavelength is comparable to the size of the black hole. For such a low energy, the particle strands are similar in shape to vacuum strands, and the motion of the scattered particle around the black hole resembles the motion of vacuum strands around a traveling black hole. This low probability agrees with expectations [71].

14 Strand predictions about general relativity, quantum gravity and gravitons

In the derivation of general relativity in Sections 7 and 8, the *cosmological horizon* was not taken into account. Strands thus imply that for *sub-galactic* distances, when the horizon has *no* influence, and for *everyday* energies, when quantum field theory plays no role, *general relativity holds exactly.* This is the precise version of the first prediction given above, in Section 8.

At sub-galactic distances, strands yield detailed predictions on different aspects of general relativity and quantum gravity, including gravitational waves and gravitons. *If any of the following* *predictions is wrong, the strand conjecture is falsified.* The first group of predictions concerns *limits*.

Pr. 54 The fundamental principle and expressions (1) imply that the Planck units c, $\hbar$ and $c^4/4G$ are *invariant limit values*, also in the presence of space curvature, be it weak or strong. This is predicted to hold locally, without any restriction, at all energies, all scales, and all positions. In particular, the gravitational constant G does not change over time, in contrast to the suggestion by Dirac [72]. The same is true for the other constants. These predictions agree with all observations.

As a consequence, there is a minimum distance, a minimum time interval, a maximum curvature, a maximum mass density in nature, and many other such limits. The limit values are given by the *corrected* Planck values, where G is replaced by $4G$. So far, they are not exceeded in any observation.

Pr. 55 As mentioned above, as a consequence of the fundamental principle there is a maximum local power or local energy flow or luminosity $c^5/4G$, a maximum local force or momentum flow $c^4/4G$, and a maximum local mass change rate or mass flow $c^3/4G$ in nature. There is also a maximum mass per length ratio $c^2/4G$ – realized by black holes. These limits yield numerous *paradoxes*, i.e., thought experiments in which higher values are apparently possible at first sight, but not possible after careful evaluation [23, 73, 74].

The paradoxes in general relativity can be solved in the same way. For defining or measuring any local flow, a physical surface must be defined first; also the flow limits only hold for flows through a physical surface. A surface is *physical* if it allows a physical observer at each of its points. In particular, a physical observer cannot be point-like, cannot be made of point masses, and cannot move faster than light. Given a physical surface, the flow limits are valid locally, for all energy scales, for all directions, at all times, at all positions, for every physical observer. Most paradoxes about the maximum flow values disappear when the impossibility of unphysical surfaces, the lack of point masses, and the lack of infinite mass densities are taken into account. Also the locality of the limit must be kept in mind; if this locality of forgotten [73], counter-examples to maximum force can easily be constructed.

Despite the availability of experimental data, experimental tests of the gravitational Planck limits are not yet discussed by Will [75]; however, discussions are beginning to appear [54]. Probing the correctness of the factor 4 in the corrected Planck limits with the help of experiments might be possible this century.

Pr. 56 Strands predict from Figure 1 that the *integrated luminosity of the universe*, at any point in space, is limited by

$$L \leq c^5/4G \tag{47}$$

This limit is predicted to apply also in case of multiple simultaneous supernovae or hypernovae or black hole mergers. So far, this prediction agrees with all observations [54].

Pr. 57 The strand conjecture of Figure 1 *prevents* infinite values of observables. Strands thus predict that there are *no singularities* in nature. Furthermore, strands predict that the evolution of space-time, despite the nonlinearities of the field equations, *cannot produce spikes*, neither in general relativity nor in quantum gravity. This agrees with all observations and expectations.

Pr. 58 The tangle model for elementary particles (see Appendix B) implies that *no* such particle can have an energy, mass or momentum *larger* than the corrected Planck values $\sqrt{\hbar c^5/4G}$, $\sqrt{\hbar c/4G}$ or $\sqrt{\hbar c^3/4G}$ also in curved space. So far, all cosmic radiation studies confirm the prediction.

Pr. 59 Expressions (1) imply that a maximum force exists in nature and in the strand conjecture. As a consequence, there is a '*gravitational indeterminacy relation*' for the measurement of the energy E and the size l of physical systems, given by

$$\frac{\Delta E}{\Delta l} \leq \frac{c^4}{4G} \ . \tag{48}$$

It appears that this relation is best tested with collisions that involve one or two black holes. So far, all observations agree with the relation. Similar relations among other observables with the same right hand side – even with other powers of c – can also be deduced.

Strands also imply predictions on *deviations* from the field equations of general relativity.

Pr. 60 Strands imply *no effect of torsion* and *no effect of higher derivatives* of the metric on the motion of massive bodies. Strands thus appear to suggest that *conformal gravity* does not apply to nature. In fact, strands exclude all theories with post-newtonian behaviour that differs from general relativity. This agrees with observations, in weak and in strong gravitational fields, including double pulsars and black hole mergers [75].

Pr. 61 Strands imply that *Palatini gravity* is not valid in nature, because the strand configuration determines all properties of space-time geometry.

Pr. 62 Strands, as Figure 7 and Figure 4 imply, predict that *parity violation* by gravity does *not* occur and that it will not be observed. So far, this agrees with observations.

Pr. 63 Strands appear to suggest that gravitation shows scale invariance, *as long as* strand diameters can be neglected. Strands thus imply that gravity shows something similar to *asymptotic safety*, as presented, e.g., in [38]. The topic needs more exploration.

Pr. 64 Deviations from general relativity occur when, instead of tethers, tangle *cores* fluctuate and are deformed. Such core deformations indeed yield the electromagnetic and the nuclear interactions, as shown in reference [18]. In other terms, in the strand conjecture, both quantum theory and the standard model of particle physics can be seen as *high-energy deviations* from or, better, as *high-energy complement* of general relativity. At high energies,

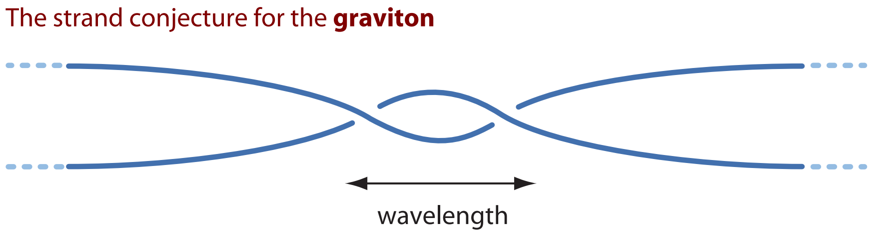

Figure 7: The strand conjecture for the graviton: a twisted pair of strands has spin 2, boson behaviour and vanishing mass. A gravitational wave is a coherent superposition of a large number of gravitons.

no *other* deviations from general relativity are predicted to occur. *For example, the observation of a fifth force, of supersymmetry, or of supergravity would falsify the strand conjecture.*

Strands thus imply a very specific gauge–gravity duality, rather different from other proposals in the literature.

The final group of predictions from the strand conjecture concerns *gravitons and quantum gravity.*

Pr. 65 Gravity is due to the exchange of virtual gravitons. The tangle model of the graviton is illustrated in Figure 7. *Gravitons have spin 2*. Indeed, gravitons return to their original state after a rotation of the tangle core along the horizontal *axis* by π, as required by a spin 2 system. *Gravitons are massless bosons.* These properties are realized by twisted pairs of strands. The topology of the graviton model also implies the existence of gravitational waves with spin 2 and velocity c. These predictions agree with expectations and observations.

Pr. 66 The graviton model agrees with expectations, because it leads to $1/r^2$ gravity, as shown in Section 9 and Figure 4 below. Simply speaking, the $1/r^2$ dependence of weak gravity arises from the twists generated in the tethers by the belt trick. The graviton model clearly visualizes the close relation between $1/r^2$ gravity and spin 2.

Pr. 67 In the strand conjecture, *single gravitons cannot be detected*, for two reasons. First, strands imply the indistinguishability between graviton observation from any other quantum fluctuation of or at the detector. Equivalently, in the strand conjecture, graviton absorption does not lead to particle emission. Secondly, even if gravitons were detectable, in the strand conjecture, they have an extremely small cross section, of the order of the square of the Planck length. The low cross section is due to the topology of the graviton tangle. This implies a low detection probability, as expected [76, 77]. The lack of graviton detection agrees with all data so far.

Pr. 68 The strand model of the graviton implies that graviton exchange is *not described by a gauge symmetry* in the way that the three gauge interactions are. Gravitons only couple to particle tangledness. Gravitons do not couple to any quantum number describing a particle; neither do they couple to any symmetry property of particles; finally, gravitons are not related to Reidemeister moves. In contrast, all these properties apply to gauge bosons.

Pr. 69 In the strand conjecture, all particles and all masses have *tethers*. Figure 6 and Figure 4 illustrate the situation. All tethers are unobservable; only their crossing switches are. In particular, strands suggests that gravity, like all other space-time effects, is due to tangle *tether* fluctuations and deformations. The existence of tethers implies that all masses show quantum effects or have components that show quantum effects. This agrees with observations.

Pr. 70 Strands and expressions (1) imply that the gravitational constant G *does not run* when energy is increased from everyday values to higher values. In the language of perturbative quantum field theory, G is *not renormalized*. This prediction agrees with expectations and with data, though the available data is sparse.

Pr. 71 Strands imply that configurations of gravitons or photons are never localized. For topological reasons, strands imply that *geons* cannot form. For the same reason, a macroscopic situation approximating a geon is expected to decay rapidly. This agrees with expectations [78].

Pr. 72 Strands imply that the wave function Ψ is the crossing density due to the tangle core – and therefore an imaginary number – whereas the gravitational potential φ is the crossing density of twisted tether pairs – and therefore a real number. Many similarities and many differences between Ψ and φ arise, including similarities and differences between *entanglement* and gravitation [79]. Strands suggest a general relation between gravity and entanglement: in a general sense, both effects are due to tether tangling. This topic is still a subject of research.

Pr. 73 Strands imply that no quantum superposition effects for gravity are observable at experimentally accessible scales, because graviton exchange destroys entanglement. This agrees with expectations [80].

Pr. 74 The strand conjecture implies that in a *double-slit experiment* with electrons, electrons pass both slits at the same time, because the core splits in two pieces during passage – though in different fractions at every passage. Therefore strands predict that the gravitational field of an electron arises on both slits, for every passage, though in different fractions at every passage. Such an experiment might be possible one day.

Pr. 75 Strands also imply that there are *no* unknown, observable *quantum corrections* to general relativity. This prediction is in contrast with many expectations, and may well be the most contentious prediction in this list. The prediction is intimately related to the impossibility to detect single gravitons or single strands. Equivalently, strands predict the lack of observable quantum effects in *semiclassical gravity.*

The result can be put in this way: *strands predict the impossibility to observe new quantum gravity effects at sub-galactic scales.* So far, this agrees with experiments; in fact, all proposals for such effects do not seem promising. In fact, strands suggest that *non-trivial quantum gravity effects* – i.e., effects other than black hole thermodynamics, particle masses, gauge interactions and dark energy – *cannot* be observed. And despite many attempts, no such effect has been detected yet.

In short, strands predict that there are *no* measurable deviations from general relativity, as described by the Hilbert action, and from known physics, at any *sub-galactic* distance. The detailed predictions agree with all observations so far. However, these predictions are *unspectacular*; the same predictions are made by many, in fact by most approaches that contain both general relativity and quantum theory as limiting cases. *Nevertheless, the future discovery of any new deviation from general relativity at sub-galactic scales would falsify the strand conjecture. The strand conjecture would also be falsified by the observation of any non-trivial quantum gravity effect, or by the observation of any additional interaction, as explained in Appendix B.*

In principle, deviations from general relativity could occur at *galactic or cosmological scales*. First predictions in this domain, on the nature of dark matter and dark energy, are given in the next sections.

15 Strand predictions about further defects in space

In the strand conjecture, particles, horizons and curved regions can be seen as *defects* in the strand network that describes flat empty space. It is legitimate to ask whether the strand network allows for additional types of defects or structures that would provide options for dark matter, for dark energy or for new physics.

In the strand conjecture, particles are *tangles*, and can thus be seen as *localized* defects in the strand network that describes nature. Neglecting their Planck size, one could call particles *zero-dimensional* defects. Horizons are weaves, thus *two-dimensional* defects in the same approximation. Are there other options? Some of the following predictions are, by their nature, less certain than those given so far.

Pr. 76 Among *zero-dimensional* strand defects, also *knotted* strands, links or tangles are imaginable. Two examples of *prime* tangles, and two examples of a *locally knotted* are shown in Figure 8. Because the strands of the strand conjecture have no ends, and *because* such configurations cannot arise by moving tethers around in space, the strand conjecture predicts that such knotted configurations are not possible. This is the most fundamental property of strands, but also their most easily questioned property. If knotted configurations were

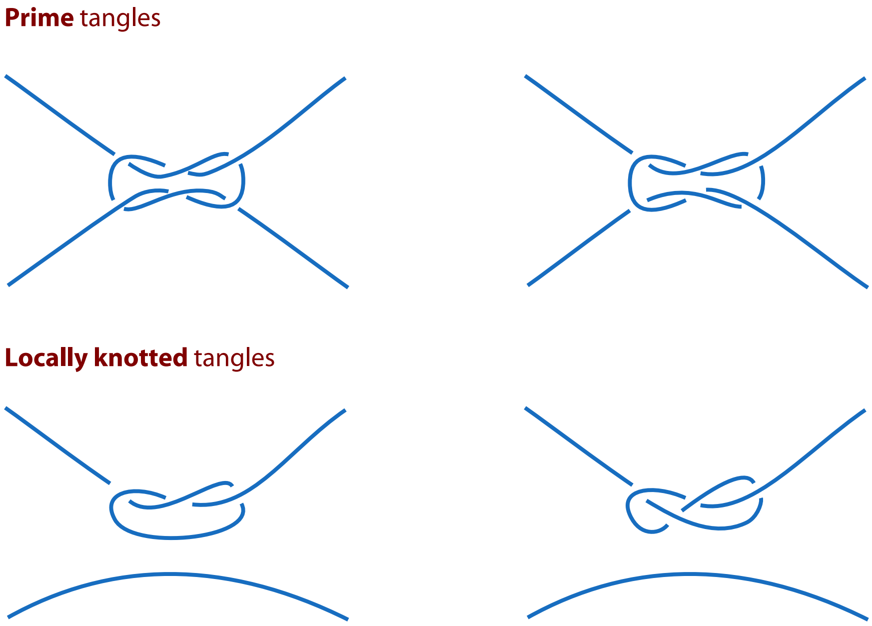

Figure 8: Top: two *prime* tangles, topologically chiral (left) and achiral (right). Bottom: two *locally knotted* tangles, again with opposite chirality. Such tangles *cannot* be formed by moving tethers around and thus do not arise in the strand description of nature – in contrast to *rational* tangles.

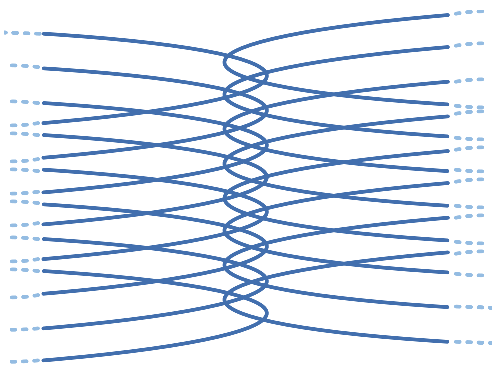

Figure 9: A hypothetical extended defect in space – here with vertical orientation.

possible, many additional objects and elementary particles would be possible in nature. So far, the strand conjecture predicts that *no* additional zero-dimensional configurations can arise.

Pr. 77 An example of a possible *one-dimensional* defect built with strands is illustrated in Figure 9. The illustration can be seen as a potential candidate for a cosmic string or a thin cylindrical black hole. Are such linear defects stable against fluctuations? The strand conjecture suggests that they are not. Such defects are expected to decay into a mixture of gravitons, matter and radiation particles – and thus not to be observable. However, the details remain a topic for research.

Pr. 78 Further *two-dimensional* strand defects could also exist. Figure 9 could also be the cross section of a two-dimensional defect, such as a domain wall. But first explorations of the stability of domain walls, of wormholes, time-like loops, toroidal black holes or black holes with other non-trivial topologies lead to negative results. Such configurations are expected to decay into elementary particles and classical black holes, due to the fluctuations of the involved strands. Two-sided weaves, i.e., weaves in which some strands leave on one side and other strands leave on the other side, are also expected to decay, mainly into elementary particles – and thus not to be observable. In fact, strands predict that, due to low probability, such configurations never form.

Pr. 79 In the strand conjecture, horizons, being weaves of fluctuating strands and being subject to damping, are *minimal* surfaces. In particular, strands predict that physical horizons have *simple* topology. Toroidal horizons appear to be unstable in the strand conjecture – and thus not to be observable.

Pr. 80 Strands allow exploring the possibility of *three-dimensional* defects. Expanding the discussion in Section 13 above, it appears that *tight* macroscopic three-dimensional defects are physically indistinguishable from two-dimensional defects, because no crossing switches are possible inside such a volume. In fact, the interior of a black hole could also be conjectured to be a tight three-dimensional defect. Given that such a structure would not be not observable, it is predicted to be of no physical importance. The question then is whether *loose* macroscopic three-dimensional defects exist. First explorations appear to suggest that all imaginable defects can be constructed from curvature, from particles and from horizons.

Pr. 81 Strands also allow statements about a frequently discussed type of volume defect: a (macroscopic) region of negative energy. Energy being action per unit time, and action being connected to crossing changes, strands do not allow the construction of regions with negative energy. In contrast, strands do allow the construction of regions with lower energy than their environment, as in the Casimir effect: in such regions, field fluctuations are simply constrained by the boundaries. In short, there does not seem to be room for additional three-dimensional defects in the strand conjecture.

Pr. 82 The strand conjecture does not allow additional spatial dimensions, and does not allow black holes or other structures in higher dimensions. This restriction is due to the inclusion of both quantum effects and gravity in the fundamental principle. *If higher dimensions are ever observed, the strand conjecture is falsified.* Conversely, mathematical results in higher dimensions *cannot* be used to falsify the strand conjecture.

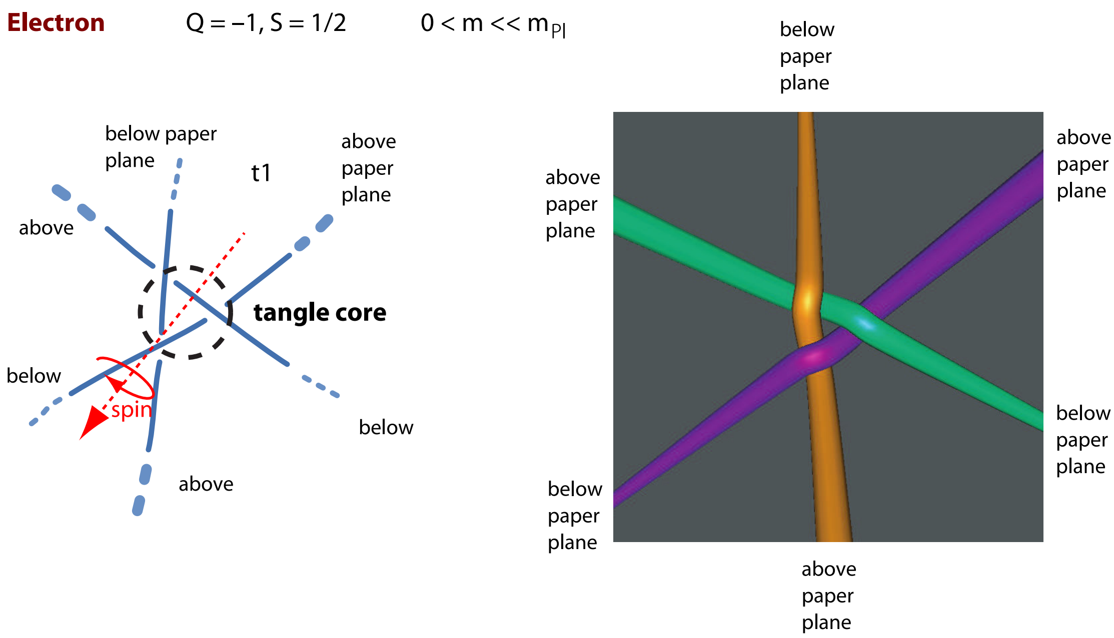

Figure 10: In the strand conjecture, elementary particles are modelled as rational tangles of strands. Single strand segments, including all tethers, are not observable. Only crossing switches are observable. Together with their fluctuating shape, tangles lead to the observation that particles are localized in the region of the tangle core. Tangles are called *rational* when they are formed by switching tethers. Only rational tangles model the observed behaviour of elementary particles. Fermion tangles, such as the one in the figure, automatically have spin 1/2.

Pr. 83 Strands predict the lack of additional elementary particles, as shown in reference [18] and summarized in Appendix B. In particular, strands predict the lack of unknown elementary dark matter particles or of other strand defects having the effects of dark matter. Conversely, strands appear to predict that gravitational lensing is always due to conventional matter or to black holes. *Any discovery of a new elementary particle would falsify the strand conjecture.*

Pr. 84 Strands predict that all energy in nature is due to crossing switches. In particular, strands appear to predict the lack of specific dark energy defects or specific dark energy configurations. *The discovery of any new or additional substance, particle or field at the origin of dark energy would falsify the strand conjecture.*

In summary, the strand conjecture appears to predict the absence of additional defects in space, whether static or dynamic. However, a *definite* topological treatment of the question, including a definite classification and complete survey of all defects, is still lacking.

16 Strand predictions about elementary particle masses

So far, the strand conjecture has not predicted anything new. However, new predictions are possible, and in particular, predictions about elementary particle masses.

Black holes are made of large numbers of woven strands. It is natural to assume that elementary particles are made of *a few* woven strands. Indeed, in the strand conjecture, all elementary particles are *rational tangles* – i.e., woven, unknotted tangles – made of one, two or three strands. Tangles made of four or more strands are composed, not elementary. An example of a rational tangle is shown in Figure 10.

Among tangles made of a few strands, those made of one strand are bosons; more precisely, they are photons. Massive elementary particles tangles are made of two or three strands.

Every fermion tangle, being a tethered structure that is tangled in the region of its tangle core, has non-vanishing mass. Every fermion tangle reproduces spin 1/2 behaviour under rotations – using Dirac's belt trick – and fermion behaviour under the exchange of positions of tangle cores. All tangles reproduce the gauge groups U(1), SU(2) and SU(3) as the result of Reidemeister moves on their tangled cores.

Only *rational* tangles – i.e., tangles that arise through the motion or braiding of their tethers – allow reproducing the transformation of particles observed in experiments. And only rational tangles allow a classification into a finite number of families that correspond to the observed elementary particles. These arguments are summarized in Appendix B and are worked out in detail in references [18] and [31].

Section 9 on gravity at low curvature has shown that mass is the property of tangles that creates virtual gravitons around them. This implies:

▷ The *particle mass* (in corrected Planck units) is the probability of strand crossing switches occurring, per Planck time, in spontaneous belt tricks of the particle tangle.

Rational tangles directly allow deducing a number of predictions about mass values of elementary particles.

Pr. 85 The strand conjecture implies that elementary particles are not black holes. Tangles woven from a few strands – elementary particles – have no horizon and cannot and do not evaporate. This is observed.

Pr. 86 Strands promise, through the analogy between thermodynamic effects and gravitational attraction, to allow calculating the gravitational mass of quantum particles. In particular, the value of gravitational mass is predicted to depend on the tangle structure and thus on the *tangle shape* of the particle – and on nothing else.

Research has shown that the average shape of a fluctuating tangle is the same as the shape of a tight tangle [81, 82]. *Therefore, the mass of an elementary particle is a function of its tight tangle shape.*

Since particle mass is due to their (tight) tangle shape, the mass values of all elementary particles are predicted to be positive, equal to that of their antiparticles, fixed, unique, calculable and constant in time and space. This agrees with data. *If particle masses would be found to vary over space or time, the strand conjecture would be falsified.*

Pr. 87 In the strand conjecture, all fermions are localizable (i.e., not trivial) tangles. Thus, fermions have positive mass. The model of the graviton implies that gravitational charge, or mass, of a fermion is defined by the (tight) *shape* of the fermion tangle core. In the strand conjecture, mass values are automatically discrete, but are not integer multiples of a smallest value. In the strand conjecture, mass thus automatically differs from the charges of the gauge interactions, which are integer multiples of a smallest value. This agrees with observation.

In the strand conjecture, as shown in references [18, 31], only particles with positive mass can have electric and weak charge. In addition, it was shown that all mass values are due to Yukwawa coupling to the Higgs. Furthermore, the tangle model, automatically, only those particles that couple to the Higgs are observed to be massive. All this agrees with observation.

Pr. 88 The tangle model of elementary particles implies that both the gravitational and the inertial mass of elementary particles are due to tether fluctuations. *Gravitational* mass describes the virtual gravitons around a mass: as explained in Section 9, virtual gravitons arise in the tethers due to the belt trick. *Inertial* mass describes how a rotating mass advances through the vacuum with the belt trick, as described in reference [18]. In the strand conjecture, it turns out that these two processes are exactly the same: both involve tether fluctuations around the core, and in particular, both involve the belt trick. Therefore, inertial and gravitational mass are equal – for infinite, flat space. Strands thus imply that the *equivalence principle* holds, in its weak and strong forms – at least for sub-galactic scales, when there is no effect of the cosmological horizon. This agrees with observations [75].

Pr. 89 Strands imply that elementary particle mass values *run* with four-momentum. The reason is that the tangles completely reproduce quantum field theory, as summarized in Appendix B: elementary particles are surrounded by virtual particle pairs; thus their mass values run with four-momentum. This agrees with observations – e.g., [83] – and expectations.

Pr. 90 It is not straightforward to estimate the entropy for the addition of an elementary particle tangle to the vacuum network. But one statement follows directly: because spontaneous tangle fluctuations leading to the belt trick are *rare*, the entropy is positive and it is *much smaller* than the maximum possible value. The gravitational mass m of elementary particles is thus predicted to be positive but also much smaller than the corrected Planck mass:

$$0 < m \ll \sqrt{\hbar c/4G} \ . \qquad (49)$$

This inequality agrees with observations and agrees with old arguments [84]. Strands thus provide a general answer to the *mass hierarchy* problem.

Pr. 91 Strands imply that falling particles are fluctuating and diffusing tangles. Describing particle mass as a thermodiffusion coefficient implies that *more complex* particle tangles have *higher* gravitational mass (for equal number of tethers). The same connection has already been deduced for inertial mass in a different way [18]. This connection yields the correct mass sequences for all hadrons and predicts normal mass ordering for neutrinos. This agrees or is compatible with observations [18]. *If neutrino masses would* not *obey normal ordering, the strand conjecture would be falsified.*

The tangle model also explains that neutrinos mix and that their mass values are stable under renormalization, as shown in references [18, 31]. Strands thus allow non-vanishing neutrino mass in the standard model of particle physics.

Pr. 92 Strands allow deducing approximations for the mass values of elementary particles. As mentioned, the mass is given by the number of crossing switches per time that occur around the particle. For a fermion, the crossing switches are generated by the tethered rotation of the particle, illustrated in Figure 11. The figure yields

$$m \approx n \cdot f \cdot p \ , \tag{50}$$

where n is the number of crossing switches arising for each belt trick, p is the probability for the initial double rotation of the core and f is the probability or frequency of the subsequent belt trick.

The factor p describes the process from the first to the second configuration in Figure 11. For a symmetric core, the rotation probability, whatever the orientation of the axis, is expected to be equal in clockwise and anticlockwise direction. In other terms, p vanishes for symmetric tangle cores. For slightly non-symmetric tangle cores, as is the case for tangles, the factor p is thus expected to be quite small. Its value will depend on the (averaged, three-dimensional, geometric) *asymmetry* of the tangle core. The asymmetry is the quantity that couples to the Higgs braid. A non-zero asymmetry leads to a non-zero mass.

The belt trick frequency f for the process that changes the second configuration in Figure 11 into the sixth configuration will also be small, as it competes with the inverse rotation of the tangle core. Interestingly, this small frequency is expected to be roughly *scale independent*: the size of the tangle core does not play an important role.

Finally, the average number n of crossing switches per belt trick plays a role. The number n counts the crossing switches among tethers and also those between the tangle core and the tethers. This number will depend on the *size* of the tangle core.

The explanation for particle mass m can be checked before any calculation or estimate is performed. As mentioned above, the resulting particle mass value is equal for particle and antiparticles, constant over space and time, and not quantized in multiples of some basic number. Gravitational and inertial mass are equal. Mass values run with energy, i.e. with the looseness of the tangle core. Mass values, via p, depend on the Yukawa coupling to the

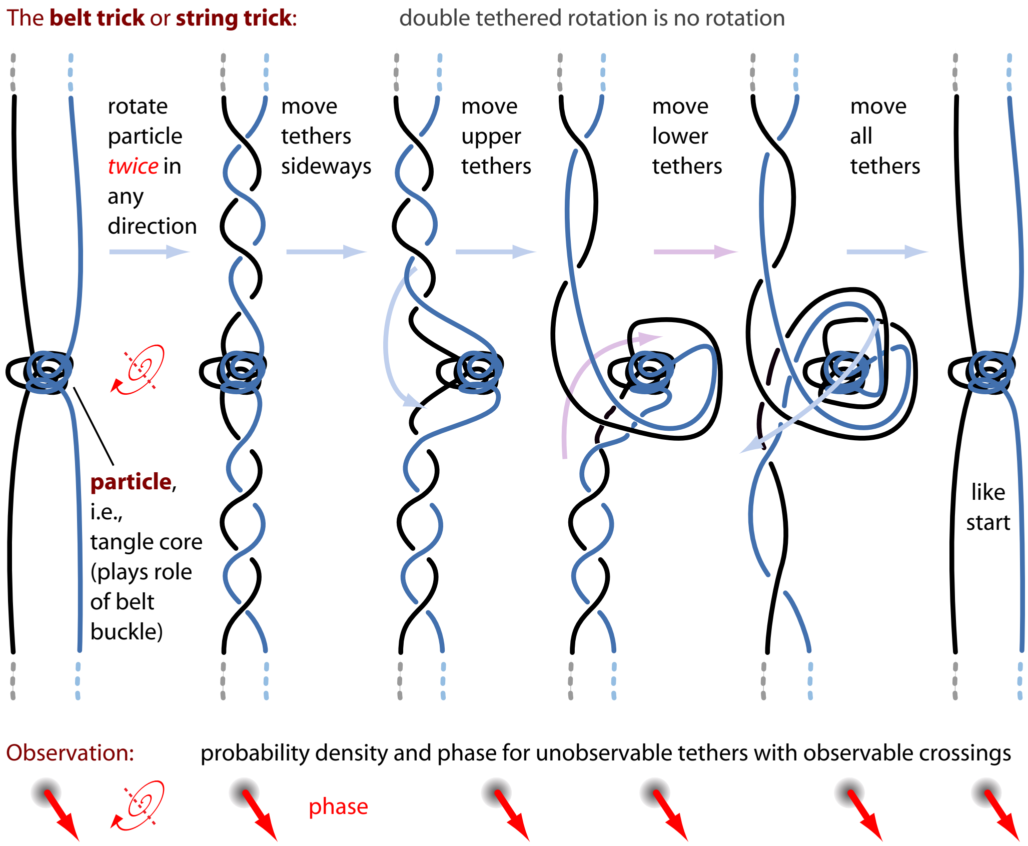

Figure 11: The *belt trick* or *string trick*, as popularized by Dirac, shows that a rotation by 4π of a tethered particle, such as a belt buckle or a tangle core, is equivalent to no rotation – when the tethers are allowed to fluctuate and untangle as shown. This equivalence, illustrated here in six configurations, allows the tethered particle to rotate forever. Untangling is impossible after a rotation by 2π only. The trick illustrates *spin* $1/2$ – if one assumes that tethers are not observable, but crossing switches are. The belt trick works for *any number* of tethers or belts. In contrast to this illustration, in the strand conjecture, leptons have six tethers, nor four; the tangle core topology determines the particle type. Above all, the belt trick allows estimating particle mass, if the probabilities for the six configurations are explored (see text).

Higgs boson. Particle mass values, due to the factor f, are much smaller that the Planck mass. Above all, as expected, particle mass values, due to the factors p and n, increase for more complex tangles, as large tangles are also more asymmetric.

At present, a direct calculation of m, even an approximate one, is still elusive. However, some numerical statements can be made.

Pr. 93 Strands allow deducing a *lower limit* for the (bare) mass values of elementary *leptons*. Leptons are made of three strands and thus have 6 tethers [18, 31]. In the mass expression (50), the number n is surely larger than 24, counting just the crossing switches in the tethers.

The rotation probability p for a neutrino results from the averaged asymmetry of its tangle core. For an electron neutrino, the asymmetry that results from the geometric chirality of the tangle is negligibly small. It is expected that the asymmetry arises only through the mixing with the other two neutrinos, and through the Yukawa term. The averaged asymmetry is hard to estimate, and expected to be larger than a part per million, so that a lower limit should be

$$p \approx 10^{-6} \quad . \tag{51}$$

A systematic error of a few orders of magnitude is expected.

For a neutrino, the belt trick frequency f for the subsequent configuration change results from the probability that the belt trick arises instead of the backwards rotation of the core. To occur, the tether configuration has to form six circles around the tangle core, all with the same orientation. The size of the six circles is not important. For each tail, the probability is roughly given by the probability to form a circle divided by the number of possible rotation axes. Thus one gets the rough estimate

$$f \approx \left(\frac{\mathrm{e}^{-2\pi}}{6 \cdot 4 \cdot 2} \right)^{6} \approx 3 \cdot 10^{-27} \quad . \tag{52}$$

where the exponent is due to the six tethers of the leptons. Again, a systematic error of a few orders of magnitude is expected.

The lower mass bound m_{ll} for leptons thus is

$$\frac{m_{\mathrm{ll}}}{\sqrt{\hbar c/4G}} = p \cdot f \cdot n \approx 10^{-31} \quad , \tag{53}$$

i.e., of the order of meV/c^2, though with a large error margin. So far, this lower limit does not seem to be in contrast with the present experimental limits on neutrino mass, which is 0.9 eV [85, 86]. However, the difficulty of deriving a reliable lower mass bound for leptons is evident.

Pr. 94 Strands allow deducing an *upper limit* for the mass value of leptons. For the heaviest lepton, the estimate $n = 24$ used for the lower mass limit will remain valid. The estimate for f will change for tangle cores that are elongated; the factor $(6 \cdot 4 \cdot 2)^6$ will be of the order of $O(10)$. Finally, the probability p due to the asymmetry will be of the order $O(1)$.

As a result, the upper mass limit for leptons will be

$$m_{\mathrm{ul}} \approx 10^{15} \cdot m_{\mathrm{ll}} \quad , \tag{54}$$

The most massive lepton, the tau lepton, has an observed mass of $1.7\,\mathrm{GeV}/c^2$. Its mass is several orders of magnitude below the upper bound. Again, the difficulty of deriving reliable particle mass estimates becomes evident.

Independently of the uncertainty in the lower lepton mass limit in the previous prediction, the factor 10^{15} is an upper limit for the mass ratio between the most massive and the least massive lepton. The actual ratio is not yet known, because the neutrino masses have not been measured yet [86]. Nevertheless, there is no reason to think that the factor is exceeded.

More precise estimates of particle masses will require the development of better approximations and of suitable computer simulation programs. This will allow determining the probability of the belt trick for each particle tangle core. At present, this challenge is still open. *The failure to reproduce the correct mass value of a single particle, at any single energy value, would falsify the strand conjecture.*

Pr. 95 Together with the predictions listed above, the strand conjecture appears to predict that elementary particle masses, mixings, couplings and the vacuum energy density are the *only* observable quantum gravity effects in nature. So far, this prediction agrees with data.

17 Strand predictions about other microscopic models

Pr. 96 The strand conjecture is not viable *in other dimensions*, because crossing switches and tangles are not possible in those cases. The strand conjecture is not viable *for branched strands*, because such strands lack the uniqueness of crossing switches. The strand conjecture is not viable *without the Planck scale*, because Dirac's equation and Einsteins's field equations would not arise. The strand conjecture is not viable *for strands with non-trivial cross sections*, such as bands, twisted ropes, triangular tubes, because twists in such bands are not observable. The strand conjecture is not viable *for knotted strands*, because such structures prevent particle reactions. The strand conjecture is not viable *for strands carrying quantum numbers or fields*, because such structures, added ad hoc, prevent the full emergence of all observables.

In other terms, modifying or generalizing the strand conjecture seems impossible. *If a modification or a generalization of the strand conjecture is found, the strand conjecture is falsified: in that case, it would not be final.*

Pr. 97 The strand conjecture is just one among a large number of approaches to quantum gravity. Other proposals include loops, superstrings, super-membranes, spin networks, tensor networks, causal sets, triangulations, graphs, microscopic wormholes or exotic manifolds. So far, these proposals did not lead to the emergence of the standard model. Strands predict the lack of inequivalent explanations for the Lagrangians of general relativity and for the standard model of particle physics. *If an alternative, non-equivalent explanation of the* *standard model is found, the strand conjecture is falsified, because a complete description must be unique.*

Pr. 98 More precisely, the strand conjecture predicts to be the only explanation for the values of the fundamental constants, i.e., particle masses, coupling constants and mixing angles. *If an alternative, non-equivalent explanation of particle masses is found, the strand conjecture is falsified.*

These are strong, almost foolish predictions. However, the predictions are required from every candidate for a complete description of nature. Not stating the predictions would be dishonest. So far, they are realized.

18 Discussion and outlook: further tests

Describing nature with the help of strands requires to change some habits of thought. On the one hand, it is not easy to think about nature as made of strands. It is also unusual to describe physical processes as made of fundamental events. On the other hand, the conjecture has the charm of deriving all observations about general relativity (at sub-galactic scales) directly from the Planck scale. Also, the complete standard model of particle physics, with its Lagrangian, arises from the Planck scale, as argued elsewhere [18, 31]. So far, no deviations from these two descriptions have been observed in any experiment.

The task of checking a conjectured description of nature is never finished. *The discovery of any new deviation from general relativity at sub-galactic scales would invalidate the strand conjecture.*

The possibility that additional quantum gravity effects are unobservable has already been explored in the past [87, 88, 89]. Strands confirm the result. They can do so because they incorporate general relativity, quantum physics and the standard model exactly.

The promise of the strand conjecture to calculate particle masses must be pursued. Such calculations will allow the most stringent test of the conjecture. So far, they also appear to distinguish the strand conjecture from other approaches to quantum gravity.

The details and consequences of the strand conjecture for cosmology, especially for the nature of dark energy and for the origin of the effects usually attributed to dark matter, should be investigated. They will allow testing the conjecture even further. A forthcoming paper will provide a first step.

Exploring rotating and charged black holes remains an ongoing research topic. The same applies to the relation between the strand conjecture for black holes, 'fuzzballs' and 'firewalls'. The calculation of black hole radiation probabilities for each tape of radiated particle is another open topic. These investigations might lead to additional tests of the strand conjecture.

Exploring additional Planck limits should also be possible. For example, deriving the limit on the ratio between viscosity and entropy density found by Kovtun, Son and Starinets [90] should be possible with the help of strands.

Strands also allow exploring the issues raised by the combining gravity and quantum mechanics. The gravitational effects of quantum superpositions – for example, the gravity of quantum particles passing double slits – should be investigated further. Also the relation between entanglement and gravitation should be explored further.

Furthermore, despite the arguments of Section 15, additional strand defects might arise in nature. The arguments need to be made more stringent. Also the experimental search for new particles and objects should continue. *Any contradiction with observations would invalidate the strand conjecture.*

Through their combination of continuity and discreteness, strands imply that various technical and mathematical problems about gravitation – see the collection by Coley [91] – acquire a different flavour. Problems about singularities and higher dimensions loose their critical status. Issues about horizons – formation, stability, and minimal mass – become more accessible. Technical difficulties, such as the motion of test particles, the Newtonian limit, the positive energy theorem, the stability of Minkowski space-time, and the definition of angular momentum appear more tractable than without strands. It seems especially promising to explore the definition and properties of ADM mass using strands. Also the question of asymptotic safety of gravity should be investigated further.

It will be also instructive to continue exploring the comparison between strands and the work in the "it from qubit" field. The field was started by Weizsäcker [28], continued by Wheeler [29], and named by Zizzi [30]. A crossing switch is a quantum two-state system and can be seen as a model for a "qubit". The emergence of all physical observables and all physical systems from crossing switches can be called the emergence of "it". In this context, the compatibility of the strand conjecture with *entropic gravity*, shown above in Section 9, is also suggestive. There is also a resemblance to the work relating space to entanglement [92].

The strand conjecture differs from *holography* in quantum gravity more than it resembles it, though deducing closer connections might be possible. In particular, the relation between strands and conformal symmetry, conformal field theory, and conformal gravity [93] should be investigated.

The strand conjecture also differs from *quantum hair* [94] because it states that strands are *not* quantum hair. However, the difference should be investigated in more detail.

One could also explore how much the strand conjecture can be related to the *twistor* approach to nature [50]. A crossing, as illustrated in Figure 12, can be seen as a four-dimensional subspace, spanned by the four angles describing the crossing, attached to a point in background space. This resembles twistor space. A certain resemblance also occurs for non-commutative space [48] and for the *aikyon* approach based on octonions [95]. It may also be possible to deduce a mapping between the work by Loll [96] on causal dynamical triangulations and the strand conjecture.

It should be added that models that are *equivalent* to strands do appear possible. Reference [74] briefly explores *funnels* as alternative to strands. Strands have a constant diameter and cross section. However, their diameter could *increase* away from the tangle core. Even the cross section could vary away from the tangle core. In this way, 'thick' strands, or (double) 'funnels', could fill up space almost completely, without changing any of the fundamental relations between topology and physics.

Strands or funnels could also be replaced by their geometric *complement*. Thus, instead of exploring the motion of strands or funnels, one could explore the motion of the space manifold between them. This approach is in line with the old aim to describe all of nature using just geometry and has been explored by Asselmeyer-Maluga [97] and by Giulini [98]. A description using strand complements would effectively deduce particles and quantum theory from the space manifold.

Finally, possible alternatives to the strand conjecture should also be explored – with energy and dedication. It remains bewildering that simple fluctuating strands describe nature.

19 Conclusion

The conjecture that nature is made of fluctuating Planck-sized strands appears to provide a quantum description of space and gravitation that is correct, complete and consistent. Though no direct observation of strands is possible, the experimental consequences of their existence and of their properties defined in Figure 1 and in expressions (1), have been explored in detail. In particular, strands imply the existence of a maximum mass flow rate $c^3/4G$, a maximum force value $c^4/4G$ and a maximum power value $c^5/4G$. Strands predict the validity of general relativity at sub-galactic scales and of all black hole properties, without any measurable deviation, up to the highest measurable energies and the smallest measurable length scales. Overall, more than 90 specific predictions covering all known aspects of gravitation were deduced. Among them, strands predict the lack of new observable quantum gravity effects. This might be the first time that this prediction arises from a quantum description of gravitation. Strands predict the lack of unknown elementary dark matter particles. So far, all the predictions deduced from the strand conjecture agree with all available data.

As a new result in the domain of quantum gravity, strands propose a solution to the mass hierarchy problem and suggest that the gravitational mass of elementary particles can be calculated ab initio from their tangle details. As long as the over 90 predictions about space, gravity and mass deduced here and the over 140 predictions about particle physics deduced in references [18, 31] are not falsified, strands remain a candidate for a complete description of nature. The deviations from general relativity that strands imply at galactic and at cosmological scales – especially about dark matter, modified Newtonian dynamics and dark energy – will be explored in a subsequent paper.

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Appendix A On the circularity of the fundamental principle

On the one hand, the crossing switch of Figure 1 is assumed to take place in space. On the other hand, space, distances and physical observables are assumed to arise from strands. The apparent circularity can be avoided – to a large degree, but not completely – by increasing the precision of the formulation.

Crossing switches take place in *background space*. In the strand conjecture, background space is defined by the observer. In contrast, *physical space*, physical distances and physical observables arise from strands and their crossing switches. When space is flat, background space and physical space coincide. Otherwise, they do not; in that case, background space is (usually) the local tangent space of physical space. A similar situation arises for the concept of time.

In nature, any observation of a *change* implies the use of (background) time; any observation of *difference* between objects or systems implies the use of separation in (background) space. Indeed, a local background space – observer-defined and usually observer-dependent – is *required* to describe *any* observation, or simply, to talk about nature. In the strand conjecture, it is equally impossible to define crossing switches or any Planck unit without a background. The strand conjecture asserts that a description of nature *without* a background space and time is impossible.

Every use of the term 'observation' or 'observable' or 'physical' implies and requires the use of a background space and time. All the illustrations of the present work are drawn in *background* space. In contrast, *physical* space – an observable in general relativity, dynamical and pseudo-Riemannian – arises through crossing switches of strands. The local background space agrees with physical space only locally, where the crossing switches being explored are taking place. In fact, the need for a background space to describe nature is rooted in a deeper issue.

Background space is what is needed to *talk* about nature. Physical space is everything that can be *measured* about space: curvature, vacuum energy, entropy, temperature etc.

There is a fundamental contrast between *nature* and its precise *description*. The properties of nature itself and the properties of a precise description *differ* and *contradict* each other. A precise *description* of nature requires axioms, sets, elements, functions, and in particular continuous space, continuous time, and points in space and time. In contrast, due to the uncertainty relations, at the Planck scale, nature itself does not provide the possibility to define points in space or time; space and time are not continuous at smallest scale, and in fact, (physical) space and time are emergent. Due to the uncertainty relations, neither sets, nor elements, nor axioms appear to exist in nature at the most fundamental level, i.e., at Planck scales. In short, observer space, or *background space*, differs in its properties from *physical space.*

Any precise description of nature thus requires a limited degree of circularity in its definition of physical time and space with the help of background time and background space. This unavoidable result has a number of implications.

An axiomatic description of *all* of nature is *impossible*. An axiomatic description is only possible for those *parts* of nature that avoid the fundamental circularity, such as quantum theory, or special relativity, or quantum field theory, or electromagnetism, or general relativity. Even though Hilbert asked for an axiomatic description of physics in his famous sixth problem, no claim for an axiomatic description of *all* of nature (all of physics) has ever appeared in the literature.

Accepting a basic circularity in physics also resolves a related issue. In physics, on the one had, *space* is defined with the help of particles – for example, via *rulers* made of matter that measure distances. On the other hand, particles are defined with the help of space – for example, via energy and spin that are *localized* in three dimensions. Distinguishing background space from physical space (almost) eliminates the circularity.

Accepting a basic circularity also resolves issues about the fundamental principle of the strand conjecture. Indeed, by using observer space / background space, the fundamental principle reproduces the circularity of physics. Strands indeed define physical space with particles and particles with physical space: physical space and gravitation can be seen to arise from strands fluctuating in a (local) background space; quantum theory arises from strand fluctuations of matter particles once (flat) physical space is defined. As a result, in the strand conjecture, locality is emergent.

Due to the use of background space, the strand conjecture (or any other unified model) *cannot* be tested by asking whether it is an *axiomatic* description of nature; it is not. In fact, *no* unified description of nature can be axiomatic. Any unified description of nature must be circular. However, the strand conjecture (or any other unified model) *can* be tested by asking whether it is a *consistent*, *complete* and *correct* description of nature. So far, this appears to be the case for strands; references [18] and [31], as well as the present work make this point.

An important example for the difference between an axiomatic description and a consistent, complete, correct – but somewhat circular – description is the dimensionality of space. The number of dimensions of (background and physical) space is not a consequence of the fundamental principle or of some axiom; the number of dimensions is assumed in the fundamental principle right from the start. Tangles only exist in three dimensions. Only three dimensions allow a description of nature that is consistent, complete, and correct: only three dimensions allow crossing switches, particle tangles, spin $1/2$, Dirac's equation and Einstein's field equations.

Appendix B From strands to quantum theory and the standard model Lagrangian

This appendix provides an extremely short summary of references [18] and [31]. They explain how quantum theory, quantum field theory and the full Lagrangian of the standard model arise from strands.

The tangle model for massive quantum particles is illustrated in Figure 12 and Figure 13. The figures visualize that crossings have properties similar to those of wave functions, and that time-averaged crossing switches have the same properties as probability densities.

Starting from the fundamental principle and Dirac's belt trick, *tangles* of fluctuating strands in flat (physical) space indeed describe matter particles and wave functions: the wave function of a particle is the *strand crossing density* of its fluctuating tangle. In other words, wave functions

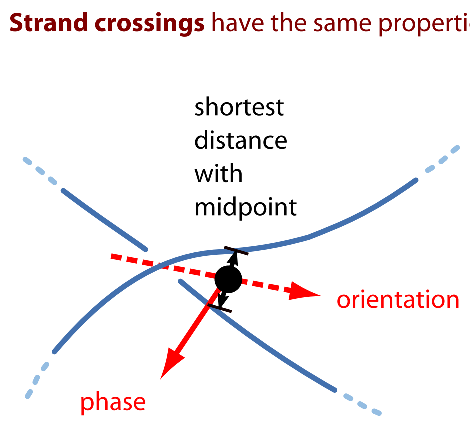

Figure 12: A configuration of two skew strands, called a *strand crossing* in the present context, allows defining density, orientation, position, and a phase. These are the same properties that characterize a wave function. The freedom in the definition of phase is at the origin of the choice of gauge. For a complete tangle, the density, the phase, and the two (spin) orientation angles define, *after spatial averaging,* the two components of the Dirac wave function Ψ of the particle and, for the mirror tangle, the two components of the antiparticle.

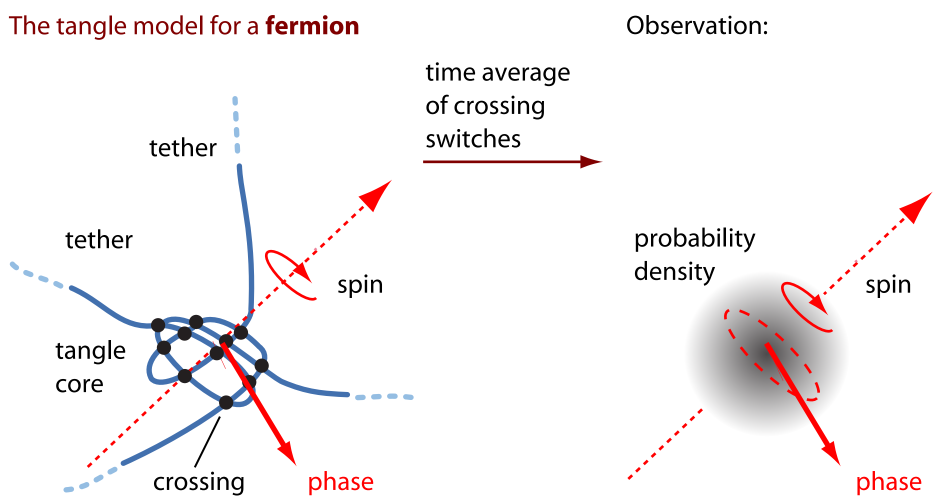

Figure 13: In the strand conjecture, the wave function and the probability density are due, respectively, to crossings and to crossing switches at the Planck scale. The wave function arises as time average of crossings in fluctuating tangled strands. The probability density arises as time average of the crossing switches in a tangle. The *tethers* – strand segments that continue up to large spatial distances – generate spin $1/2$ behaviour under rotation and fermion behaviour under particle exchange. The tangle model also ensures that fermions are massive and move slower than light (see text).

arise as *local time averages* of strand crossings. More specifically, to get the value of the wave function at a certain position in space, the local time average of the strand crossings at that position is taken, averaging over a time scale of (at least) a few Planck times. In this way, a density and a phase can be defined, for each 'position' in space. As usual for quantum theory, also in the strand conjecture physical space and time have to be defined *before* defining the concept of wave function. The probability density for a particle is the local time average of the *crossing switch density* of its fluctuating tangle. A detailed exploration [18, 31, 74] shows that strands produce a Hilbert space, the quantum phase, interference, contextuality, and freedom in the definition of the absolute phase value.

Moving particles are advancing rotating tangles. Antiparticles are mirror tangles rotating in the opposite direction. Fluctuating rational tangles made of two or more strands imply spin $1/2$ behaviour under rotation and, above all, Dirac's equation [16]. For systems of several particles, tangles reproduce fermion behaviour and entanglement. Tangles of strands are fully equivalent to textbook quantum theory and predict the lack of any extension or deviation, up to Planck energy. For example, the principle of least action is the *principle of fewest crossing switches*. In this way, strands also explain the origin of the principle of least action [31].

No new physics arises in the domain of quantum theory. Strands only *visualize* quantum theory; they do not modify it. *Every quantum effect is due to crossing switches* – and vice versa. The visualization of quantum effects with strands requires that strands remain unobservable in principle, whereas their crossing switches are observable.

Tangles also allow deducing quantum *field* theory. Exploring all possible tangles, it appears that *rational*, i.e., unknotted tangles reproduce the known spectrum of elementary particles and their properties [18, 31]. Every *massive* elementary particle is represented by an infinite *family* of rational tangles made of either two or three strands. Quarks are made of two strands; all other massive elementary particles are made of three strands. Three generations for quarks and for leptons arise. The Higgs itself is represented by a braid. The family members for each elementary particle differ among them only by the number of attached braids. The structure of each elementary particle tangle explains the spin value, parity, charge and all other quantum numbers.

Models for the *massless bosons* also arise. In particular, a photon is a single, twisted strand. Photons are emitted or absorbed by topologically chiral tangles, i.e., by fermion tangles that are electrically charged. Figure 14 illustrates the strand conjecture for quantum electrodynamics. Only three kinds of massless bosons arise, each kind due to one Reidemeister move. The boson generator algebras turn out to be the well-known U(1), broken SU(2) and SU(3) of the three gauge interactions [18, 31]. The violation of parity in the weak interaction and the way that the massless bosons of SU(2) acquire mass are also explained.

A detailed investigation shows that tangles reproduces every propagator and every Feynman vertex observed in nature – and no other ones. Particle mixing appears naturally. The correct couplings between fermions and bosons also arise. As a result, the full Lagrangian of the modern standard model arises, term by term, including PMNS mixing of Dirac neutrinos, without any addition or modification [18, 31].

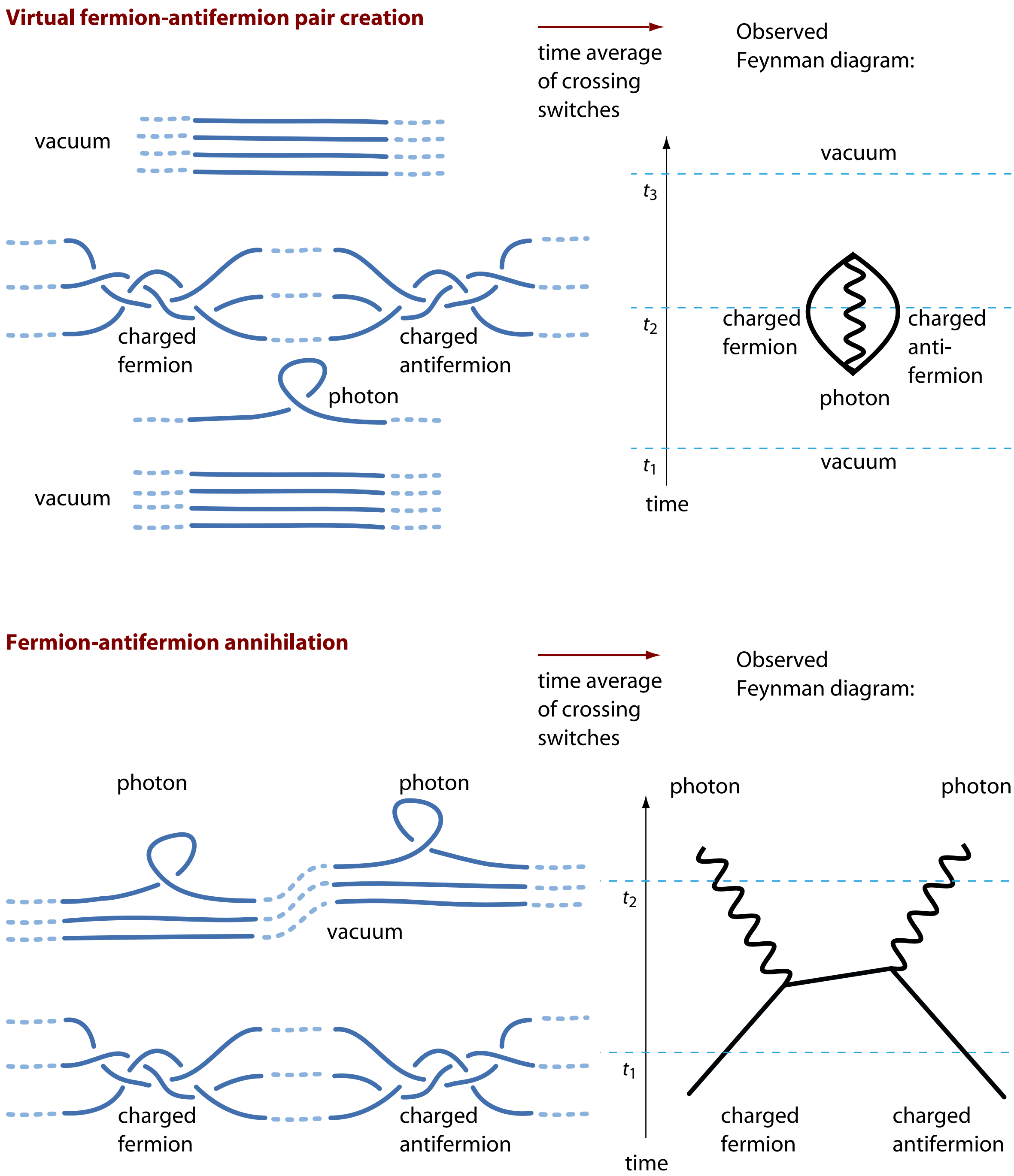

Figure 14: An illustration of two Feynman diagrams of quantum electrodynamics in the tangle model.

Interestingly, rational tangles also promise to explain the values of the *inertial mass* of particles. Inertial mass, like gravitational mass, is related to the *complexity* of the tangle core. More complex tangles have higher masses than less complex tangles. The predicted mass sequences agree with observations [18, 31]. Neutrinos are predicted to be massive Dirac fermions with PMNS mixing and normal mass ordering. The experimental confirmation is still open.

In short, strands imply that no new particles, no new interactions, no other energy scales, no new quantum numbers, now new symmetries, no new dimensions and no new quantum effects are observable in nature. *Any observed deviation from the standard model of particle physics with massive neutrinos and PMNS mixing would falsify the strand conjecture.* Strands predict the lack of any physics beyond the standard model.

So far, the strand conjecture is the only model in the research literature that predicts the particle spectrum, the interaction symmetries, and the fundamental constants. A detailed investigation also shows that quantum field theory does not affect or modify gravity at any sub-galactic energy scale, and vice versa. *Any newly discovered influence between quantum field theory and gravitation at sub-galactic scales – apart from the cosmological constant, the particle masses and the other constants of the standard model, including their running with energy – would falsify the strand conjecture.* This terse summary of the implications of strands for quantum field theory allows proceeding with the exploration of space and gravity.

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