Testing a conjecture on the origin of space, gravity and mass

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Abstract

A Planck-scale model for the microscopic degrees of freedom of space and gravity, based on a fundamental principle that involves fluctuating one-dimensional strands, is tested. Classical and quantum properties of space and gravitation, from the field equations of general relativity to gravitons, are deduced. Predictions include the lack of any change to general relativity at all sub-galactic scales, the validity of black hole thermodynamics, the lack of singularities and the lack of unknown observable quantum gravity effects. So far, all predictions agree with observations, including the validity of the maximum luminosity or power value $c^5/4G$ for all processes in nature, from microscopic to astronomical. Finally, it is shown that the strand conjecture implies a model for elementary particles that allows deducing ab-initio upper and lower limits for their mass values.

Keywords: general relativity; quantum gravity; particle mass; strand conjecture.

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1 The quest to uncover the microscopic aspects of space and gravity

What are the microscopic degrees of freedom of black holes, the microscopic nature of the vacuum, and the microscopic details of curvature? Many possibilities have been proposed and explored [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. In order to show that any proposed microscopic degrees of freedom are candidates for a description of nature, it is necessary to show that they *reproduce* space, curvature, mass and gravitation in all their macroscopic and microscopic aspects. It is equally necessary to show that strands provide *additional* results about gravitation that go beyond the usual description of space as a continuous manifold made of points.

The so-called *strand conjecture* proposes a microscopic model for black holes, particles, space and gravity that is based on one-dimensional fluctuating constituents that are called *strands*. The model is based on a single fundamental principle that describes nature at the Planck scale. It will appear that the strand conjecture agrees with all observations about general relativity and quantum gravity at all sub-galactic scales. Strands also provide a model for elementary particles and their gauge interactions, and suggest a way to estimate their mass values.

2 The origin of the strand conjecture

When Max Planck discovered the quantum of action $\hbar$ in 1899, he found the underlying quantity that explains the observation of all quantum effects in nature [11]. Bohr described quantum theory as consequence of the minimum observable action value $\hbar$ [12]. Heisenberg introduced the canonical commutation relation and Schrödinger introduced the wave function. Pauli included spin and Dirac the maximum energy speed c. From around 1929 onwards, Dirac regularly made use of the so-called *string trick* or *belt trick* in his lectures. The trick, illustrated below in Figure 9, assumes that fermions are connected to spatial infinity by tethers that are unobservable, but whose crossings are observable. With help of the trick, Dirac used to describe spin $1/2$ behaviour as result of tethered rotation. Nevertheless, he never published anything about this connection. Answering a letter from Gardner, Dirac wrote that the trick demonstrates that angular momenta below $\hbar/2$ are not possible in nature [13]. A smallest angular momentum $\hbar/2$ still implies a smallest observable action value $\hbar$.

Historically, tethers were the first hint that nature might be built from unobservable extended constituents. It took several decades to understand that also the complete Dirac equation could be deduced from unobservable tethers. This was first achieved by Battey-Pratt and Racey in 1980 [14]. Independently, in 1987, Kauffman conjectured a direct relation between the canonical commutation relation – and thus Planck's constant $\hbar$ – and a crossing switch of tethers [15]. Again, without stating so explicitly, the assumption was that tethers are unobservable, whereas their crossings are. In the early twenty-first century, independently of the work by Battey-Pratt and Racey and of that by Kauffman, Dirac's trick again led to the discovery of the relation between crossing switches of unobservable tethers, $\hbar$, wave functions, and the Dirac equation [16]. In short, Dirac's trick of invisible tethers implies Dirac's equation. It thus appeared that *every quantum effect* can be thought as being due to unobservable extended constituents – called *strands* in the following – whose crossings are observable.

A question arises naturally: can unobservable strands also explain gravity? The finite value of black hole entropy and its surface dependence provided first hints [17, 18]. Indeed, it turns out that both the properties of black holes and Einstein's field equations can be deduced from crossing switches of unobservable strands. This deduction is repeated below. It thus appeared that *every gravitational effect* can be thought as being due to unobservable extended constituents whose crossings are observable.

The strand conjecture for fundamental physics appears promising also from another perspective. The central parts of quantum field theory can be summarized by the statements that all observable action values obey $W \geq \hbar$ and that all observable energy speeds obey $v \leq c$. General relativity can be summarized by the statement that all observable power values obey $P \leq c^5/4G$, as discussed in various publications [19, 20, 21, 22]. This suggests that nature is fundamentally simple and that a complete and unified description of motion could be based on inequalities.

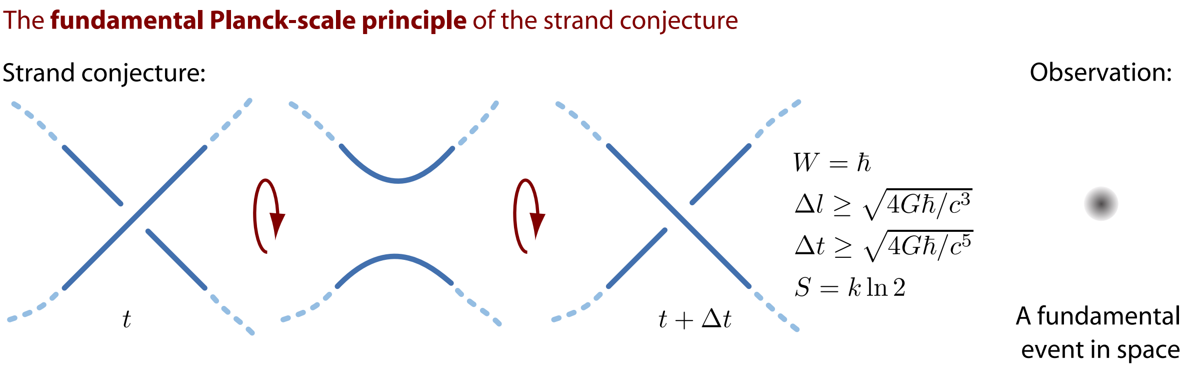

Figure 1: The fundamental principle of the strand conjecture specifies the simplest observation – a simplified version of Dirac's trick, shown in Figure 9, taking place at the Planck scale – that is possible in nature: the almost point-like *fundamental event* results from a *skew strand switch*, or *crossing switch*, at a position in three-dimensional space. The strands themselves are *not observable*. They are impenetrable and are best imagined as having Planck size radius. The observable switch defines the action unit $\hbar$. The double Planck length limit and the double Planck time limit arise, respectively, from the smallest and from the fastest crossing switch possible. The paper plane represents background space, i.e., the local tangent Euclidean space defined by the observer.

3 The fundamental principle and the claims of the strand conjecture

The strand conjecture states: all physical systems found in nature – matter, radiation, space and horizons – are made of strands that fluctuate at the Planck scale but remain unobservable.

▷ A *strand* is defined as smooth curved line – a one-dimensional, open, continuous, everywhere infinitely differentiable subset of $\mathbb{R}^3$ or of a curved 3-dimensional Riemannian space, with trivial topology and without endpoints – that is surrounded by a perpendicular disk of Planck radius $\sqrt{\hbar G/c^3}$ at each point of the line, and whose shape is randomly fluctuating over time.

The strand conjecture is then formulated in the following way:

▷ Strands are unobservable. However, crossing switches of skew strands – exchanges of over- and underpasses – are observable. Crossing switches determine the Planck units G, c and $\hbar$; this fundamental principle is illustrated in Figure 1.

The defining Figure 1 thus combines the essence of Dirac's trick with the Planck limits.

The strand conjecture claims: the fundamental principle of Figure 1 contains all observations, all equations of motions and all Lagrangians. In particular, the fundamental principle implies:

- ▷ Physical space is a (three-dimensional) *network* of fluctuating strands – i.e., of strands that are neither woven nor tangled nor knotted, as illustrated in Figure 2).
- ▷ Horizons are (two-dimensional) *weaves* of fluctuating strands – i.e., similar to a fabric made of woven threads, and illustrated in Figure 3.
- ▷ Particles are (localized) *rational tangles* of fluctuating strands – using the term from topological knot theory, defined and illustrated in Figure 8.
- ▷ Physical motion *minimizes* the number of observable crossing switches of fluctuating unobservable strands.

Using Figure 1, the strand conjecture appears to imply general relativity, fermions, bosons and the gauge interactions, with all their observed properties. The figure also illustrates the most fundamental event and the most fundamental process in nature, from which all other processes are built, including all motion in nature. The following sections check these claims in detail for gravitation at sub-galactic scales. After all the checks are passed successfully, a number of new results are presented.

The implications of the strand conjecture for particle physics, gauge interactions, and the standard model have been explored elsewhere [16, 23]. Tangles of strands imply the particle spectrum, Reidemeister moves imply the three gauge groups and couplings, and tether exchanges imply particle mixings. The complete Lagrangian of the modern standard model follows from strands. In the following, however, only gravitation is explored.

The fundamental principle of the strand conjecture of Figure 1 states that action, length, time and entropy are *limited from below*:

$$W \geq \hbar \ , \ \ \Delta l \geq \sqrt{4G\hbar/c^3} \ , \ \ \Delta t \geq \sqrt{4G\hbar/c^5} \ , \ \ S \geq k \ln 2 \ . \tag{1}$$

Strands realize and visualize these inequalities. Together with Figure 1, they contain everything that is needed to deduce the rest of the present work. The number $\ln 2$ in the minimum entropy is due to the 2 strand configurations. One notes that the minimum length and time are given by *twice* the Planck values. In order to visualize the minimum length, it is easiest to visualize strands as having Planck-size radius.

Apart from their crossings in space – a strand segment passing over another – strands have *no* observable properties. Strands have no colour, no tension, no mass, and no energy. Due to the impossibility of observing strands, strands have no meaningful equation of motion. Indeed, all results in the following are *independent* of the detailed fluctuating motion one might imagine for strands – as long as crossing switches reproduce observations. This aspect eliminates any apparent arbitrariness of the description of space, horizons and matter with fluctuating strands.

Strands *cannot be cut*; they are not made of parts. Strands cannot interpenetrate; they *never* form an actual crossing. When the term 'crossing' is used in the present context, only the two-dimensional projection shows a crossing. In three dimensions, strands are *always at a distance*. Like in Dirac's trick, a crossing *switch* – the change from an overpass to an underpass – cannot arise through strand interpenetration, but only via strand deformation.

In the strand conjecture, *all physical observables* – action, momentum, energy, mass, velocity, length, surface, volume, tension, entropy, field intensities, quantum numbers, etc. – arise from combinations of crossing switches. No physical observable is a property of strands themselves; all physical observables arise from *shape configurations* of *several* strands. In short: all physical observables *emerge* from strand crossings.

4 Deducing physical space from strands

Because strands are unobservable, it is *not* possible to describe them with differential equations, and it is *not* possible to speak about their motion or their dynamics. The *only* observable aspect of strands – as in Dirac's trick – are their crossing switches, and thus, for example, the distribution of crossing switches. To relate strands to physics, it is important to deduce the behaviour of crossing

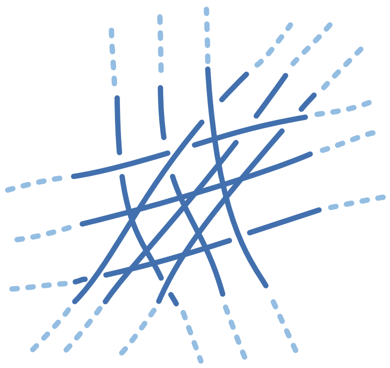

Figure 2: A simplified and idealized illustration of the strand conjecture for a flat vacuum, i.e., for flat physical space. The space of the picture is *background* space. *Physical* space is generated by strand crossings. Strands fluctuate in all directions. (Typical strand distances are many orders of magnitude larger than their diameters.) For sufficiently long time scales, the lack of crossing switches leads to a vanishing energy density; for short time scales, particle–antiparticle pairs, i.e., rational tangle–antitangle pairs, arise in the vacuum due to the shape fluctuations of the strands, as illustrated in Figure 12 below. The difference between background space and physical space is discussed in Appendix A.

switches from the fundamental principle. This is done now, starting with physical space.

In the strand conjecture, a *network* of fluctuating strands is conjectured to yield *physical* flat empty space. A strand network is illustrated in Figure 2. The picture uses background space to define physical space. *Background* space is what is needed to *talk* about nature. *Physical* space is everything that can be *measured* about space: curvature, vacuum energy, entropy, temperature etc. The circularity issues that arise are discussed in Appendix A.

A network of *untangled*, *unwoven* and *unknotted* strands models *empty and flat* physical space. The time-average of the fluctuations, on a scale of a few Planck times or more, yields three-dimensional physical flat space, including its continuity, homogeneity, isotropy and Lorentz-invariance. On sufficiently long time scales, there are (on average) *no* crossing switches, and thus neither matter nor energy – just empty space. Strands thus imply that *no deviation* from the continuity, homogeneity, isotropy, dimensionality and Lorentz-invariance of (physical) flat space can be observed – at any energy – *despite* the existence of a smallest length $l_{\text{min}} = \sqrt{4G\hbar/c^3}$.

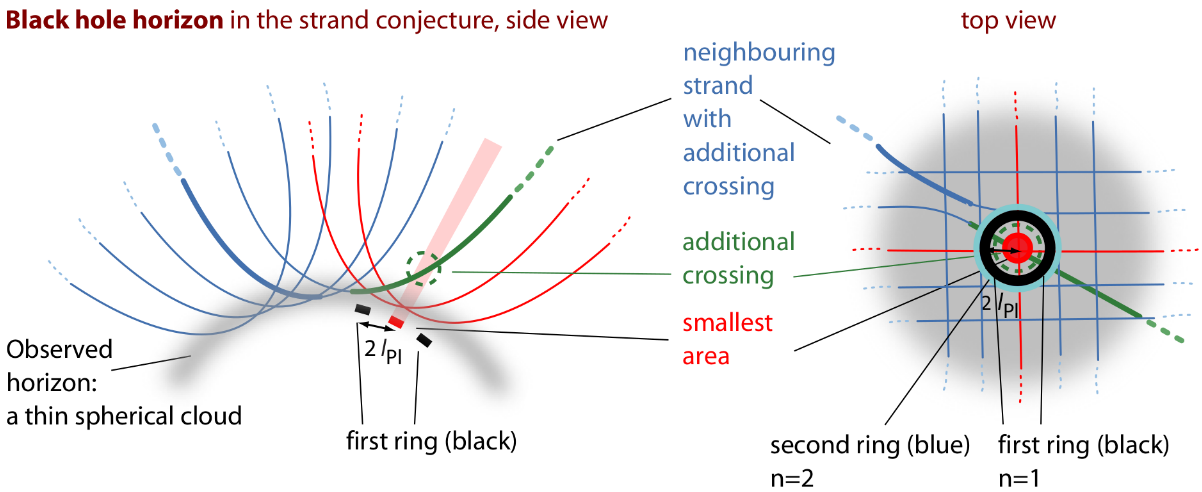

Figure 3: The strand conjecture illustrated for a Schwarzschild black hole, as seen by a distant observer at rest: the horizon is a cloudy or fuzzy surface produced by the crossing switches of the strands woven *tightly* into it. Due to the additional crossings *above* the horizon, the number of microstates per smallest area is larger than 2, and given by the base e of the natural logarithms (see text). This yields the entropy of black holes.

5 Deducing horizons and black holes from strands

In the strand conjecture, *woven* fluctuating strands define horizons. More precisely, the strand conjecture implies that

▷ Horizons are one-sided, tight *weaves*.

In this statement, *one-sided* means that all strands leave the horizon on one side, the side of the observer. One-sidedness means that there is 'nothing', not even an unobservable strand, on the other side of the horizon. A schematic illustration of a Schwarzschild black hole, shown both as a cross section and as a top view for a distant observer at rest, is given in Figure 3. For a black hole, and for any other horizon, all strands come in from far away, are *woven* into the horizon, and leave again to far away. If strands are imagined as having Planck radius, the weave of strands forming a horizon is as *tight* as possible: seen from above, there is one crossing for each smallest area.

At a larger scale, a weave becomes a two-dimensional surface. For a distant observer at rest, a one-sided weave also implies that no space and no events are observable behind it. The weave thus acts as a *limit* to observation. For a falling observer, the strands do not form a weave, but continue on the other side and form a (distorted) network, i.e., curved vacuum. Such an observer does not notice anything special when approaching the horizon, as seen by an observer at spatial infinity, or when crossing it, in its own reference frame. A one-sided weave thus shows the qualitative properties that characterize a horizon.

The strand conjecture for horizons allows determining the *energy* and thus the *mass* of a spherical, non-rotating horizon. Energy E has the dimension action per time. Because every crossing switch is associated with an action $\hbar$, the horizon energy is found by determining the number N_{cs} of crossing switches, multiplied by $\hbar$, that occur per unit time. This number will depend on the surface area of the horizon. In a horizon, crossing switches *propagate* from one crossing to the next, over the surface of the whole (tight) weave. Since the horizon weave is *tight*, the propagation speed is one smallest crossing per shortest switch time: switch propagation thus occurs at the speed of light c. Since the horizon weave is *tight*, each crossings has the size of the minimum length squared, given by $A_{\mathrm{cPl}} = 4\,G\hbar/c^3$. In the time T needed to circumnavigate a *spherical*, non-rotating horizon of area $A = 4\pi R^2$ at the speed of light, *all* crossings of the horizon switch. This yields:

$$E = \frac{N_{\mathrm{cs}}\hbar}{T} = \frac{A/(4G/c^3)}{2\pi R/c} = \frac{c^4}{2G}\,R \;. \qquad (2)$$

The woven strand model of a horizon thus reproduces the relation between the energy – or mass – and the radius of a Schwarzschild black hole.

Strands also determine the number of microstates per horizon area. Figure 3 shows that for a smallest area on the horizon, i.e., for an area that contains just one strand crossing, the effective number N of microstates *above* that smallest area is *larger* than 2. The number would be two if each smallest area would contain just one crossing, with its 2 possible signs. However, a number larger than 2 occurs because also fluctuating neighbouring strands sometimes *cross above* that smallest area.

The probability for a neighbouring strand to cross above a given (central) smallest area will depend on the distance at which the neighbouring strand leaves the lowest woven layer of the horizon. To calculate the probability, one imagines the central crossing surrounded by an infinite series of rings, each with a smallest area value $A_{\rm cPl} = 4\,G\hbar/c^3$. As illustrated in Figure 3, the rings are numbered with a number n. The central crossing corresponds to $n = 0$. Ring number n therefore *encloses* n times the smallest area $A_{\rm cPl}$. The probability that a strand from ring 1 reaches the centre, forming an additional crossing above it, is

$$p_1 = \frac{1}{2} = \frac{1}{2!} \quad . \tag{3}$$

The probability that a strand from ring n reaches the centre and forms an additional crossing is

$$p_n = \frac{1}{n+1}\, p_{n-1} = \frac{1}{(n+1)!} \quad , \tag{4}$$

because the strand has to continue in the correct direction above every ring on its way to the centre. This expression is a result of the *extension* of strands; it would not arise if the fundamental constituents of the horizon would not be extended – in short, if they would not be strands. The expression yields an effective number N of microstates above the central crossing given by

$$N = 2\; +\; \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ... + \frac{1}{n!} + ... = \mathrm{e} = 2.718281... \tag{5}$$

In this expression, the term 2 is due to the two options at the central point; the term $1/2!$ arises from the first ring around it, as shown in Figure 3; the following terms are due to the subsequent rings. Expression (5) implies that the average number N of strand microstates for each smallest area, i.e., for each *corrected Planck area* $A_{\rm cPl} = 4\,G\hbar/c^3$ on the black hole horizon, is given by $N = \mathrm{e}$. In the strand conjecture, every corrected Planck area therefore contains *more* than 1 bit of information (which would correspond to $N = 2$).

The calculation of the entropy of the complete black hole horizon starts with the usual definition

$$S = k\;\ln N_{\rm total} \quad , \tag{6}$$

where k is the Boltzmann constant and N_{total} is total number of microstates of the complete horizon. Because the full horizon area A can be seen as composed of *many* corrected Planck areas, the total number of microstates is given by the product of the number of states for every corrected Planck area:

$$N_{\text{total}} = N^{A/(4\,G\hbar/c^3)} \quad . \tag{7}$$

So far, only standard thermodynamics was used. The next step is to insert the result (5) due to strands. This yields

$$N_{\text{total}} = \mathrm{e}^{A/(4\,G\hbar/c^3)} \quad . \tag{8}$$

This total number of horizon microstates can be inserted into expression (6) for the entropy. The horizon entropy S of a black hole with surface A is

$$\frac{S}{k} = \frac{A}{4\,G\hbar/c^3} \quad . \tag{9}$$

This is the expression discovered by Bekenstein [17]. In the strand conjecture, the *finiteness* of the entropy is thus due to the *discreteness* of the microscopic degrees of freedom. Strands also imply that both the *surface dependence* of the entropy and the factor $1/4$ – including the lack of factors like $\ln 2$ or of a Barbero-Immirzi parameter – are due to the *extension* of the microscopic degrees of freedom.

In short, strands imply the energy E and the entropy S of Schwarzschild black holes. As usual, the ratio $E/2S$ determines the *temperature* of such black holes [24]:

$$T_{\text{BH}} = \frac{\hbar c}{4\pi k}\,\frac{1}{R} = \frac{\hbar}{2\pi k c}\,a \quad . \tag{10}$$

In the last equality, the surface gravitational acceleration $a = GM/R^2 = c^2/2R$ was introduced, using expression (2). In short, black holes are warm.

The finite temperature value implies that *black holes radiate*. As a consequence, strands reproduce black hole *evaporation*. Radiation and evaporation are due to strands detaching from the horizon. If a single strand detaches, a photon is emitted. If a tangle of two or three strands detaches, a graviton or a fermion is emitted. When all strands have detached, the complete black hole has evaporated.

The expressions (1) and the fundamental principle of Figure 1 contain a further result of interest. The gravitational acceleration on the surface of a black hole is $a = GM/R^2 = c^2/2R$; this is the maximum value possible. The value of black hole energy (2) implies that the black hole mass is given by $M = Rc^2/2G$. Taken together, this yields a limit on force $F = Ma$ given by

$$F \leq \frac{c^4}{4G} = 3.0 \cdot 10^{43}\,\mathrm{N} \quad . \tag{11}$$

This is the *maximum force* that can be observed at a single point. The existence of a maximum force is inextricably tied and equivalent to the minimum size of masses in nature. All derivations of its value make use of this connection; for example, $c^4/4G$ is also the maximum possible gravitational force between two black holes [19, 20, 21, 22].

6 Deducing general relativity from thermodynamics

In 1995, in a path-breaking paper, Jacobson showed that the thermodynamic properties of the microscopic degrees of freedom of space and of black holes imply Einstein's field equations of general relativity [25]. He started with three thermodynamic properties:

- the entropy–area relation $S = A\,kc^3/4G\hbar$,
- the temperature–acceleration relation $T = a\,\hbar/2\pi kc$,
- the relation between heat and entropy $\delta Q = T\delta S$.

Using these three properties, the basic thermodynamic relation

$$\delta E = \delta Q \quad , \tag{12}$$

which is valid *only* in case of a horizon, yields the first principle of horizon mechanics

$$\delta E = \frac{c^2}{8\pi G} a\,\delta A \quad . \tag{13}$$

This expression can be rewritten, using the energy–momentum tensor T_{ab}, as

$$\int T_{ab}\, k^a \mathrm{d}\Sigma^b = \frac{c^2}{8\pi G} a\,\delta A \quad , \tag{14}$$

where $\mathrm{d}\Sigma^b$ is the general surface element and k is the Killing vector that generates the horizon. The Raychaudhuri equation [26] – a purely geometric relation – allows rewriting the right-hand side as

$$\int T_{ab}\, k^a \mathrm{d}\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab}\, k^a \mathrm{d}\Sigma^b \quad , \tag{15}$$

where R_{ab} is the Ricci tensor that describes space-time curvature. This equality between integrals implies that the integrands obey

$$T_{ab} = \frac{c^4}{8\pi G} \left(R_{ab} - (\frac{R}{2} + \Lambda)\, g_{ab} \right) \quad , \tag{16}$$

where R is the Ricci scalar and Λ is an undetermined constant of integration. These are Einstein's field equations of general relativity. The value of the cosmological constant Λ is thus *not determined* by the thermodynamic properties of horizons.

As Jacobson explained, the field equations are valid everywhere and for all times, because a suitable coordinate transformation can position a horizon at any point in space and at any instant of time. Achieving this just requires a change to a suitable accelerating frame of reference.

Given that horizons and black holes are thermodynamic systems, so is curved space. In other words, the field equations result from *thermodynamics of space*. Jacobson's argument thus shows that space is made of microscopic degrees of freedom, and that gravity is due to the same microscopic degrees of freedom.

7 Deducing general relativity from strands

As explained in Section 5 above, strands imply the existence of horizons and of black holes. Above all, strands imply their thermodynamic properties: strands reproduce the entropy relation (6) of black holes, the temperature (10) of black holes, and their heat–entropy relation from (2). These are the three conditions for using Jacobson's argument to derive general relativity. Strands thus fully reproduce the argument. Therefore,

▷ Fluctuating strands lead to general relativity.

However, the result must be taken with caution. Jacobson's deduction of the field equations is *independent* of the details of the fluctuations and *independent* of the microscopic model of space, as long as the three thermodynamic properties given at the start are valid. After Jacobson's result, various kinds of microscopic degrees of freedom for space have been conjectured, including those found in references [1, 2, 3, 4, 5, 6, 8, 9, 10]. These explorations have shown that finding the *correct* microscopic degrees of freedom of physical space among all the proposals in the literature is *not possible* using arguments from quantum gravity alone.

Any promising candidate for the microscopic degrees of freedom of space and gravitation must also reproduce the standard model of particle physics and, above all, explain particle masses and the other fundamental constants. This seems *the only way* to differentiate between the various microscopic models of gravitation. Given that strands appear to reproduce the Lagrangian of the standard model – as argued in references [16] and [23] – it is worth exploring them also in the domain of gravitation. *Any experiment finding a deviation from the standard model would falsify the strand conjecture.*

In summary, in the strand conjecture, the field equations – and thus the Hilbert action – appear as consequences of fluctuations of impenetrable, featureless, unobservable strands. The first prediction of strands in the domain of gravitation is:

Pr. 1 *No deviations* between general relativity and the strand conjecture arise.

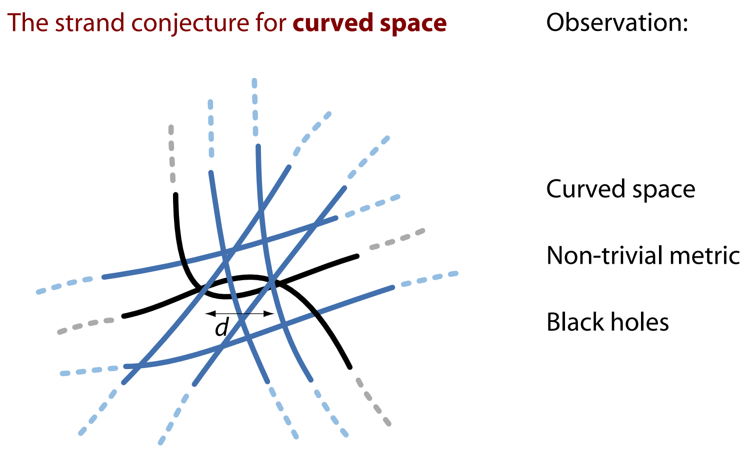

Figure 4: An illustration of the strand conjecture for a *curved* vacuum. The strand and crossing configuration is *not* homogeneous and is midway between that of a flat vacuum and that of a horizon. Strands in black differ in their configuration from those in a flat vacuum. The value of the curvature is inversely proportional to the distance d.

As a result, all processes described by general relativity are reproduced by strands. This includes $1/r^2$ gravity, as illustrated in Figure 5. Therefore, *the smallest deviation between general relativity and observations would falsify the strand conjecture.* Below, in Section 11, the lack of deviations is made more precise: the prediction is limited to sub-galactic distances. The validity for galactic and cosmological distances will be explored in a separate paper.

8 Deducing curvature from strands

Strands not only visualize flat space; strands also visualize curvature. The fundamental principle of the strand conjecture implies:

▷ *Flat space* is a homogeneous network of fluctuating strands.

▷ *Curvature* is an inhomogeneous crossing (switch) density in the vacuum network.

An illustration of spatial curvature is given in Figure 4. The strand configuration differs from that of flat space: certain strands break the isotropy and homogeneity. The main curvature value depends on the configuration of the strands leading to the inhomogeneity. The curvature can evolve over time. This strand model for curved space implies that curved space-time is, locally, a Minkowski space. Thus, strands lead to a *pseudo-Riemannian space-time*.

In short, strands visualize space, black holes, gravity and curvature. It is now time to test the strand conjecture in detail.

9 Strand predictions about physical space

Pr. 2 Because tangling of strands is not possible in other dimensions, strands predict that flat physical space is *three-dimensional*, *unique* and *well-behaved* at all scales. Flat physical space is a three-dimensional *continuum* that is *homogenous* and *isotropic*, without observable deviations.

So far, these predictions about physical space agree with expectations [27] and with the most recent observations [28, 29]. Any evidence for other dimensions, other topologies, quantum foam, different vacuum states, different vacuum states, cosmic strings, domain walls, regions of negative energy, ‘space-time noise’, ‘particle diffusion’, ‘space viscosity’ or crystal behaviour of space, or any other deviation from a well-behaved pseudo-Riemannian space-time manifold would directly *falsify* the strand conjecture.

Pr. 3 As a consequence of the fundamental principle in Figure 1 and of the expressions (1), the maximum local energy speed in nature is c. This applies at all energy scales, in all directions, at all times, at all positions, for every physical observer. In short, the strand conjecture predicts no *observable* violation of Lorentz-invariance, for all energies and all physical systems. It predicts no variable speed of light, no time-dependent speed of light, no time-dependent energy of light, i.e., no ‘tired’ light, no energy-dependent speed of light, no helicity-dependent speed of light, no ‘double’ and no ‘deformed special relativity’. Strands predict the lack of dispersion, birefringence and opacity of the vacuum. So far, all this agrees with observations [30].

Pr. 4 The strand conjecture for the vacuum illustrated in Figure 2 predicts the *lack of trans-*

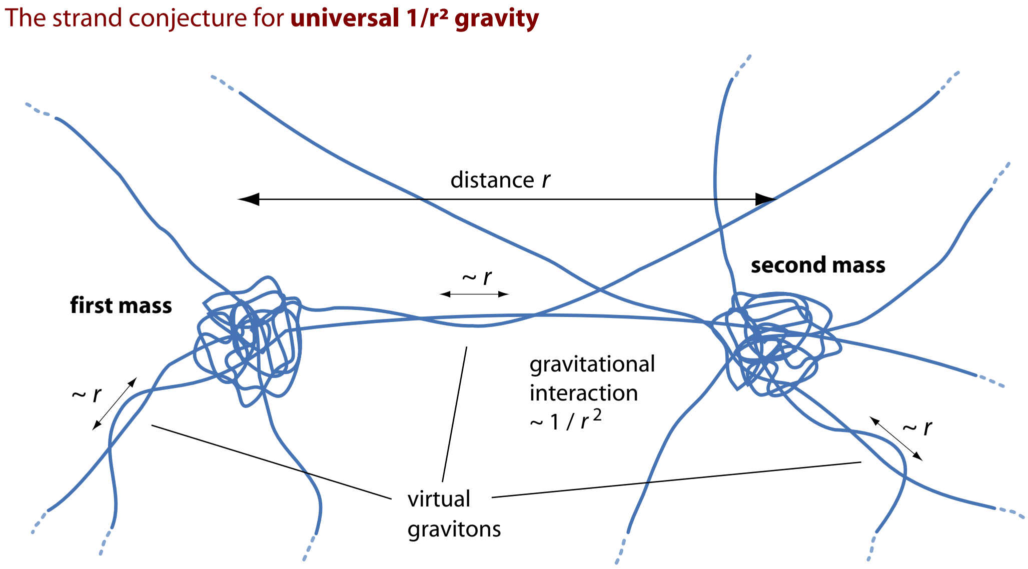

Figure 5: Gravitational attraction results from strands. More precisely, everyday *gravitation* is due to tether pair twists and their influence on tether fluctuations. When speeds are low and spatial curvature is negligible, as illustrated here, twisted tether pairs – i.e., virtual gravitons – from any mass lead to a $1/r^2$ attraction of other masses. The average length of twisted pairs of tethers scales with r. As a consequence, the curvature around such a mass scales as $1/r^3$. These results are valid for infinite, approximately flat space.

Planckian effects. For example, the existence of a *minimal measurable length* given by the corrected Planck length

$$l_{\min} = \sqrt{4G\hbar/c^3} \tag{17}$$

is predicted. If *any effect* due to space intervals smaller than the minimal length can be observed – for example in electric dipole moments [31], in higher order effects in quantum field theory, or in the discreteness of space – the strand conjecture is falsified. The same holds for time intervals shorter than the corrected Planck time. So far, no observation exceeds the corrected Planck limits.

Pr. 5 Strand crossings resemble fermionic or anti-commuting coordinates as used in supergravity, resemble non-commutative space [32, 33], resemble Clifford algebras, and even resemble the internal spaces of the *aikyon* approach based on octonions [34]. A crossing, as illustrated in Figure 10, can also be seen as a four-dimensional subspace, spanned by the four angles describing the crossing, specific to a point in background space, and thus resembles twistor space [35]. Though strands resemble these internal spaces, they do so only at *certain* points in space and at *certain* instants in time, because strands fluctuate. In short, Figure 2 implies that strands do *not* produce fixed internal spaces. This result agrees with data so far, but could be falsified in the future.

Pr. 6 An untangled strand network generates flat physical space. In contrast, a weakly tangled network, as illustrated in Figure 4, generates *curved physical space*. As a result, also curved space has *three* dimensions, at all measurable distance scales, in all directions, at all times, at all positions, for every physical observer. So far, this is observed.

Pr. 7 The value of spatial curvature κ around a mass is due to the tether crossing switch density induced by the mass. In the strand conjecture, the crossing switch density decreases with distance r from the mass. So does the strand inhomogeneity. As illustrated in Figure 5, this yields the proportionality

$$\kappa \sim \frac{1}{r^3} \quad . \tag{18}$$

This relation agrees with expectations. A factor $1/r^2$ is due to Gauss' law, and a factor $1/r$ is due to the average size of twisted pairs of tethers – the virtual gravitons due to Dirac's trick – around the mass. The third power in the decrease of the curvature around a mass is thus due to the three dimensions of space and to the extension of strands. Without extended constituents, an explanation of the $1/r^3$ dependence does not seem possible.

Pr. 8 Strands and expressions (1) imply a *limit* to curvature κ. The limit is given by the inverse of the smallest length:

$$\kappa \leq \frac{1}{l_{\min}} = \sqrt{\frac{c^3}{4G\hbar}} \quad . \tag{19}$$

This limit implies the lack of singularities in nature, of any kind. So far, this prediction is not in contrast with observations.

10 Strand predictions about black holes

The fundamental principle of the strand conjecture – Figure 1 and expressions (1) – allows drawing numerous testable conclusions about black holes. *If any of the following predictions is wrong, the strand conjecture is falsified.*

Pr. 9 The fundamental principle of the strand conjecture – in particular the expressions (1) – implies that in all processes, near or far from horizons, the power or luminosity limit

$$P \leq c^5/4G = 9.0709(3) \cdot 10^{51}\,\mathrm{W} \tag{20}$$

and the force or momentum flow limit

$$F \leq c^4/4G = 3.0257(2) \cdot 10^{43}\,\mathrm{N} \tag{21}$$

are always valid, at every point in space. These limits – one quarter Planck mass per Planck time, or $50\,756(12)$ solar masses per second, times c and times c^2 – are predicted to hold for every local process in nature [19, 20, 21, 22]. (A solar mass of $1.9885(5) \cdot 10^{30}\,\mathrm{kg}$ is used.)

No Earth-bound process approaches the force and power limit, by far. Astrophysical observations are necessary to check the limits. Galaxies, quasars, galaxy clusters, and blazar jets all emit below 10^{-5} solar masses per second. In supernovae and hypernovae, both accretion and emission are below 10^{-2} solar masses per second. Gamma ray bursts emit at most 1 solar mass per second. The fastest observed and simulated accretion processes achieve 10 solar masses per second. The highest observed luminosities so far are those observed in black hole mergers by LIGO and VIRGO [36]. At present, the highest peak powers were observed for the events GW170729 and GW190521. They showed values of $4.2(1.5) \cdot 10^{49}\,\mathrm{W}$ or 230 ± 80 solar masses per second [37] and of $3.7(9) \cdot 10^{49}\,\mathrm{W}$ or 207 ± 50 solar masses per second [38]. All these values are well below the (corrected)

Planck limit of $50\,756(12)$ solar masses per second.

Present data therefore does not yet allow distinguishing between the corrected Planck luminosity limit $P \leq c^5/4G$ and the conventional Planck limit $P \leq c^5/G$ that is four times larger. *Future discoveries might allow a direct test of this aspect common to general relativity and the strand conjecture.*

Similar limits arise for mass flow $\mathrm{d}m/\mathrm{d}t \leq \frac{c^3}{4G}$ and for energy density $E/V \leq \frac{c^7}{16\,G^2\hbar}$, again preventing singularities.

Pr. 10 The strand conjecture implies that a *rotating* black hole realizes a belt trick that involves a *huge* number of tethers. Animations illustrating such a process were available on the internet [39] before the strand conjecture formulated this equivalence, programmed by Jason Hise. Figure 6 shows such a configuration during rotation. In this description, the *ergosphere* is the region in which the crossing switches take place during the belt trick. In contrast to the figure, the horizon of a rotating black hole is flattened at the poles, and so is its ergosphere.

11 Strand predictions about quantum gravity and gravitons

In the derivation of general relativity in Sections 6 and 7, the *cosmological horizon* was not taken into account. Strands thus imply that for *sub-galactic* distances, when the horizon has *no* influence, *general relativity holds exactly.* This is the precise version of the first prediction given above in Section 7. But strands also make predictions on quantum gravity.

Pr. 11 Gravity is due to the exchange of virtual gravitons. The tangle model of the graviton, a twisted pair of strands, is illustrated in Figure 7. Indeed, gravitons surround masses: twisted strand pairs arise in Dirac's trick. The model also predicts that gravitons have spin 2, because gravitons return to their original state after a rotation of the tangle core by π. Gravitons are predicted to be massless, because their core is not localized. Gravitons are predicted to be bosons, because cores can swap positions along the strands. As a result,

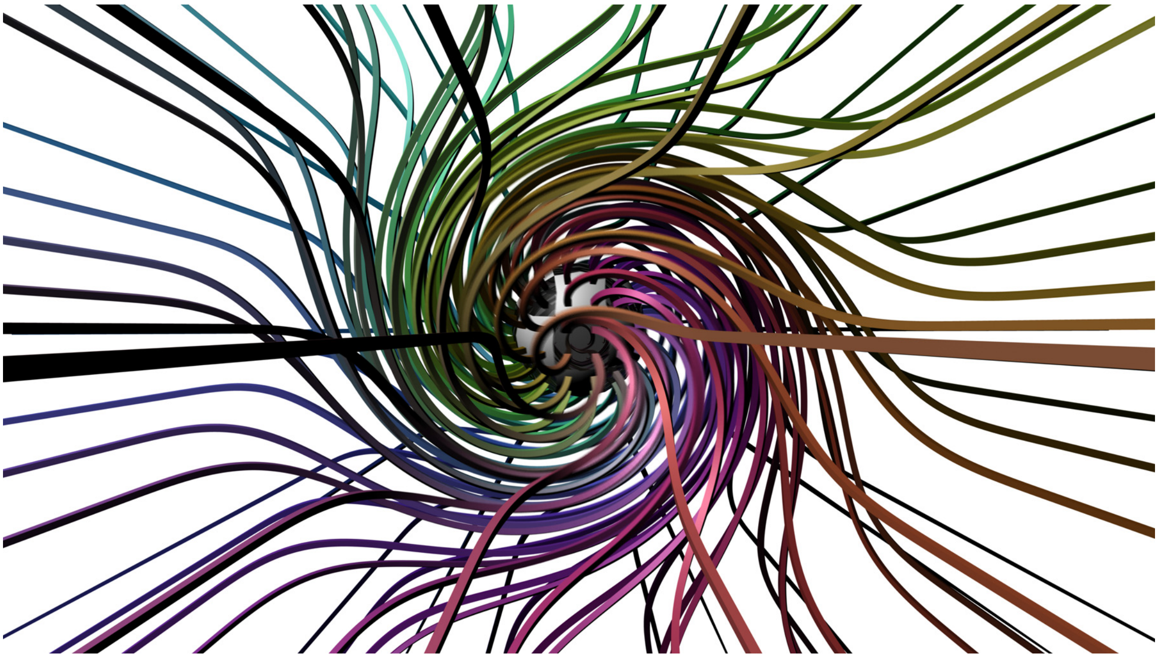

Figure 6: The strand conjecture for a rotating black hole rotating about the vertical axis (© Jason Hise). The flattening of the horizon at the poles is not shown. For an animation, see the online video at reference [39].

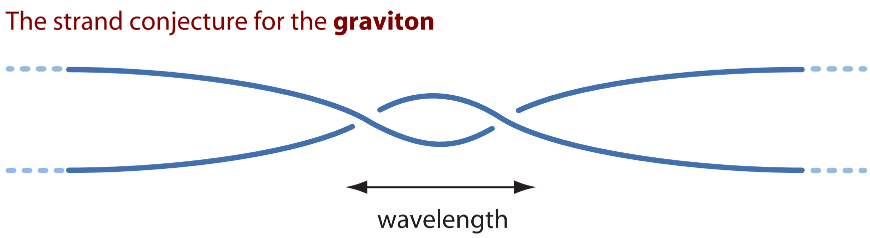

Figure 7: The strand conjecture for the graviton: a twisted pair of strands has spin 2, boson behaviour and vanishing mass. A gravitational wave is a coherent superposition of a large number of gravitons.

coherent gravitons are predicted to yield gravitational waves with spin 2 and velocity c., as observed.

Pr. 12 In the strand conjecture, *single gravitons cannot be detected*, for two reasons. First, strands imply the indistinguishability between graviton observation from any other quantum fluctuation of or at the detector. Equivalently, in the strand conjecture, graviton absorption does not lead to particle emission. Secondly, even if gravitons were detectable, in the strand conjecture, they have an extremely small cross section, of the order of the square of the Planck length. The low cross section is due to the topology of the graviton tangle. This implies a low detection probability, as expected [40, 41].

Pr. 13 Strands and expressions (1) imply that the gravitational constant G *does not run* when energy is increased from everyday values to higher values. In the language of perturbative quantum field theory, G is *not renormalized*. This prediction agrees with expectations and with data, though the available data is sparse.

Pr. 14 Strands imply that no quantum superposition effects for gravity are observable at experimentally accessible scales, because graviton exchange destroys entanglement. This agrees with expectations [42].

Pr. 15 The strand conjecture implies that in a *double-slit experiment* with electrons, the electrons pass both slits at the same time, because the core splits in two pieces during passage – though in different fractions at every passage. Therefore strands predict that the gravitational field of an electron arises on both slits, for every passage, though in different fractions at every passage. Such an experiment might be possible one day.

Pr. 16 The impossibility to detect single gravitons and single strands implies that there are *no* unknown, observable *quantum corrections* to general relativity. This prediction is in contrast with many expectations, and may well be the most contentious prediction in this list. Equivalently, strands predict the lack of observable quantum effects in *semiclassical gravity.*

So far, these predictions are not contradicted by any experiment. *The future discovery of any new deviation from general relativity at sub-galactic scale, including any non-trivial quantum gravity effect, would falsify the strand conjecture.*

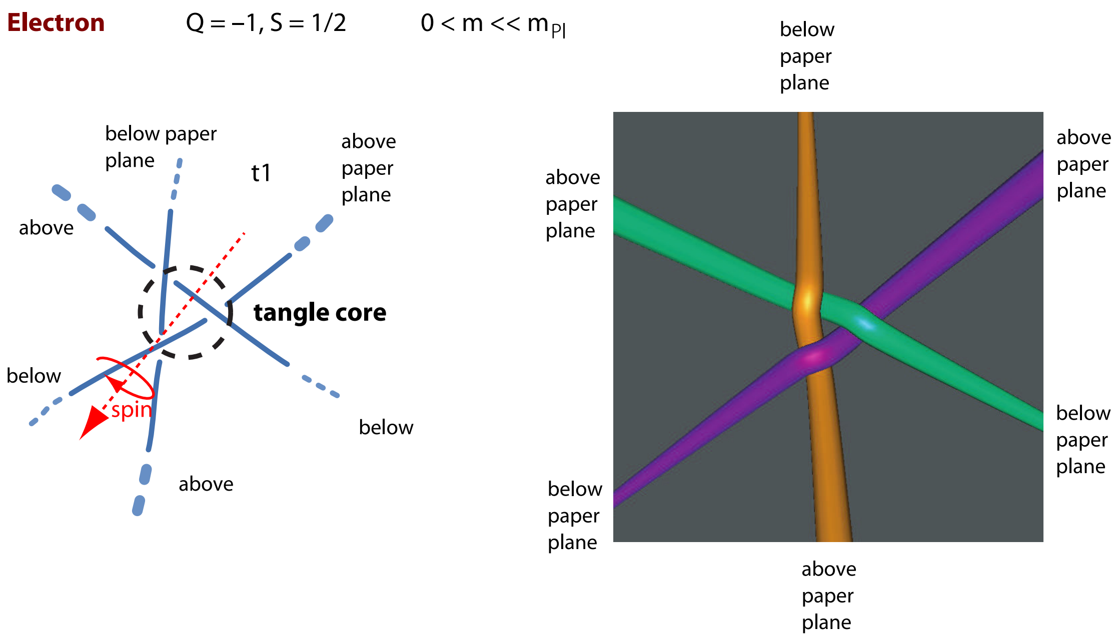

Figure 8: In the strand conjecture, elementary particles are modelled as rational tangles of strands. Strand segments, including tethers, are not observable; only crossing switches are observable. Together with their fluctuating shape, tangles lead to the observation that particles are localized in the region of the tangle core. Tangles are called *rational* when they are formed by switching tethers. Only rational tangles model the observed behaviour of elementary particles. Fermion tangles, such as the one in the figure, automatically have spin 1/2.

12 Strand predictions about elementary particle masses

So far, the strand conjecture has not predicted anything new. However, new predictions are possible, and in particular, predictions about elementary particle masses.

Black holes are made of large numbers of woven strands. It is natural to assume that elementary particles are made of *a few* woven strands. Indeed, in the strand conjecture, all elementary particles are *rational tangles* – i.e., woven, unknotted tangles – made of one, two or three strands [16, 23]. Tangles made of four or more strands are composed, not elementary. An example of an elementary rational tangle is shown in Figure 8.

Among tangles made of a few strands, those made of one strand are bosons; more precisely, they are photons. Massive elementary particles tangles are made of two or three strands.

Every fermion tangle, being a tethered structure that is tangled in the region of its tangle core, has non-vanishing mass. Every fermion tangle reproduces spin 1/2 behaviour under rotations – using Dirac's belt trick – and fermion behaviour under the exchange of positions of tangle cores. All tangles reproduce the gauge groups U(1), SU(2) and SU(3) as the result of Reidemeister moves on their tangled cores.

Only *rational* tangles – i.e., tangles that arise through the motion or braiding of their tethers – allow reproducing the transformation of particles observed in experiments. And only rational tangles allow a classification into a finite number of families that correspond to the observed elementary particles. These arguments are summarized in Appendix B and are worked out in detail in references [16] and [23].

Mass is the property of tangles that creates virtual gravitons around them. This implies:

▷ The *particle mass* (in corrected Planck units) is the probability of strand crossing switches occurring, per Planck time, in spontaneous belt tricks of the particle tangle.

Rational tangles directly allow deducing a number of predictions about mass values of elementary particles.

Pr. 17 Strands promise, through the analogy between thermodynamic effects and gravitational attraction, to allow calculating the gravitational mass of quantum particles. The value of gravitational mass is predicted to depend on the *tangle shape* of the particle – and on nothing else.

Research has shown that the average shape of a fluctuating tangle is the same as the shape of a tight tangle [43, 44]. *Therefore, the mass of an elementary particle is determined by its tight tangle shape.*

Since particle mass is due to their (tight) tangle shape, the mass values of all elementary particles are predicted to be zero or positive, equal to that of their antiparticles, fixed, unique, calculable and constant in time and space. This agrees with data. *If particle masses would be found to vary over space or time, the strand conjecture would be falsified.*

Pr. 18 In the strand conjecture, all fermions are localizable (i.e., not trivial) tangles. Thus, fermions have positive mass. The model of the graviton implies that gravitational charge, or mass, of a fermion is defined by the (tight) *shape* of the fermion tangle core. In the strand conjecture, mass values are automatically discrete, but are not integer multiples of a smallest value. In the strand conjecture, mass thus automatically differs from the charges of the gauge interactions, which are integer multiples of a smallest value. This agrees with observation.

In the strand conjecture, as shown in references [16, 23], only particles with positive mass can have electric and weak charge. In addition, it was shown that all mass values are due to Yukwawa coupling to the Higgs. Only those particles that couple to the Higgs are observed to be massive. All this agrees with observation.

Pr. 19 The tangle model of elementary particles implies that both the gravitational and the inertial mass of elementary particles are due to tether fluctuations. *Gravitational* mass describes the virtual gravitons around a mass: they arise in the tethers due to the belt trick. *Inertial* mass describes how a rotating mass advances through the vacuum with the belt trick, as described in reference [16]. In the strand conjecture, these two processes are exactly the same: both involve tether fluctuations around the core, and in particular, both involve the belt trick. Therefore, inertial and gravitational mass are equal – for infinite, flat space. Strands thus imply that the *equivalence principle* holds, in its weak and strong forms – at least for sub-galactic scales, when there is no effect of the cosmological horizon. This agrees with observations [45].

Pr. 20 Strands imply that elementary particle mass values *run* with four-momentum. The reason is that the tangles completely reproduce quantum field theory, as summarized in Appendix B: elementary particles are surrounded by virtual particle pairs; thus their mass values run with four-momentum. This agrees with observations – e.g., [46] – and expectations.

Pr. 21 Because spontaneous tangle fluctuations leading to the belt trick are *rare*, the gravitational mass m of elementary particles is predicted to be positive and to be much smaller than the corrected Planck mass:

$$0 < m \ll \sqrt{\hbar c/4G} \ . \tag{22}$$

This inequality agrees with observations and agrees with old arguments [47]. Strands thus provide a general answer to the *mass hierarchy* problem.

Pr. 22 Strands imply that falling particles are fluctuating and diffusing tangles. This implies that *more complex* particle tangles have *higher* gravitational mass (for equal number of tethers). The same connection has already been deduced for inertial mass in a different way [16]. The connection yields the correct mass sequences for all hadrons and predicts normal mass ordering for neutrinos. *If neutrino masses would* not *obey normal ordering, the strand conjecture would be falsified.*

The tangle model also explains that neutrinos mix and that their mass values are stable under renormalization, as shown in references [16, 23]. Strands thus allow non-vanishing neutrino mass in the standard model of particle physics.

Pr. 23 Strands allow deducing approximations for the mass values of elementary particles. As mentioned, the mass is given by the number of crossing switches per time that occur around the particle. For a fermion, the crossing switches are generated by the tethered rotation of the particle, illustrated in Figure 9. The figure yields

$$m \approx p \cdot f \cdot n \ , \tag{23}$$

where p is the probability for the initial double rotation of the core, f is the probability or frequency of the subsequent belt trick, and n is the number of crossing switches arising.

The factor p describes the process from the first to the second configuration in Figure 9.

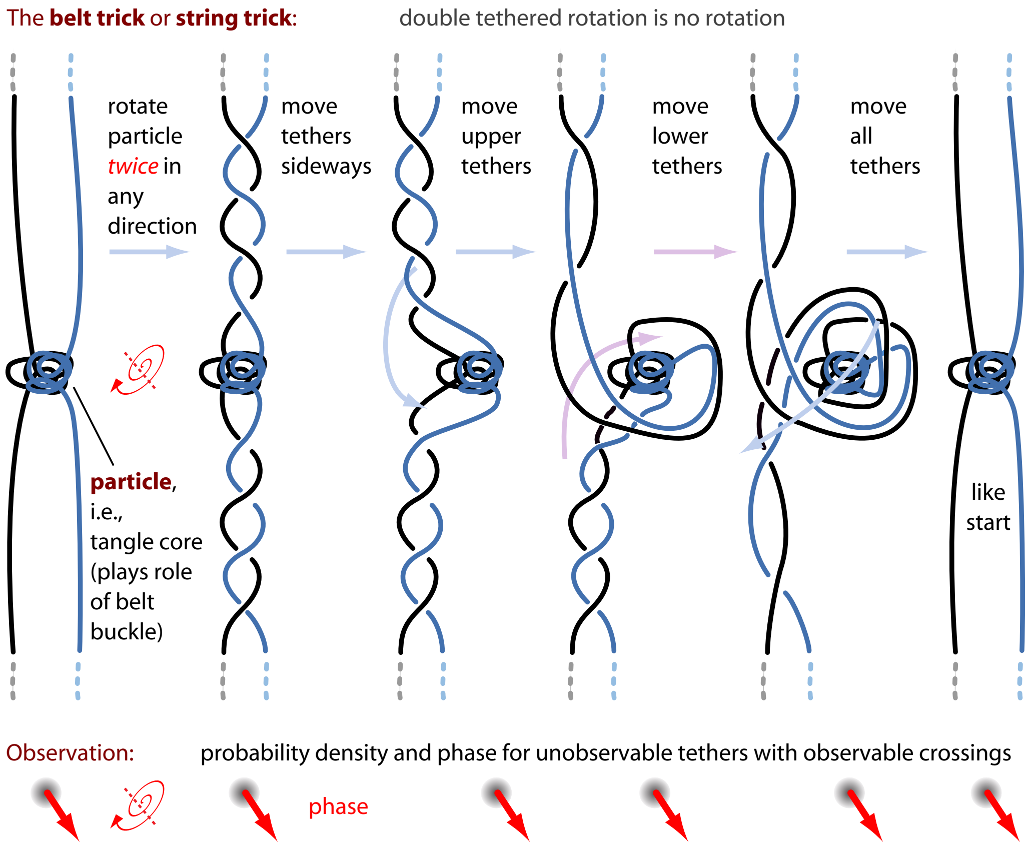

Figure 9: The *belt trick* or *string trick*, as popularized by Dirac, shows that a rotation by 4π of a tethered particle, such as a belt buckle or a tangle core, is equivalent to no rotation – when the tethers are allowed to fluctuate and untangle as shown. This equivalence, illustrated here in six configurations, allows the tethered particle to rotate forever. Untangling is impossible after a rotation by 2π only. The trick illustrates *spin* $1/2$ – if one assumes that tethers are not observable, but crossing switches are. The belt trick works for *any number* of tethers or belts. In contrast to this illustration, in the strand conjecture, leptons have six tethers, nor four; the tangle core topology determines the particle type. The belt trick also allows estimating particle mass, if the probabilities for the six configurations are explored (see text).

For a symmetric core, the rotation probability, whatever the orientation of the axis, is expected to be equal in clockwise and anticlockwise direction. In other terms, p vanishes for symmetric tangle cores. For slightly non-symmetric tangle cores, as is the case for tangles, the factor p is thus expected to be quite small. Its value will depend on the (averaged, three-dimensional, geometric) *asymmetry* of the tangle core. The asymmetry is the quantity that couples to the Higgs braid. A non-zero asymmetry leads to a non-zero mass.

The belt trick frequency f for the process that changes the second configuration in Figure 9 into the sixth configuration will also be small, as it competes with the inverse rotation of the tangle core. Interestingly, this small frequency is expected to be roughly *scale independent*: the size of the tangle core does not play an important role.

Finally, the average number n of crossing switches per belt trick plays a role. The number n counts the crossing switches among tethers and also those between the tangle core and the tethers. This number will depend on the *size* of the tangle core.

The strand explanation (23) for particle mass m can be checked before any calculation or estimate is performed. As mentioned above, the resulting particle mass value is equal for particle and antiparticles, constant over space and time, and not quantized in multiples of some basic number. Gravitational and inertial mass are equal. Mass values run with energy, i.e. with the looseness of the tangle core. Mass values, via p, depend on the Yukawa coupling to the Higgs boson. Particle mass values, due to the factor f, are much smaller that the Planck mass. Above all, as expected, particle mass values, due to the factors p and n, increase for more complex tangles, as large tangles are also more asymmetric.

At present, a direct calculation of m, even an approximate one, is still elusive. However, some numerical estimates can be deduced.

Pr. 24 Expression (23) allows estimating the mass ratio between the most massive and the least massive leptons.

Leptons tangles are made of three strands and thus have 6 tethers [16, 23]. For a neutrino, the belt trick frequency f for the subsequent configuration change in Figure 9 results from the probability that the belt trick arises instead of the backwards rotation of the core. To occur, the tether configuration has to form six circles around the tangle core, all with the same orientation. The size of the six circles is not important. For each tail, the probability is roughly given by the probability to form a circle divided by the number of possible rotation axes. Thus one gets the rough estimate

$$f \approx \left(\frac{\mathrm{e}^{-2\pi}}{6 \cdot 4 \cdot 2}\right)^6 \approx 3 \cdot 10^{-27} \quad . \tag{24}$$

where the exponent is due to the six tethers of the leptons. An error of a few orders of magnitude is expected. In the mass expression (23), the number n for neutrinos is surely larger than 24, which just counts the crossing switches in the tethers.

For the most massive lepton, the estimate $n = 24$ for the neutrinos will be approximately doubled. The estimate for f will increase for tangle cores that are elongated; the value $(6 \cdot 4 \cdot 2)^{-6}$ will be replaced by a number of the order $O(1/10)$. The asymmetry p for massive leptons is surely larger than that for neutrinos. As a result, the mass ratio r between the most massive and the least massive leptons will be

$$r \approx 2\ O(1/10)\ (6 \cdot 4 \cdot 2)^6\ \frac{p_{\mathrm{mm}}}{p_\nu} \quad . \tag{25}$$

The lepton mass ratio r is thus surely larger than $2 \cdot 10^9$. The observed value for the lepton mass hierarchy r is above 10^9. Better estimates for r require to determine p_{mm}/p_ν. This is not yet possible. In short, only a precise determination of the neutrino mass – both from experiments and from strand calculations – will allow a definite test of this prediction.

Pr. 25 Strands will allow deducing a *lower limit* for the (bare) mass values of *leptons*.

The rotation probability p for a neutrino results from the averaged asymmetry of its tangle core. For an electron neutrino, the asymmetry that results from the geometric chirality of the tangle is negligibly small. It is expected that the asymmetry arises only through the mixing with the other two neutrinos, or through the Yukawa term, or both. The averaged asymmetry is hard to estimate. It is expected to be larger than one part per million, thus

$$p \approx 10^{-6} \quad . \tag{26}$$

A systematic error of a few orders of magnitude is expected. This estimate yields a lower mass bound $m_{\rm ll}$ for leptons of

$$\frac{m_{\rm ll}}{\sqrt{\hbar c/4G}} = p \cdot f \cdot n \approx 10^{-31} \quad , \tag{27}$$

which is of the order of meV$/c^2$, though with a large error margin. So far, this lower limit is not in contrast with the present experimental limit on neutrino mass, which is below 0.9 eV [48]. Again, the difficulties of deriving a reliable lower mass bound for leptons are evident.

Pr. 26 Strands will also allow deducing an *upper limit* for the mass value of leptons. For heavy leptons, the probability p due to the asymmetry of the tangle core is expected to be of order $O(1)$. As a result, the upper mass limit for leptons will be

$$m_{\rm ul} \approx 10^{17} \cdot m_{\rm ll} \quad , \tag{28}$$

again with a large error margin. The most massive lepton, the tau lepton, has an observed mass of 1.7 GeV$/c^2$. It indeed is several orders of magnitude below the upper bound.

More precise strand estimates of particle masses will require the development of better approximations and of suitable computer simulation programs. *The failure to reproduce the correct mass value of a single particle, at any single energy value, would falsify the strand conjecture.*

13 Discussion and outlook

On the one hand, it is not easy to think about nature as made of strands. It is also unusual to describe physical processes as made of fundamental events. On the other hand, the conjecture has the charm of deriving all observations about general relativity (at sub-galactic scales) directly from the Planck scale. Also, the complete standard model of particle physics, with its Lagrangian, arises from the Planck scale, as argued elsewhere [16, 23]. So far, no deviations from these two descriptions have been observed in any experiment. *The discovery of any new deviation from general relativity at sub-galactic scales would invalidate the strand conjecture.*

The possibility that additional quantum gravity effects are unobservable has already been explored in the past [49, 50, 51]. Strands confirm the result. They can make such a prediction because they incorporate general relativity, quantum physics and the standard model exactly.

As a new result in the domain of quantum gravity, strands propose a solution to the mass hierarchy problem and promise that the gravitational mass of elementary particles can be calculated ab initio from their tangle details. Such calculations will allow the most stringent tests of the conjecture. As long as the strand predictions about space, gravity, particles and mass are not falsified, strands remain a candidate for a complete description of nature.

14 Acknowledgments and declarations

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Appendix A On the circularity of the fundamental principle

On the one hand, the crossing switch of Figure 1 is assumed to take place in space. On the other hand, space, distances and physical observables are assumed to arise from strands. The apparent circularity can be avoided – to a large degree, but not completely – by increasing the precision of the formulation.

Crossing switches take place in *background space*. In the strand conjecture, background space is defined by the observer. In contrast, *physical space*, physical distances and physical observables arise from strands and their crossing switches. When space is flat, background space and physical space coincide. Otherwise, they do not; in that case, background space is (usually) the local tangent space of physical space. A similar situation arises for the concept of time.

In nature, any observation of a *change* implies the use of (background) time; any observation of *difference* between objects or systems implies the use of separation in (background) space. Indeed, a local background space – observer-defined and usually observer-dependent – is *required* to describe *any* observation, or simply, to talk about nature. In the strand conjecture, it is equally impossible to define crossing switches or any Planck unit without a background. The strand conjecture asserts that a description of nature *without* a background space and time is impossible.

Every use of the term 'observation' or 'observable' or 'physical' implies and requires the use of a background space and time. All the illustrations of the present work are drawn in *background* space. In contrast, *physical* space – an observable in general relativity, dynamical and pseudo-Riemannian – arises through crossing switches of strands. The local background space agrees with physical space only locally, where the crossing switches being explored are taking place. In fact, the need for a background space to describe nature is rooted in a deeper issue.

Background space is what is needed to *talk* about nature. Physical space is everything that can be *measured* about space: curvature, vacuum energy, entropy, temperature etc.

There is a fundamental contrast between *nature* and its precise *description*. The properties of nature itself and the properties of a precise description *differ* and *contradict* each other. A precise *description* of nature requires axioms, sets, elements, functions, and in particular continuous background space, continuous background time, and points in background space and background time. In contrast, due to the uncertainty relations, at the Planck scale, nature itself does not provide the possibility to define points in physical space or time; physical space and time are not continuous at smallest scale, and in fact, physical)space and time are emergent. In short, observer space, or *background space*, differs in its properties from *physical space.*

Any precise description of nature thus requires a limited degree of circularity in its definition of physical time and space with the help of background time and background space. Therefore, an axiomatic description of *all* of nature is *impossible*. An axiomatic description is only possible for those *parts* of nature that avoid the fundamental circularity, such as quantum theory, or special relativity, or quantum field theory, or electromagnetism, or general relativity. Even though Hilbert asked for an axiomatic description of physics in his famous sixth problem, no claim for an axiomatic description of *all* of nature (all of physics) has ever appeared in the literature. Any unified description of nature must be circular. The strand conjecture, like or any other unified model, can be tested by asking whether it is a *consistent*, *complete* and *correct* description of nature. So far, this appears to be the case for strands.

An important example for the difference between an axiomatic description and a consistent, complete, correct – but somewhat circular – description is the dimensionality of space. The number of dimensions of (background and physical) space is not a consequence of the fundamental principle or of some axiom; the number of dimensions is assumed in the fundamental principle right from the start. Tangles only exist in three dimensions. Only three dimensions allow a description of nature that is consistent, complete, and correct: only three dimensions allow crossing switches, particle tangles, spin $1/2$, Dirac's equation and Einstein's field equations.

Appendix B From strands to quantum theory and the standard model Lagrangian

This appendix provides an extremely short summary of references [16] and [23], which explain how quantum theory, quantum field theory and the full Lagrangian of the standard model arise from strands. The tangle model for massive quantum particles is illustrated in Figure 10 and Figure 11. The figures visualize that crossings have properties similar to those of wave functions, and that time-averaged crossing switches have the same properties as probability densities.

Starting from the fundamental principle and Dirac's belt trick, *tangles* of fluctuating strands in flat (physical) space indeed describe matter particles and wave functions: the wave function of a particle is the *strand crossing density* of its fluctuating tangle. In other words, wave functions arise as *local time averages* of strand crossings. More specifically, to get the value of the wave function at a certain position in space, the local time average of the strand crossings at that position is taken, averaging over a time scale of (at least) a few Planck times. In this way, a density and a phase can be defined, for each 'position' in space. As usual for quantum theory, also in the strand conjecture physical space and time have to be defined *before* defining the concept of wave function. The

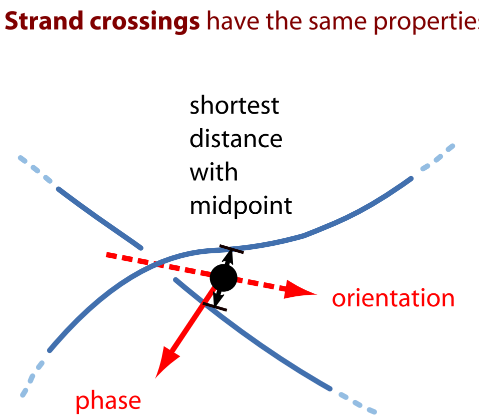

Figure 10: A configuration of two skew strands, called a *strand crossing* in the present context, allows defining density, orientation, position, and a phase. These are the same properties that characterize a wave function. The freedom in the definition of phase is at the origin of the choice of gauge. For a complete tangle, the density, the phase, and the two (spin) orientation angles define, *after spatial averaging,* the two components of the Dirac wave function Ψ of the particle and, for the mirror tangle, the two components of the antiparticle.

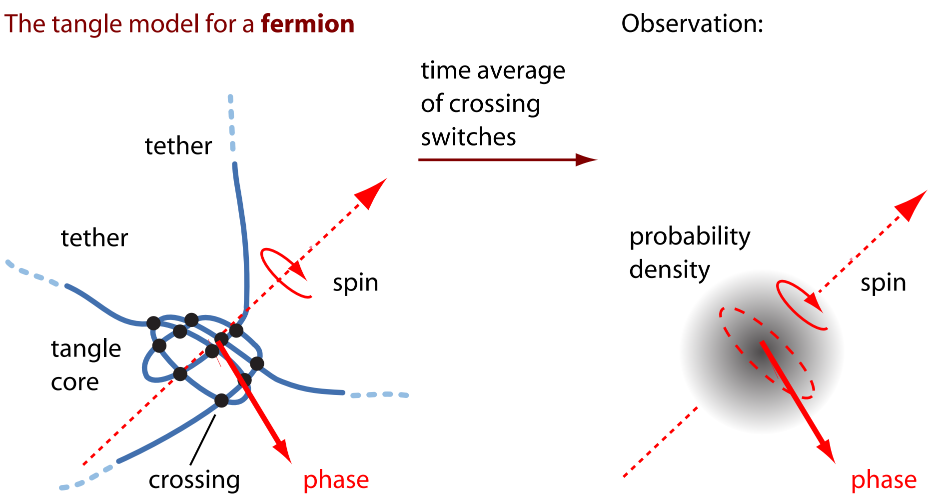

Figure 11: In the strand conjecture, the wave function and the probability density are due, respectively, to crossings and to crossing switches at the Planck scale. The wave function arises as time average of crossings in fluctuating tangled strands. The probability density arises as time average of the crossing switches in a tangle. The *tethers* – strand segments that continue up to large spatial distances – generate spin $1/2$ behaviour under rotation and fermion behaviour under particle exchange. The tangle model also ensures that fermions are massive and move slower than light (see text).

probability density for a particle is the local time average of the *crossing switch density* of its fluctuating tangle. A detailed exploration [16, 23] shows that strands produce a Hilbert space, the quantum phase, interference, contextuality and freedom in the definition of the absolute phase value.

Moving particles are advancing rotating tangles. Antiparticles are mirror tangles rotating in the opposite direction. Fluctuating rational tangles made of two or more strands imply spin $1/2$ behaviour under rotation and, above all, Dirac's equation [14]. For systems of several particles, tangles reproduce fermion behaviour and entanglement. Tangles of strands are fully equivalent to textbook quantum theory and predict the lack of any extension or deviation, up to Planck energy. For example, the principle of least action is the *principle of fewest crossing switches*. In this way, strands also explain the origin of the principle of least action [23].

No new physics arises in the domain of quantum theory. Strands only *visualize* quantum theory; they do not modify it. *Every quantum effect is due to crossing switches* – and vice versa. The visualization of quantum effects with strands requires that strands remain unobservable in principle, whereas their crossing switches are observable.

Tangles also allow deducing quantum *field* theory. Exploring all possible tangles, it appears that *rational*, i.e., unknotted tangles reproduce the known spectrum of elementary particles and their properties [16, 23]. Every *massive* elementary particle is represented by an infinite *family* of rational tangles made of either two or three strands. Quarks are made of two strands; all other massive elementary particles are made of three strands. Three generations for quarks and for leptons arise. The Higgs itself is represented by a braid. The family members for each elementary particle differ among them only by the number of attached braids. The structure of each elementary particle tangle explains the spin value, parity, charge and all other quantum numbers.

Models for the *massless bosons* also arise. In particular, a photon is a single, twisted strand. Photons are emitted or absorbed by topologically chiral tangles, i.e., by fermion tangles that are electrically charged. Figure 12 illustrates the strand conjecture for quantum electrodynamics. Only three kinds of massless bosons arise, each kind due to one Reidemeister move. The boson

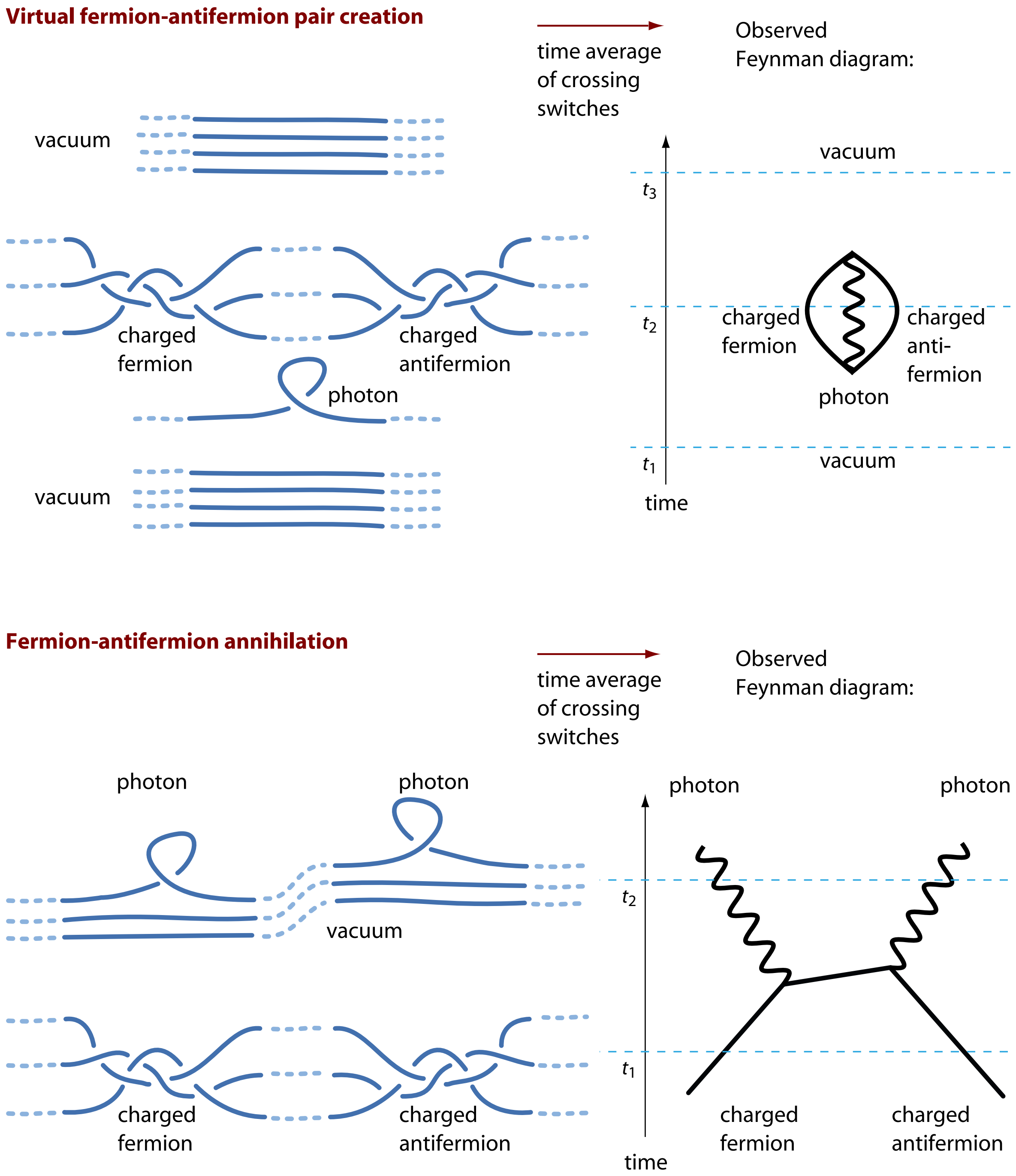

Figure 12: An illustration of two Feynman diagrams of quantum electrodynamics in the tangle model.

generator algebras turn out to be the well-known U(1), broken SU(2) and SU(3) of the three gauge interactions [16, 23]. The violation of parity in the weak interaction and the way that the massless bosons of SU(2) acquire mass are also explained.

A detailed investigation shows that tangles reproduces every propagator and every Feynman vertex observed in nature – and no other ones. Particle mixing appears naturally. The correct couplings between fermions and bosons also arise. As a result, the full Lagrangian of the modern standard model arises, term by term, including PMNS mixing of Dirac neutrinos, without any addition or modification [16, 23].

In short, strands predict the lack of any physics beyond the standard model. *Discovering such an effect or any new influence between quantum field theory and gravitation at sub-galactic scales – apart from the cosmological constant, the particle masses and the other constants of the standard model, including their running with energy – would falsify the strand conjecture.* This terse summary of the implications of strands for quantum field theory allows proceeding with the exploration of space and gravity.

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