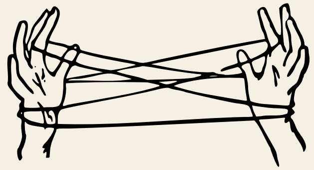
Nature is topological: it plays cat’s cradle



From tangles of strands to elementary particles, wave functions, gauge groups and the standard model, as well as to space, curvature and general relativity

Christoph Schiller, April 2024

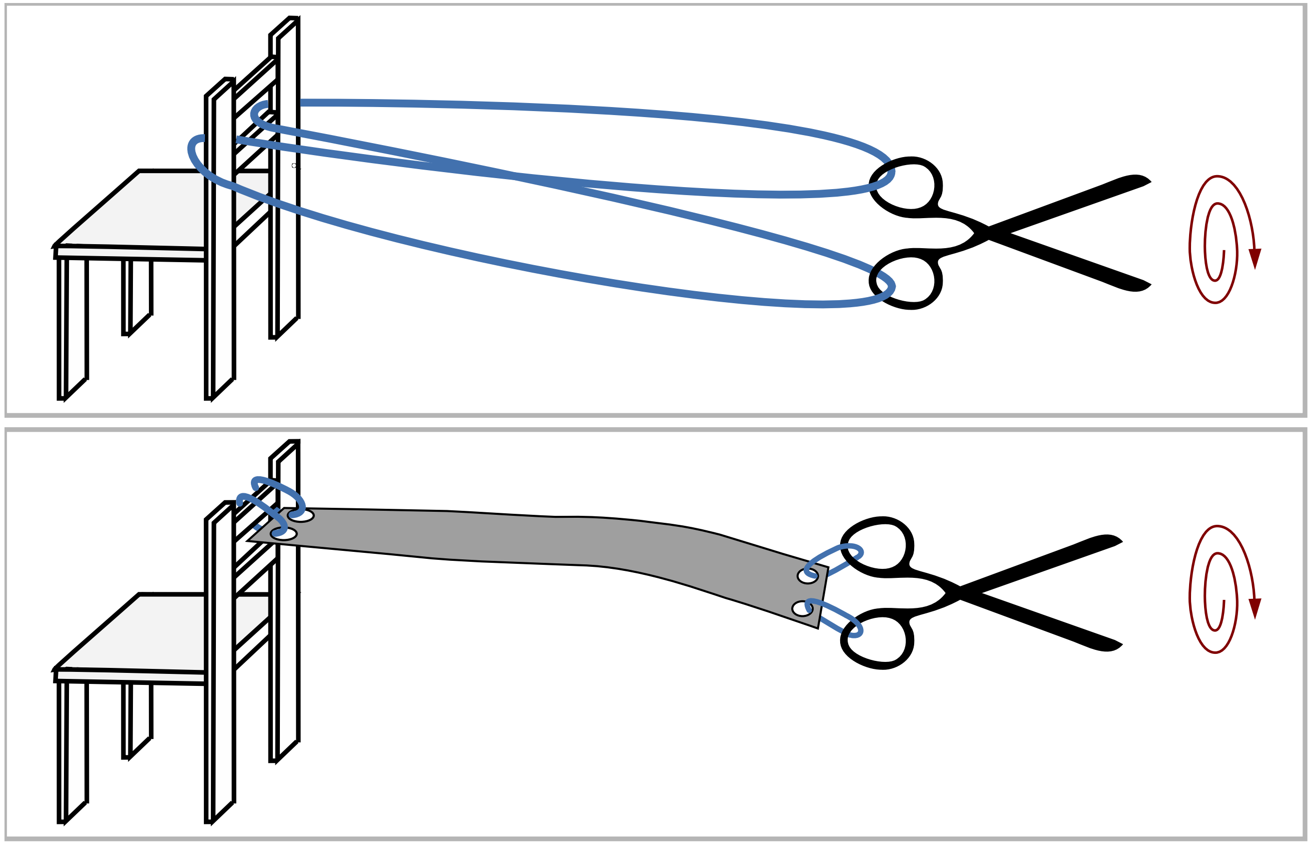
The State of Physics: 9 Lines Describe Nature

(1)	$\mathbf{d}\boldsymbol{W} = \mathbf{0}$	Action $W = \int L \, \mathrm{d}t$ is minimized in local motion. The other lines fix the two fundamental Lagrangians L.
(2)	$\boldsymbol{v} \leqslant \boldsymbol{c}$	Local energy speed v is limited by the speed of light c. This implies special relativity and restricts the possible Lagrangians.
(3)	$\boldsymbol{F} \leqslant \boldsymbol{c}^{\mathbf{4}}/\mathbf{4}\boldsymbol{G}$	Local force F is limited by c and by the gravitational constant G. This implies general relativity and fixes its Lagrangian.
(4)	$\boldsymbol{W} \geqslant \boldsymbol{\hbar}$	Action W is never smaller than the quantum of action $\hbar$. This implies quantum theory and restricts the possible Lagrangians.
(5)	$\boldsymbol{S} \geqslant \boldsymbol{k}\,\mathbf{ln2}$	Entropy S is never smaller than $\ln 2$ times the Boltzmann constant k. This implies thermodynamics.
(6)	**U(1)**	is the gauge group of the electromagnetic interaction. It yields its Lagrangian.
(7)	**SU(3) and broken SU(2)**	are the gauge groups of the two nuclear interactions, yielding their Lagrangians.
(8)	**18 elementary particles**	– gauge bosons, the Higgs boson, quarks, leptons, and the undetected graviton – with all their quantum numbers, make up everything in nature and, with their interactions, fix the standard model Lagrangian.
(9)	**Finally, 27 numbers**	– dimensions, cosmological constant, coupling constants, particle mass ratios, mixings and phases – complete the two fundamental Lagrangians. They determine all observations and all colours.

(Link to details and to a paper that summarizes about half a million publications in the past 50 years.)

Lines 6, 7, 8 and 9 need explanations. This talk explains lines 6, 7 and 8.

Dirac's Lecture Trick – According to Penrose



R. Penrose (Dirac's student) & W. Rindler, *Spinors and space-time*, vol. I (1984).

The scissors represent a spin 1/2 fermion.

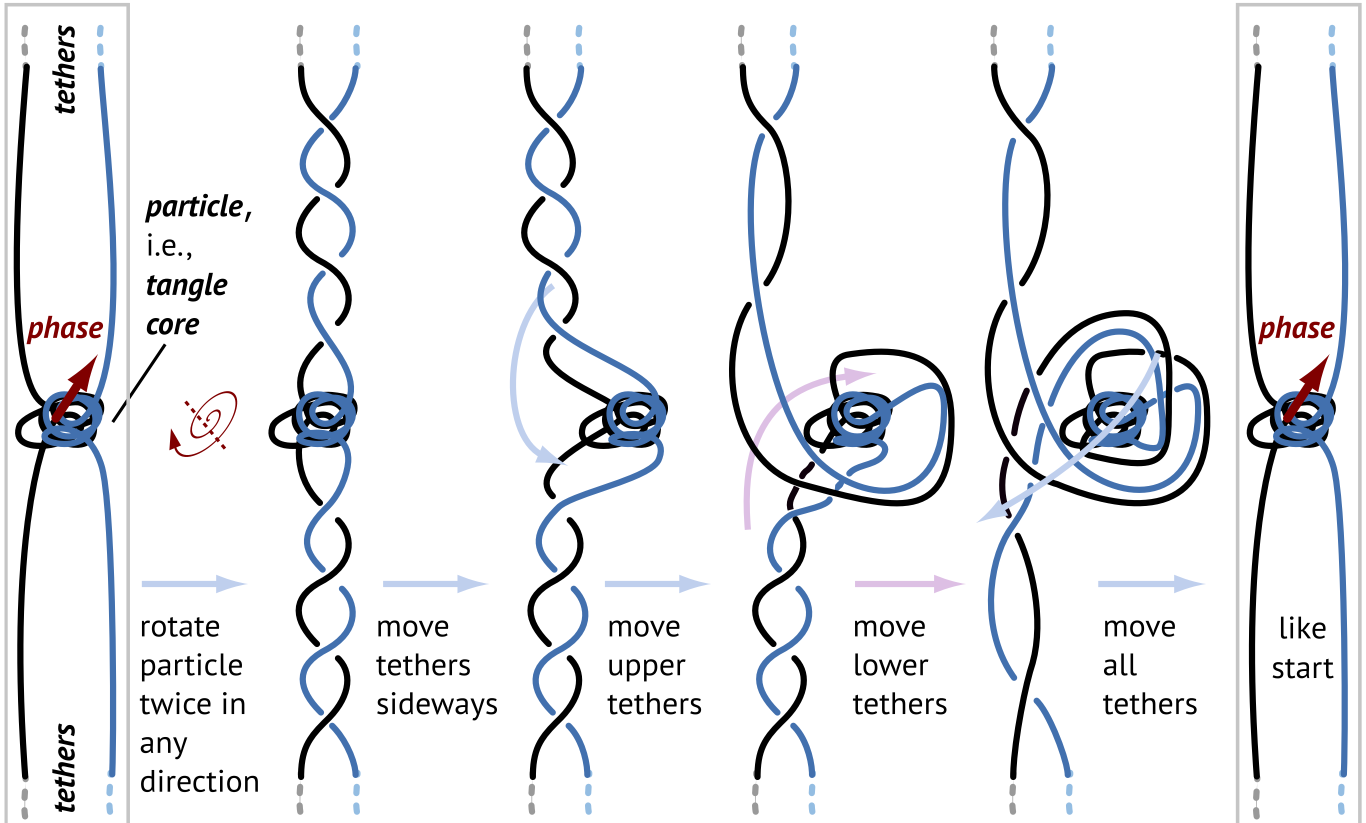
The chair represents the cosmological horizon.

Only a (scissor) rotation by 4π leads back to the original situation. 2π does not.

Is every particle tethered (attached) to the cosmological horizon? Yes.

Strands and Belts Explain Spin 1/2

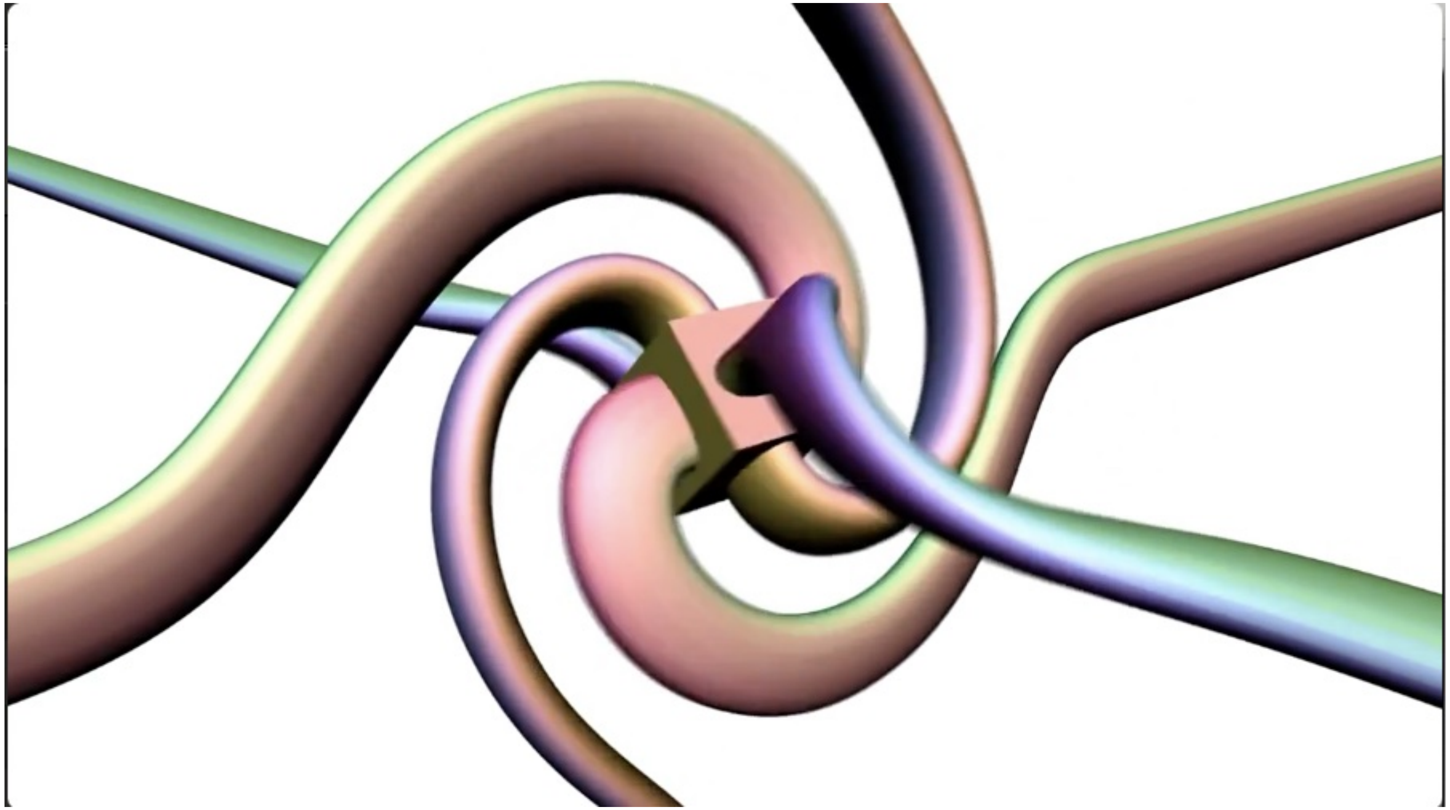
Dirac's belt trick or string trick: Double tethered particle rotation is no rotation.



Resulting observation. Time averaging unobservable tethers but observable crossing switches leads to a probability density and phase:

Core/particle rotation by 4π is equivalent to no rotation, for 2 or more strands.

Spin Is Rotation



© Jason Hise

The cubic centre represents a lepton tangle core.

As illustrated below, leptons have six tethers.

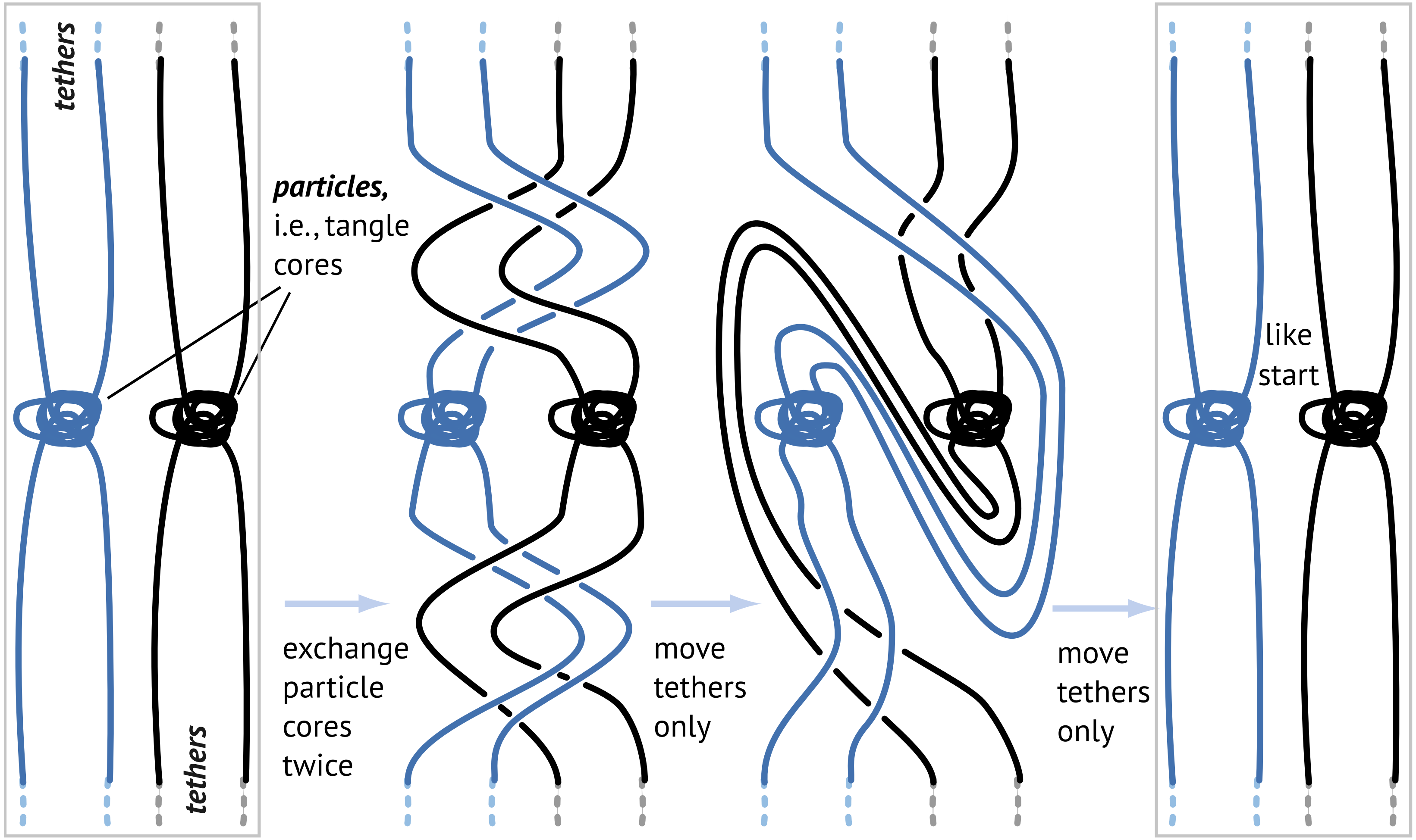
A spinning particle is a rotating tangle core.

Dirac's trick works with any number of tethers equal or larger than 3.

Strands and Belts Explain Fermions

The fermion trick: Double tethered particle exchange is no exchange.

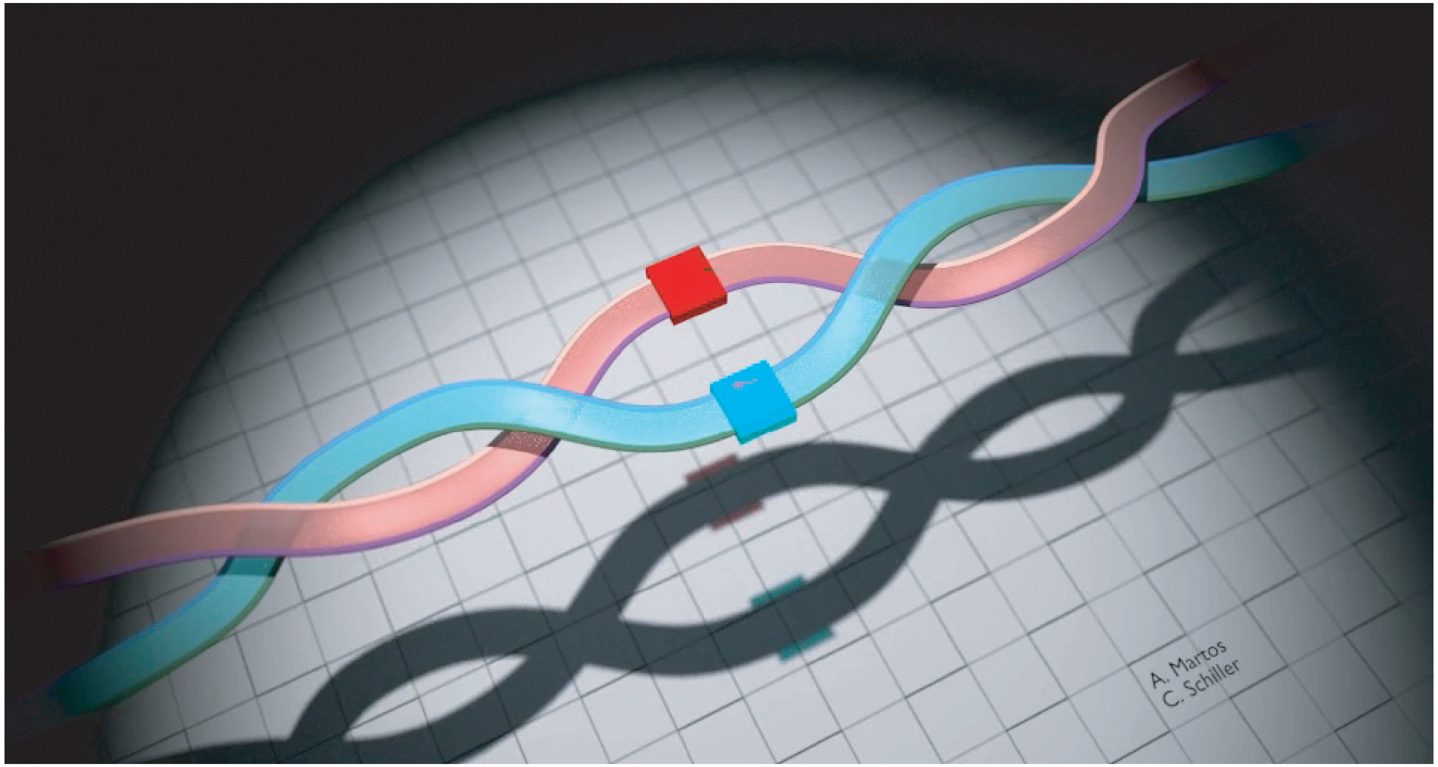
The trick also works if some or all the strands *connect* one tangle core to the other core.



Double exchange is no exchange.

The Dirac trick yields the fermion trick.

Fermion Behaviour Allows Orbiting Particles



The fermion trick works for any number of tethers.

Spin 1/2 particles are fermions. This is (half) the spin-statistics theorem.

Paul Dirac's Letter to Martin Gardner

Dear Mr. Gardner:

I am sorry I was too busy to answer your letter earlier. I first thought of the problem of the strings about 1929. I used it to illustrate a property of rotations, that two rotations of a body about an axis can be continuously deformed, through a set of motions which each end up with the original position, into no motion at all.

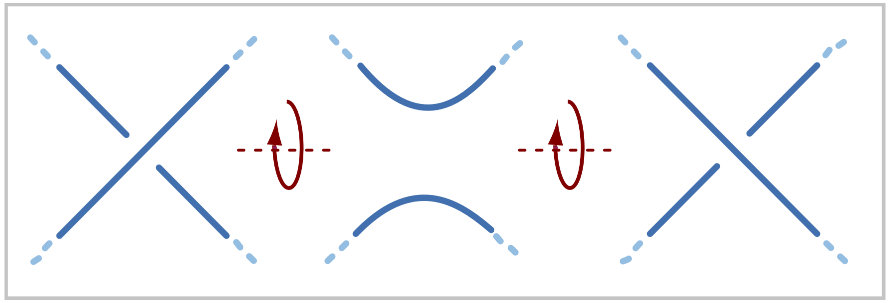
It is a consequence of this property of rotations that a spinning body can have half a quantum of angular momentum, but cannot have any other fraction of a quantum.

Yours sincerely
P.A.M. Dirac

M. Gardner, *Riddles of the Sphinx and Other Mathematical Puzzle Tales* (1987), page 47.

Rotations of tethered particles produce crossing switches.

A *crossing switch* is a change of overpass and underpass:

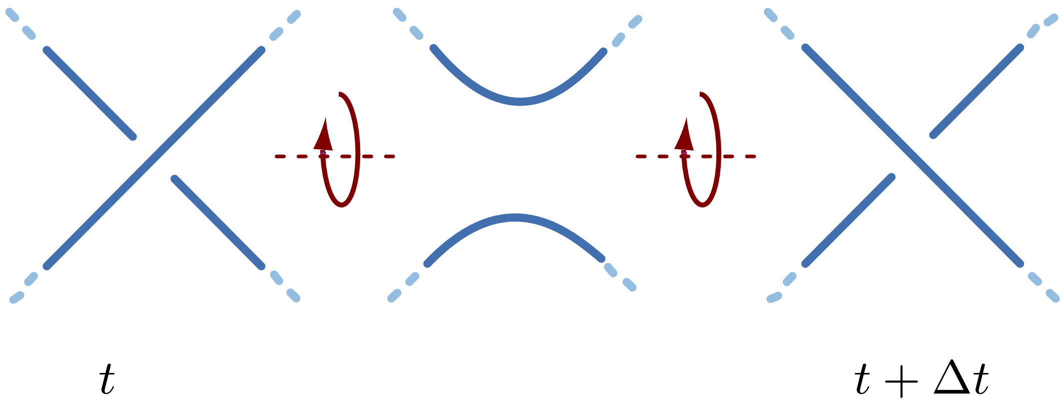


Therefore, crossing switches yield Planck's quantum of action $\hbar$.
(L. Kauffman, 1987)

Crossing Switches Define Planck's $\hbar$

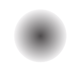
The fundamental, Planck-scale principle of the strand tangle model

Strand description:



$$W = \hbar$$
$$\Delta t \geq \sqrt{4\hbar G/c^5}$$
$$\Delta l \geq \sqrt{4\hbar G/c^3}$$
$$S = k \ln 2$$

Resulting observation:



A fundamental event localized in space-time within Planck scales

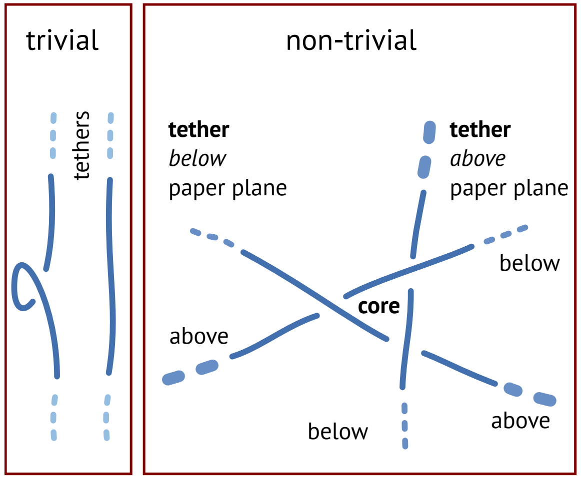
Strands have **Planck radius**. Strands are **unobservable**, impenetrable and featureless: no mass, no tension, no torsion, no branches, no fixed length, no ends.
A **crossings** is the region of the smallest distance between two strands.
Every **event** is a **crossing switch** characterized by $\hbar$.
All observables are defined and measured in terms of crossing switches.

Thesis: This fundamental principle implies all of physics.
The principle implies general relativity (via $F \leqslant c^4/4G$) and the standard model, with the three gauge groups and the known particles. And not more.

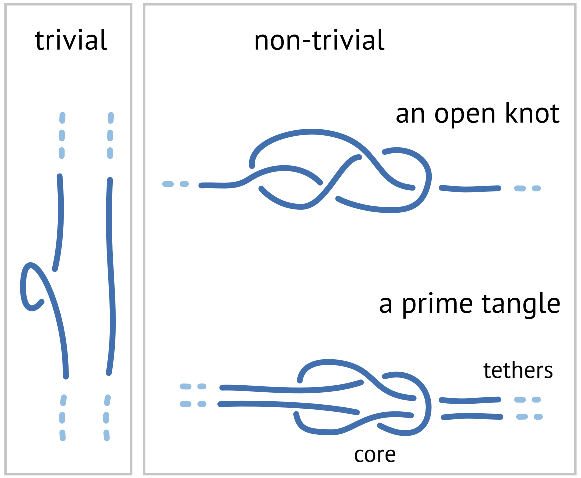
(Link to details and publications.)

Rational 3d Tangles Are Special

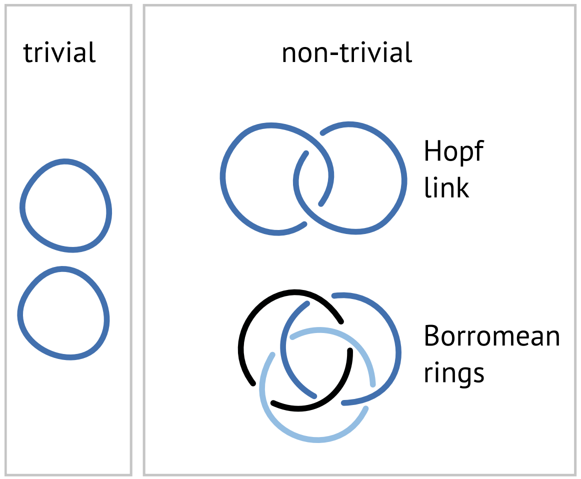
Rational (3d) tangles – or (3d) braids



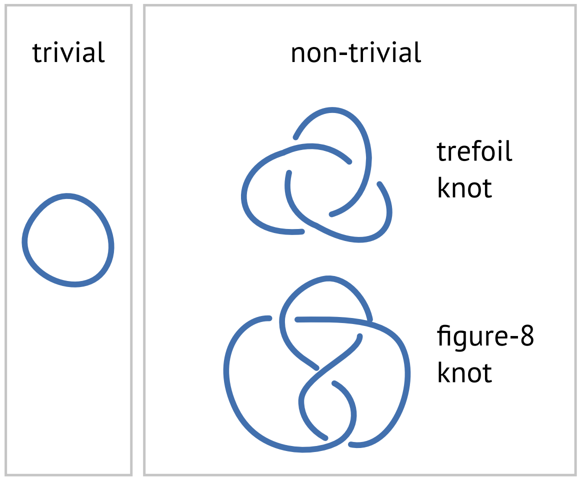
Knotted tangles



Links



Knots



Only rational 3d tangles reproduce *particle reactions* and *transformations*.

In the strand model, particles are rational 3d tangles.

Quantum of Action
Wave Functions
❖ Rational 3d tangles
❖ Elementary fermions
❖ Wave functions
❖ Spinning electron
Gauge interactions
Gravitation
Conclusion
Bonus Material

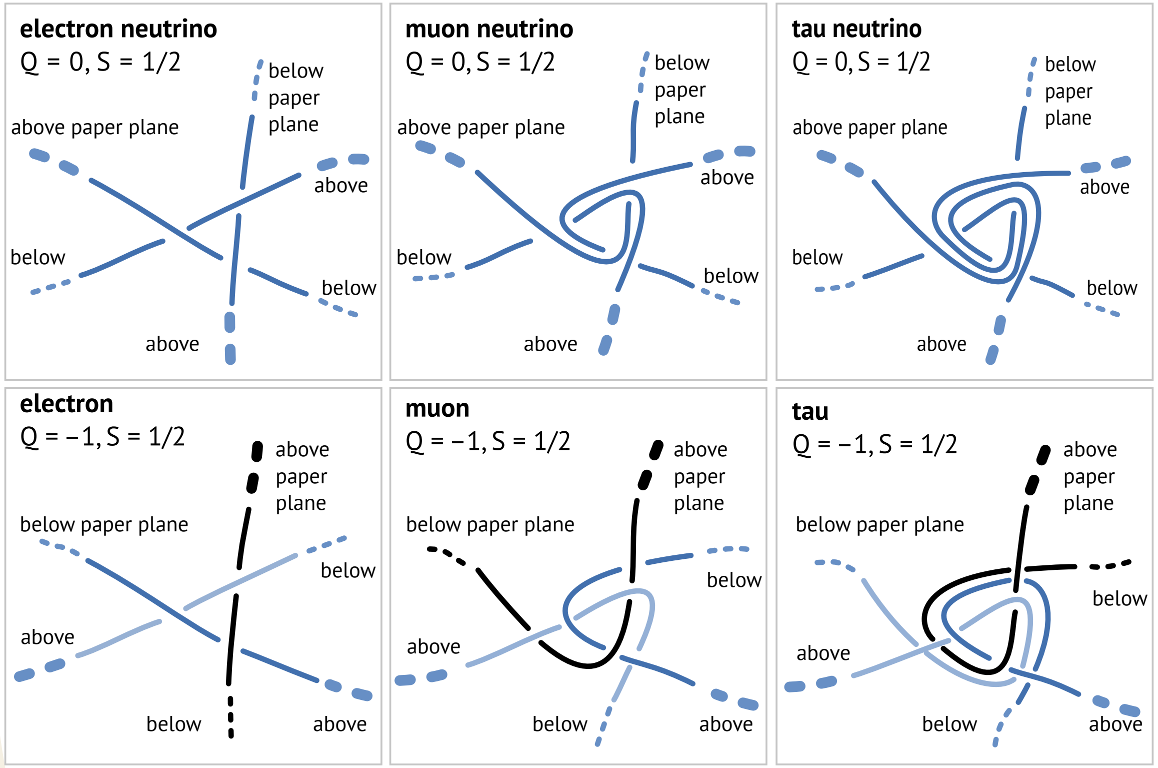
Elementary Fermions Are Rational 3d Tangles

Quarks - `tetrahedral' tangles made of two strands with four tethers (only simplest family members)

Parity P = +1, Baryon number B = +1/3, Spin S = 1/2
Charge Q = −1/3

Charge Q = +2/3

Leptons - `cubic' tangles made of three strands along cordinate axes (only simplest family members)



'Elementary' means 1 to 3 strands.

'Fermion' means localizable tangle with 2 or more strands.

These simplest tangles reproduce all quantum numbers.

No additional elementary fermions are possible.

No other explanation of the particle spectrum exists.

(Pedagogical link.)

Wave Functions Are Crossing Densities

Quantum of Action

Wave Functions
- ❖ Rational 3d tangles
- ❖ Elementary fermions
- ❖ Wave functions
- ❖ Spinning electron

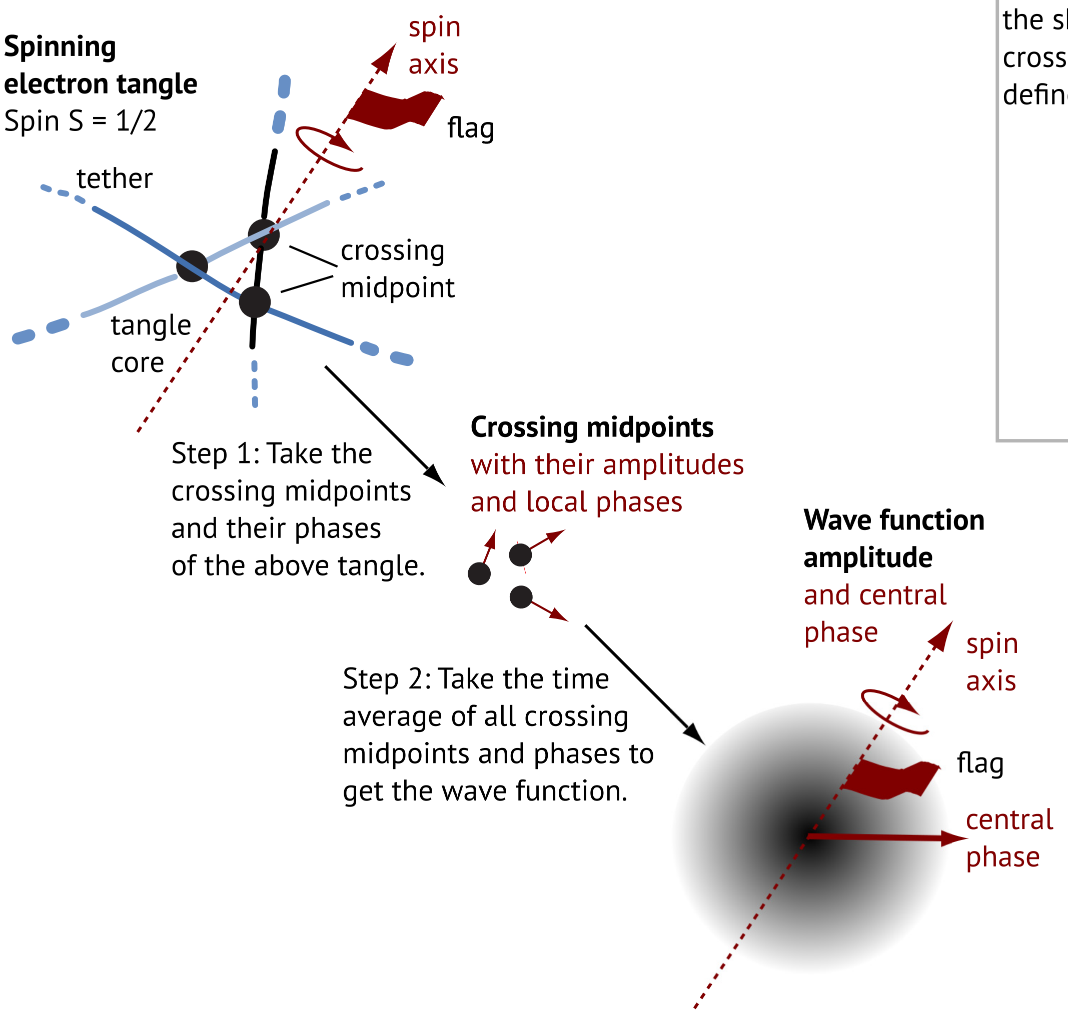
Gauge interactions

Gravitation

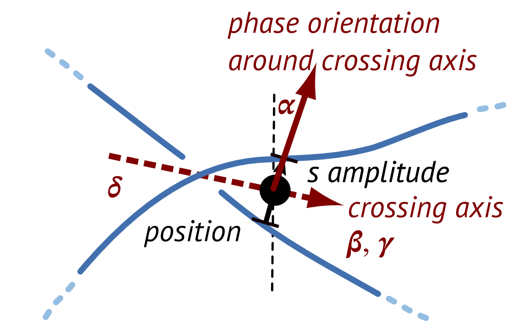
Conclusion

Bonus Material

The strand tangle model for **wave functions**



The black dot specifies the crossing *position*, the shortest distance s determines the crossing *amplitude*, and the angle α defines the crossing *phase*.

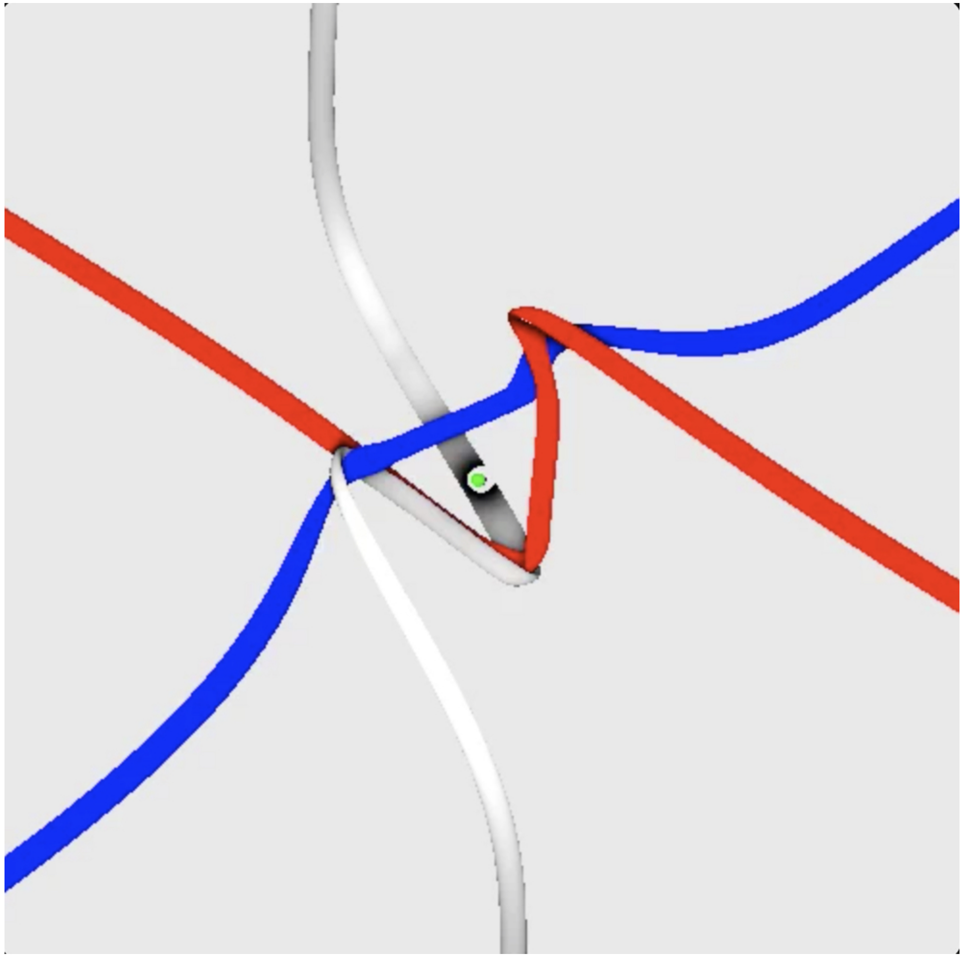


Tangles are skeletons of wave functions.

Tangles follow the free Dirac equation.
(Battey-Pratt & Racey, 1980.)

Crossings have *amplitudes* (inverse distance) and *phases*.
Crossing densities of fluctuating tangles are **wave functions:** they yield Hilbert spaces, interference, decoherence, collapse, and entanglement. **(Pedagogical link.)**
Dirac's equation is the infinitesimal version of Dirac's trick.

The Spinning Electron (slightly incorrect)

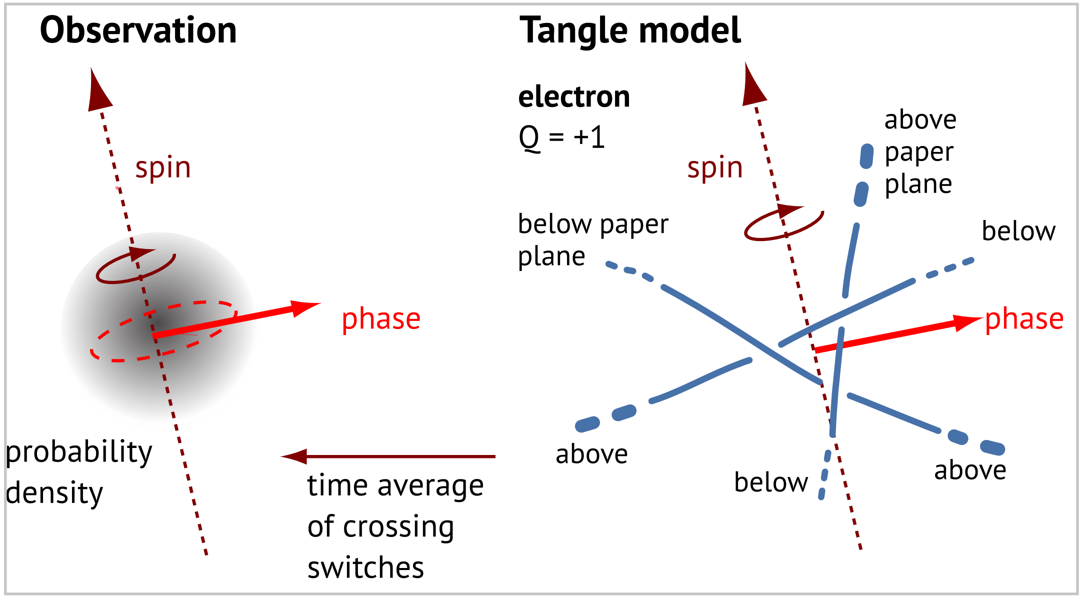


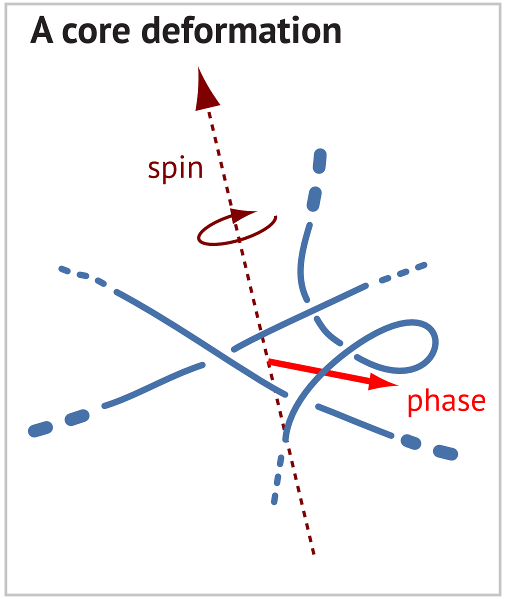
Rotation details depend on the speed:
***mass* arises.**

Mass calculation requires estimating the number of Dirac tricks per time.
A challenge!

In any case,
the masses of elementary particles are small:
$m \ll m_{\mathrm{Pl}}$.

Interactions Are Tangle Core Deformations





Free* propagating particles are cores that *rotate:

Core rotation axis → spin axis
Core orientation → phase of wave function
Tether deformations for rigid cores → space-time symmetries

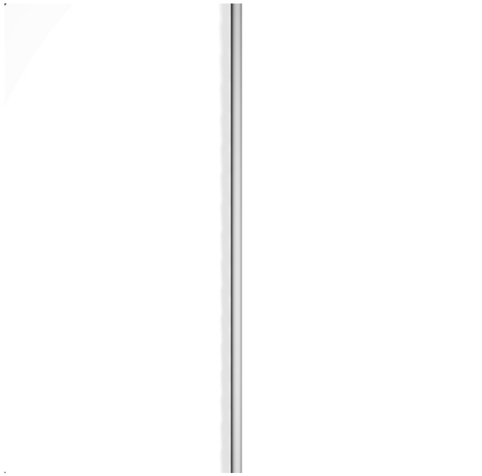
Interacting* fermions are cores being *deformed:

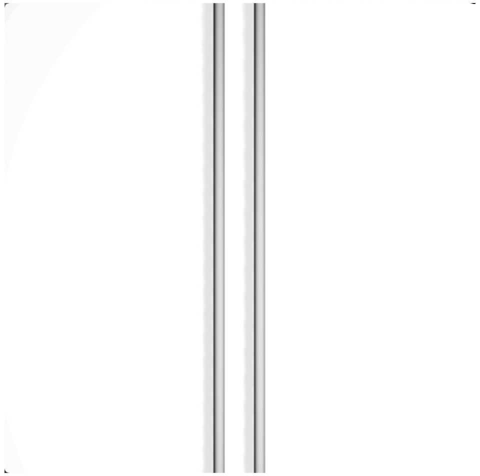
Core deformations change the phase → interactions
Freedom in the definition of phase → freedom of gauge

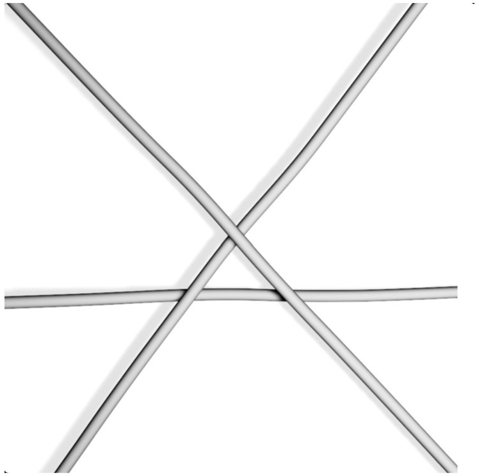
Surprise: All observable deformations can be built from 3 basic types.

Animated Reidemeister Moves

Double click.







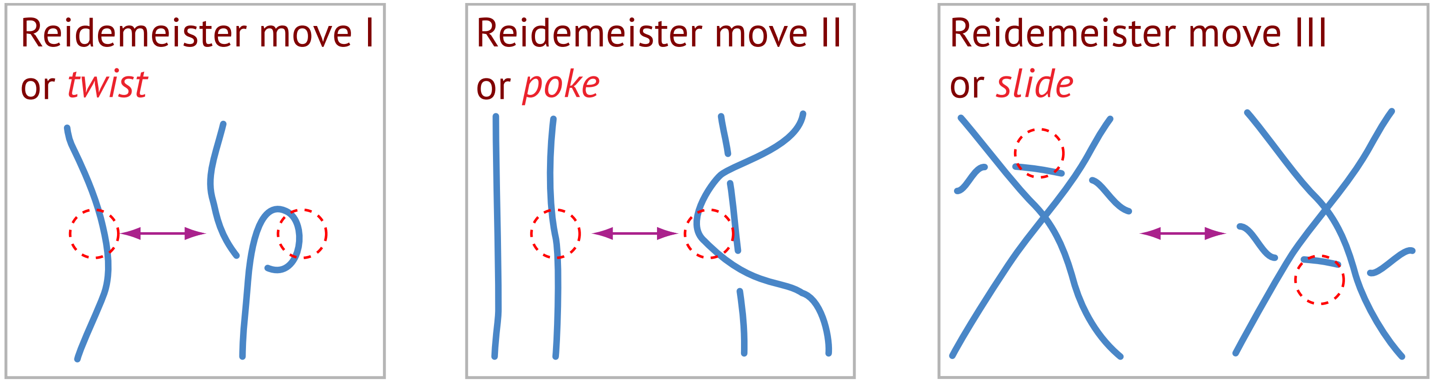
Reidemeister moves are related to crossing switches.

Reidemeister moves in tangles cores are thus physically observable.

Reidemeister moves are the only physically observable deformations.

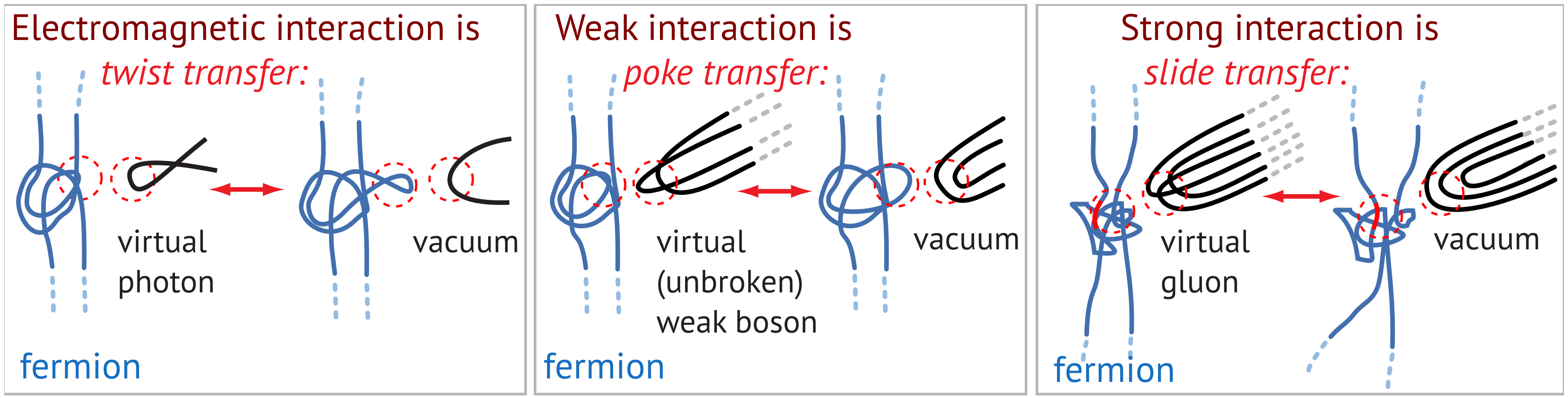
Reidemeister Moves Classify Interactions

Every tangle core deformation is built from **three basic types**: (Reidemeister 1926)



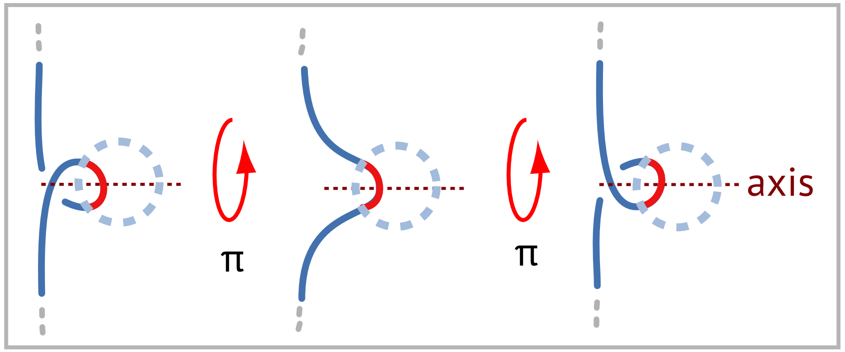
Twists generate **U(1)**, pokes generate **SU(2)**, parity violation and symmetry breaking, while slides generate **SU(3)**. (Schiller 2009, 2019, 2024 **link.**)

Gauge interactions are (statistical) crossing transfers:



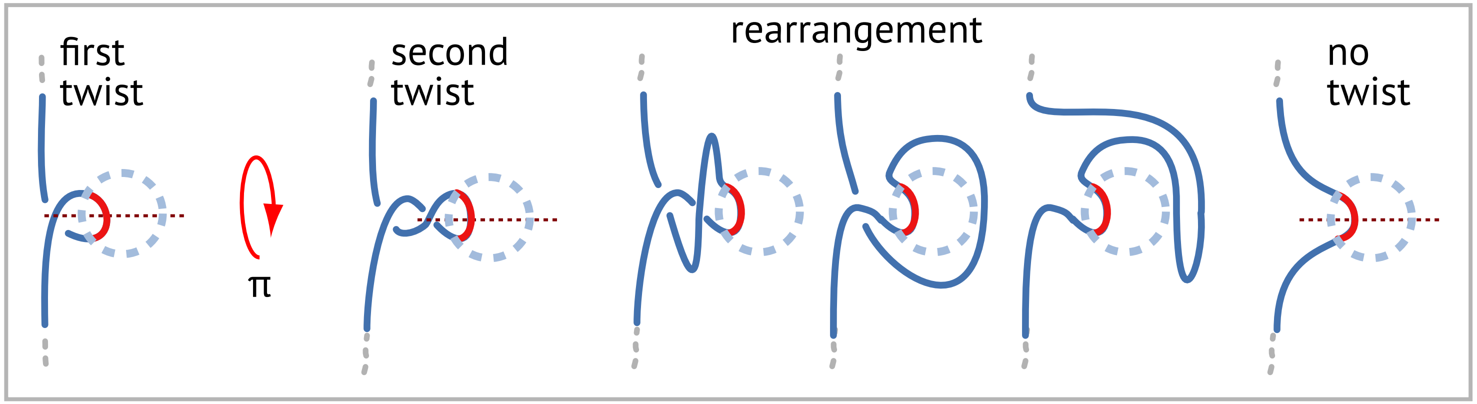
Twists Generate Local U(1)

The **twist**, or **first Reidemeister** move, is related to a crossing switch:



Twists, performed by rotating the **encircled segment**, are thus **observable**.

A **double twist** of the **encircled segment** can be rearranged to an untwisted strand, keeping the encircled segment fixed in space: **no** twist:

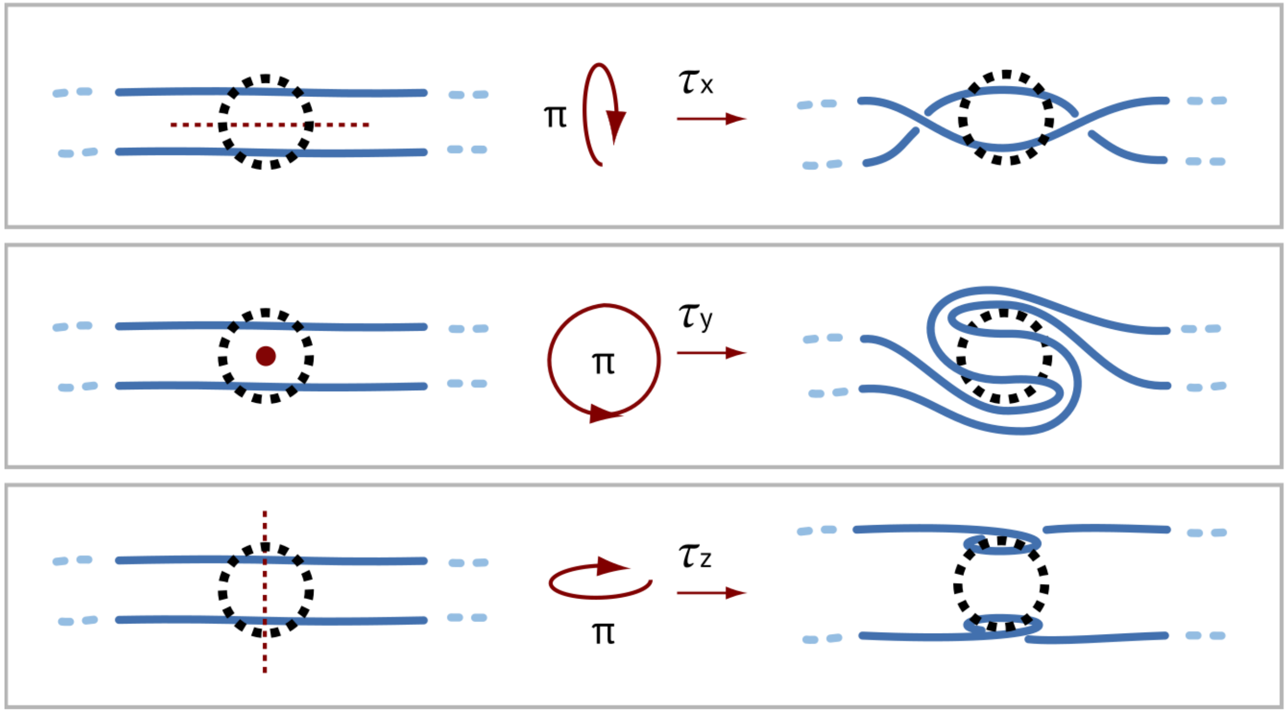


In a fermion, the twist around a given axis thus **generates a local U(1) Lie group.**

Twists rotate the dotted circle by π. *Generalized* twists rotate the dotted circle by *arbitrary* angles. They form the local Lie group U(1). **Rotating twists also yield a model for the photon.** (More later on.)

Pokes Generate SU(2) via the Belt Trick

The poke, or **second Reidemeister move**, on pairs of strands generates an SU(2) Lie group, because the three rotations by π generate the algebra of SU(2):



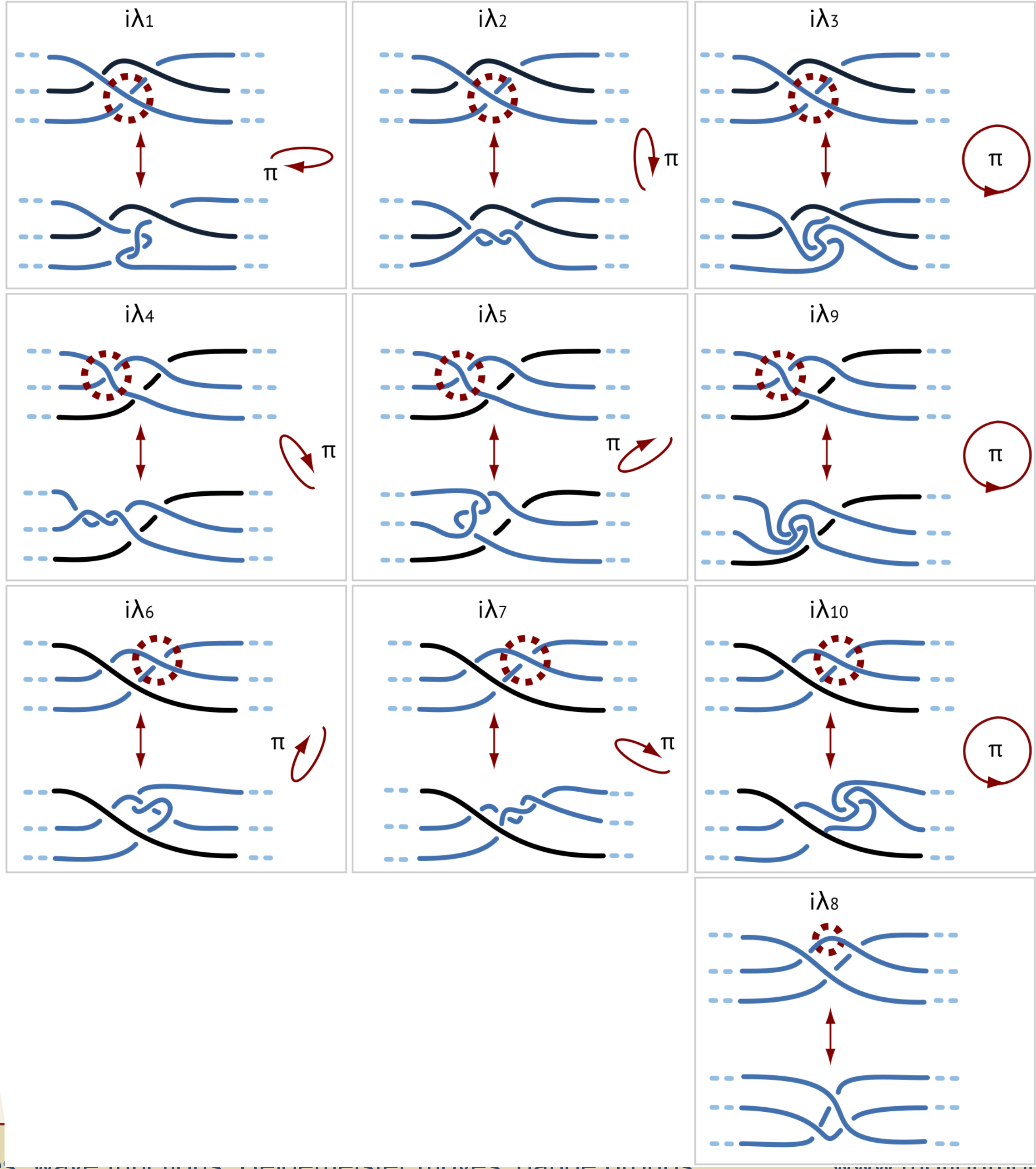
Pokes, like belts, yield the Pauli matrices, i.e., the Lie algebra of SU(2):

$$\tau_x = i\sigma_x = i\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\ ,\ \tau_y = i\sigma_y = i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\ ,\ \tau_z = i\sigma_z = i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Generalized pokes, by *arbitrary* angles, yield the full local Lie group SU(2). Maximal parity violation and SU(2) breaking also follow (see bonus material).

Slides Generate Three Belt Tricks and SU(3)

Slides, or **third Reidemeister moves**, acting on strand pairs in three-strand structures, can be generalized to the generators of the Lie group SU(3).



Slides rotate the dotted circle by π.

The deformations allow reading off the matrix representations (see next page).

λ_3, λ_9 and λ_{10} are not linearly independent.

Traditionally, λ_3 and λ_8 are used.

λ_8 is the slide prototype.

Slides Generate SU(3)'s Gell-Mann Matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_9 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$\text{and} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Of the ten slide deformations, only the first eight are linearly independent.

These eight deformations yield the Gell-Mann matrices.

The eight deformations generate the algebra of SU(3) – and describe gluons.

These eight generators also yield the relations $\operatorname{tr} \lambda_n = 0$ and $\operatorname{tr}(\lambda_n \lambda_m) = 2\delta_{nm}$.

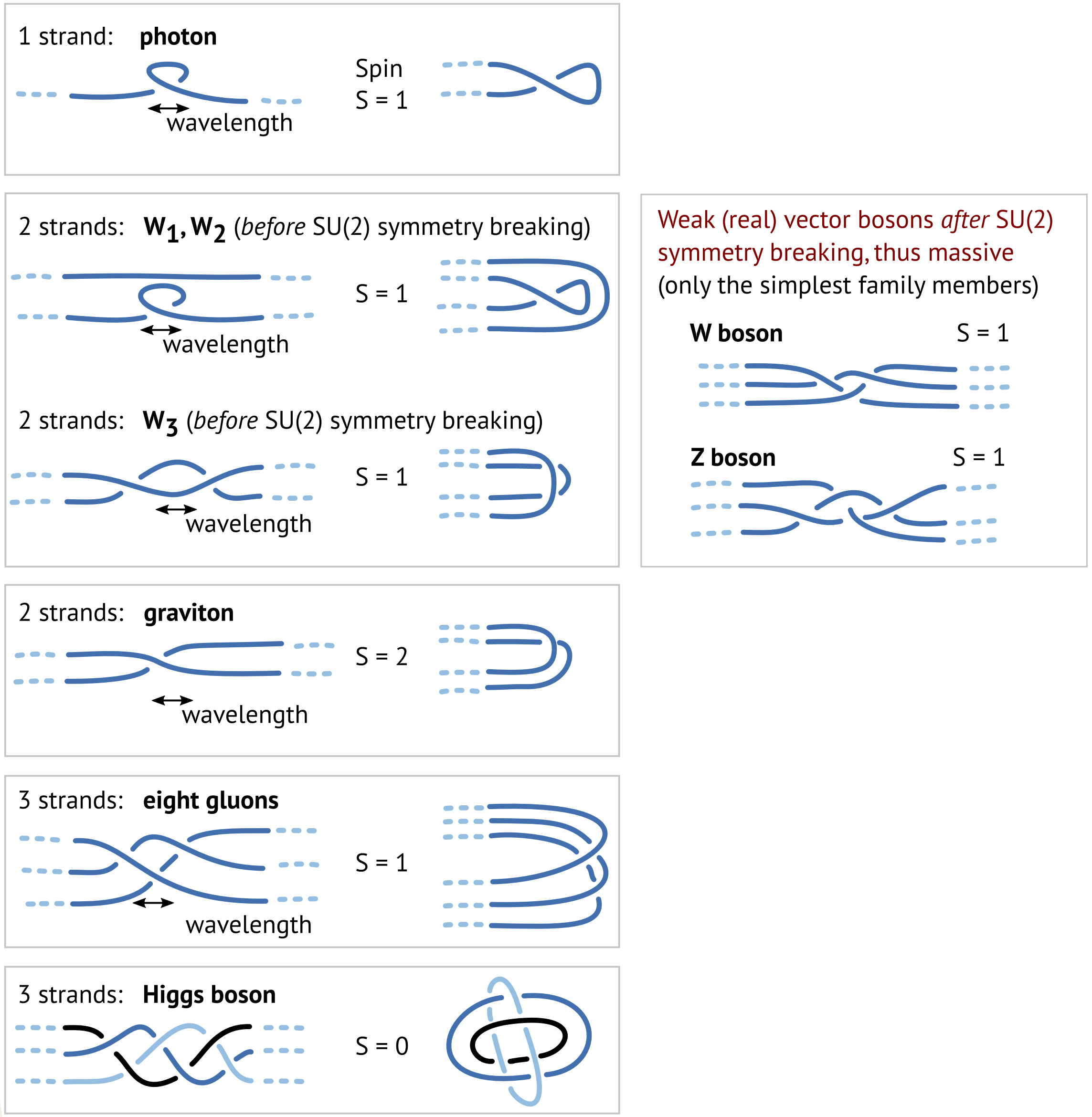
SU(3) has three linear independent SU(2) subgroups – one in each row.

Generalized slides, by *arbitrary* angles, yield the full Lie group SU(3). (**Publication link.**)

Elementary Bosons Follow

Elementary bosons are simple configurations of 1, 2 or 3 strands that propagate:

Real bosons: Virtual bosons:



'Elementary' means 1, 2 or 3 strands.

'Boson' means unlocalizable tangle.

The gauge bosons tangles reproduce all quantum numbers.

No additional gauge bosons are possible.

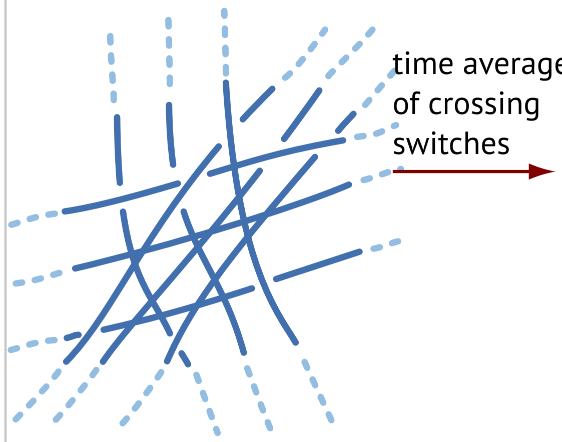
No other explanation of the gauge spectrum exists.

(Pedagogical link.)

A Planck-Scale Model of Almost Everything

The flat vacuum – a ***homogeneous strand aggregate***

Observation:

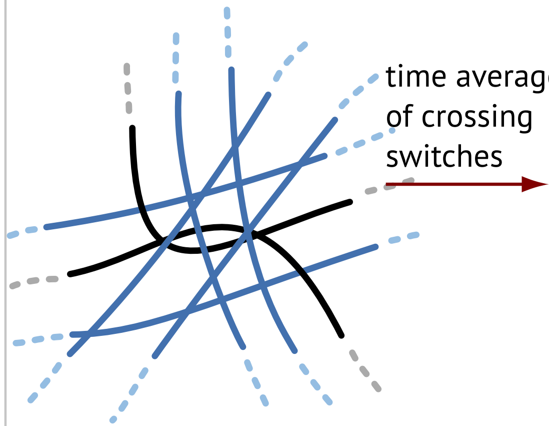


Nothing (for long observation times)

Virtual pairs (for short observation times)

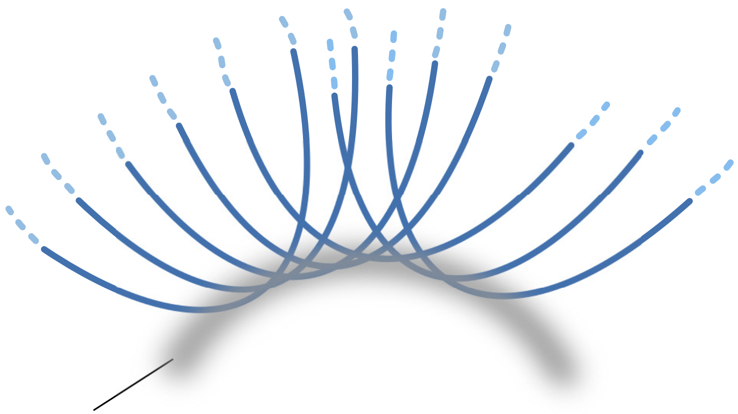
Curved vacuum – an ***irregular strand aggregate***

Observation:



Curved space with non-trivial metric

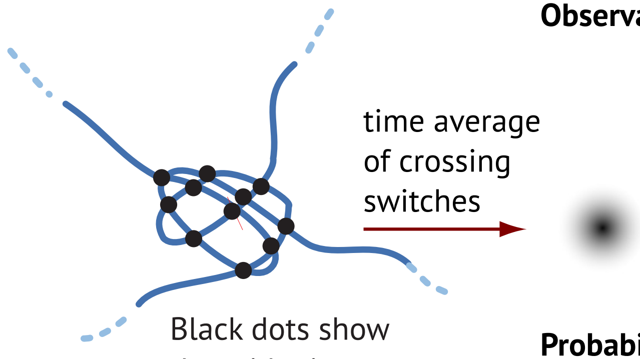
A black hole horizon – a ***weave*** of strands



Observation after time average of crossing switches: a **horizon**, i.e., a thin spherical cloud, with mass, moment of inertia, entropy, and temperature.

A particle – a ***tangle*** of strands

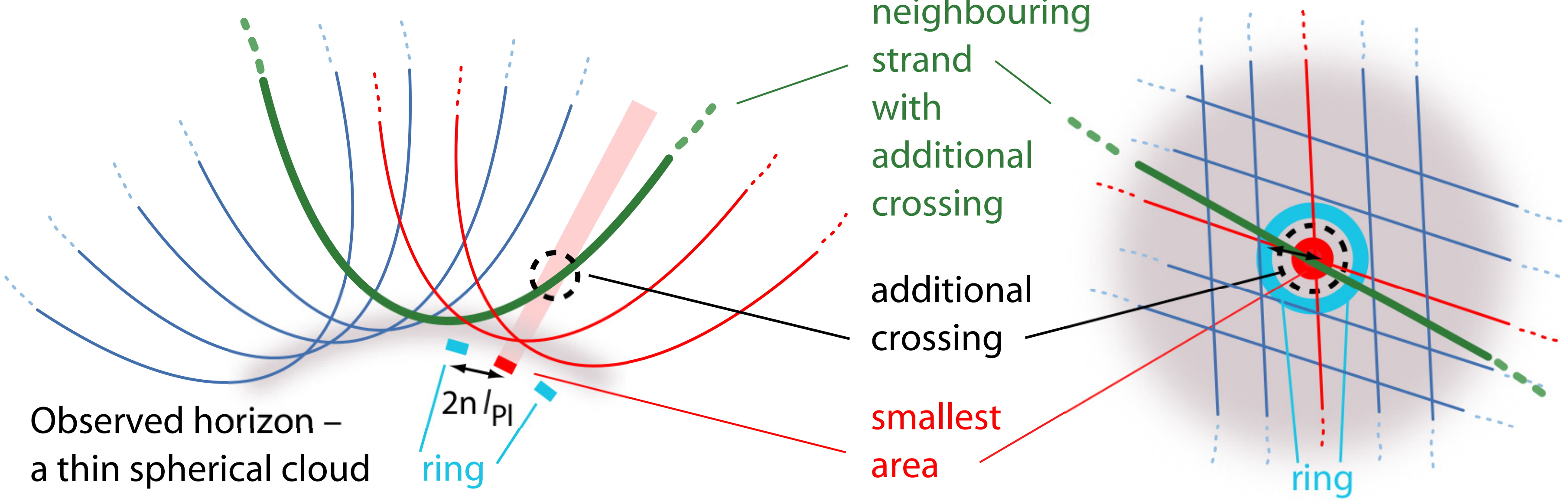
Observation:



Black dots show the midpoints of crossings.

Probability density

Black hole horizon in the strand conjecture:



The effective number n of possible microstates per smallest area:

$$n = 2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + ... = \mathrm{e} = 2.71828...$$

yields an entropy value S that depends on the area A: (Schiller 2009, 2019, 2023)

$$\frac{S}{k} = \frac{A}{4\,\hbar G/c^3} \quad - \mathcal{O}(\ln \frac{A}{4\,\hbar G/c^3})$$

The fundamental principle implies **black hole entropy**, energy, temperature – and evaporation: strands detach. Strands imply general relativity.

Strands imply **force** $\boldsymbol{F \leqslant c^4/4G}$, **power** $\boldsymbol{P \leqslant c^5/4G}$, **mass/length** $\boldsymbol{m/l \leqslant c^2/4G}$, etc. Strands again imply **pure general relativity**.

Thus, **no** singularities, negative energy regions, wormholes, black hole hair, torsion, time-like loops, running of G, or new quantum gravity effects.

Black Holes Can Rotate

Strands are not observable, only crossing switches are.
Black holes have a finite moment of inertia; mass is distributed over the horizon.

The Main Results

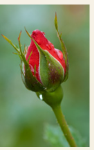
Only fluctuating tangles of strands explain **wave functions.**

Only fluctuating tangles of strands explain **elementary particles** – and their quantum numbers and properties – from tangle classification.

Only fluctuating tangles of strands explain the **gauge groups** – and all the interaction properties – using the Reidemeister moves.

The fascinating aspect is due to the **simplicity** of the fundamental principle and to the **uniqueness** of the explanations:

- The fundamental principle implies **observed particle physics only**.
- The fundamental principle implies **observed general relativity only**.
- **Only the fundamental principle** provides these explanations.
- There is **no way to modify or to generalize** the fundamental principle or the tangle model – and their predictions.



Predictions – Beyond The Standard Model

- Planck length and Planck time are the smallest measurable intervals. **Space is neither continuous nor discrete.** No new quantum gravity effects.
- **3 dimensions. No supersymmetry. No non-commutative space.**
- Planck momentum and energy are the highest measurable values for elementary particles. $c^4/4G$ and $c^5/4G$ are maximum force and luminosity. Maximum values for probability densities, electric fields, magnetic fields, strong and weak fields exist. **No trans-Planckian effects.**
- **3 gauge interactions. Only. They are fundamental. No GUT.**
- **3 generations. No new particles. No unknown elementary dark matter.** No axions, no WIMPS, no sterile neutrinos, no monopoles, etc.
- **No measurable deviations from the standard model.** Only known Feynman diagrams. Scattering amplitudes, running, $g-2$, and electric dipole moments are as predicted. No proton decay. No baryon number violation. CPT holds. **Dirac neutrinos with normal mass order.**

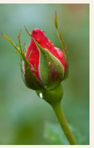
- ***No physics* beyond the standard model with massive Dirac neutrinos.**

!

And

- **Masses, mixing angles and coupling constants *can be calculated.***

Mathematical Outlook And Challenges



Prove, clarify or disprove:

• No visualization of SU(2) or SU(3) without tethers is possible in 3 dimensions.

• No visualization of SU(n) with strands (or without strands) for $n > 3$ is possible in 3 dimensions. (This has profound consequences for physics.)

• The rational tangle classification is mathematically complete and leaves no room for additional elementary fermions or bosons.

• The rational tangle classification is mathematically complete and leaves no room for additional defects in space that are neither fermions nor bosons.

Determine:

• How does the probability of belt-trick-like rotation for a tethered ball depend on the chirality and size of the tethered structure and on the number of ropes? Use ideas from hydrodynamics of viscous liquids.

• Use the result to estimate neutrino masses. Ideally, before they are measured.

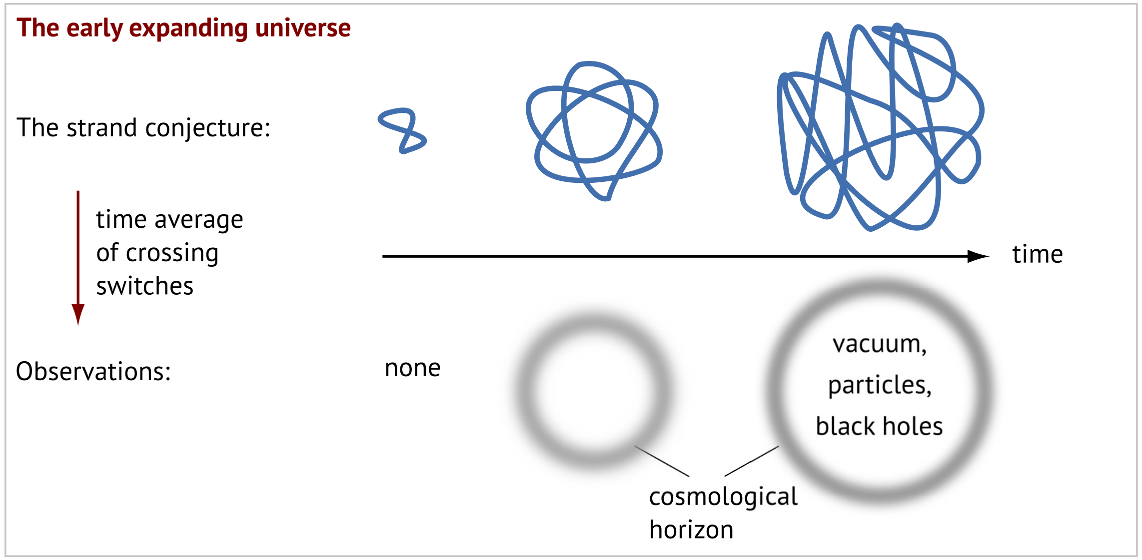
• Calculate the three gauge coupling constants from the average tangle shape.

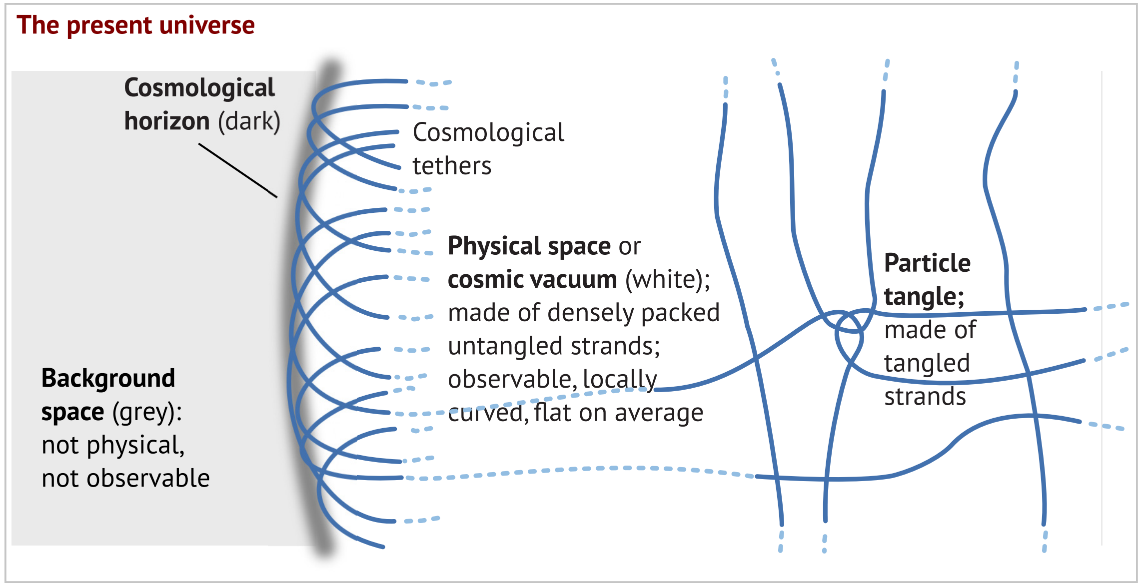
Earn prizes for specific math problems on knot theory:

• See www.motionmountain.net/charge-mass.html

The Universe

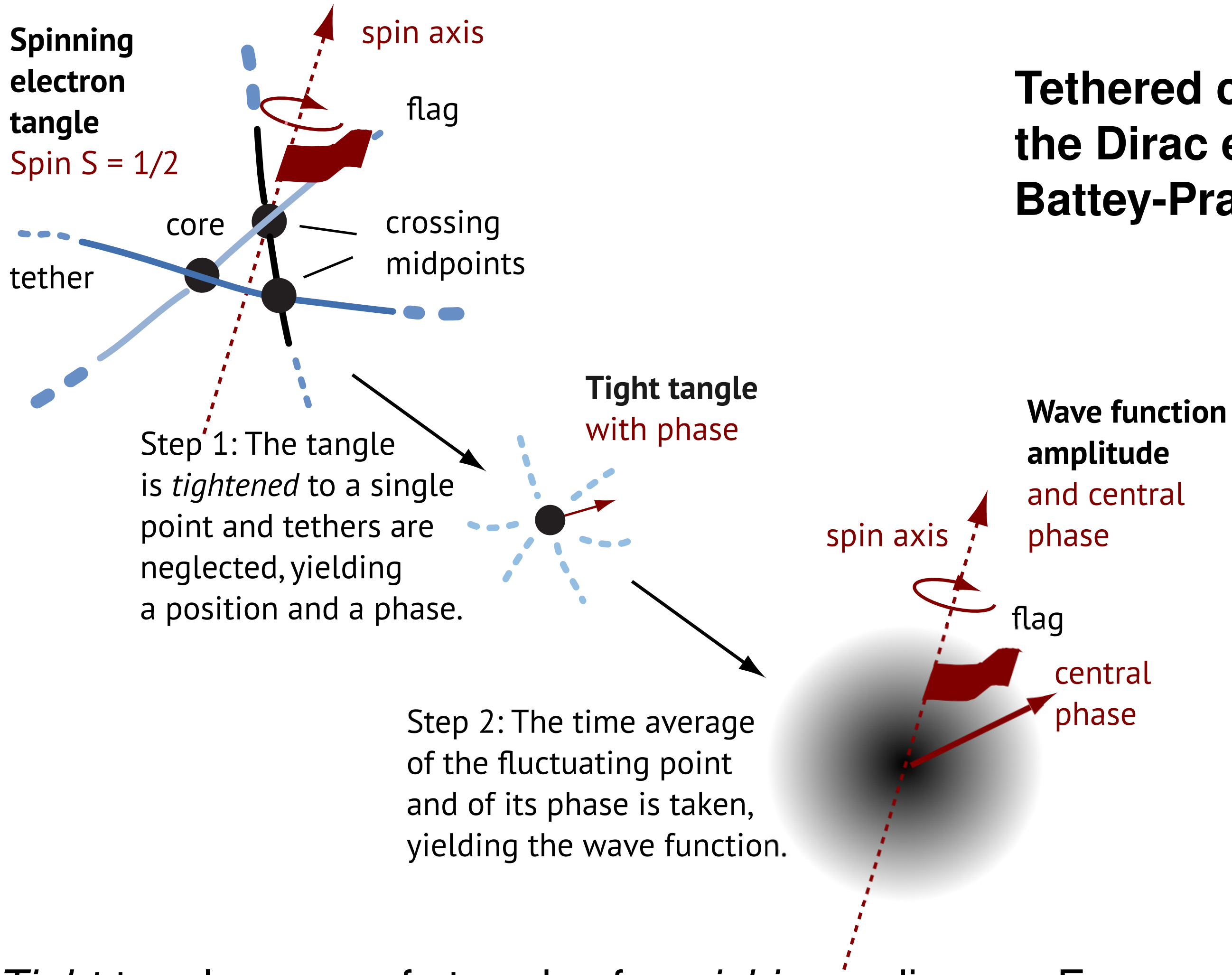
Nature is a wobbly criss-crossing strand woven into the night sky.
The universe plays cat's cradle.





Tangles Also Yield Path Integrals

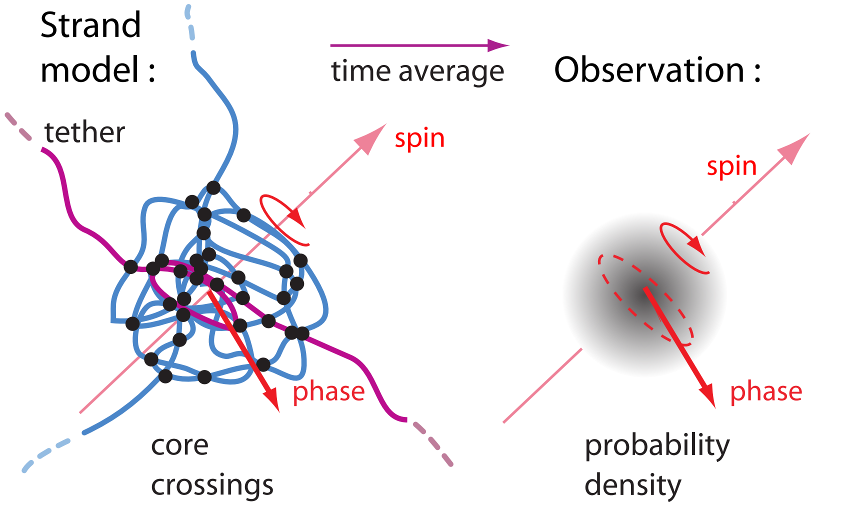
The strand tangle model for a fermion in the path-integral formulation

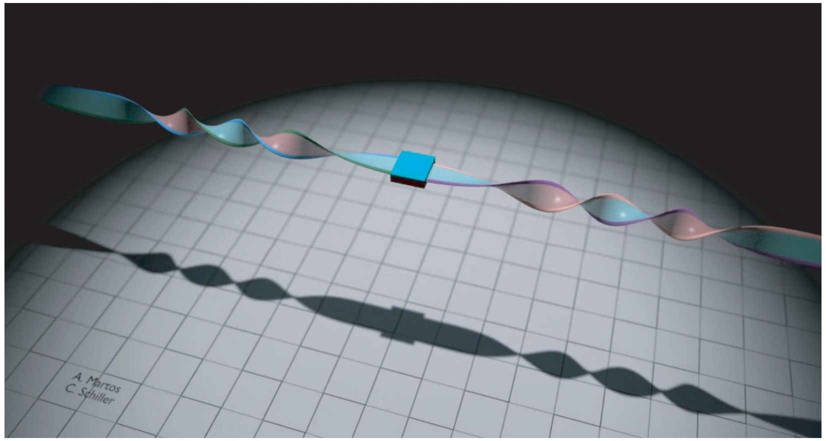


Tethered cores follow the Dirac equation. Battey-Pratt & Racey, 1980.

Tight tangle cores of strands of *vanishing* radius are Feynman's point particles. Their phase (arrow / flag) rotates when advancing. Their crossing (midpoint) density yields Dirac's equation. **(Pedagogical link.)**

Spin 1/2, the Belt Trick and Dirac's Equation





Free particles (spinors) are (blurred) spinning tangle cores.
Dirac's belt trick allows continuous (tethered) rotation (see film © by Antonio Martos).
Spin is rotation; spin value is due to strand number and tangle details.
Antiparticles are mirror tangles with opposite belt trick.
Particle momentum and energy are core wavelength and rotation frequency.

Quantum phase is 1/2 of the orientation angle of the tangle core.
The **wave function** is the time-averaged ("blurred") tangle crossing density.
Maximum speed c and minimum action $\hbar$ hold.

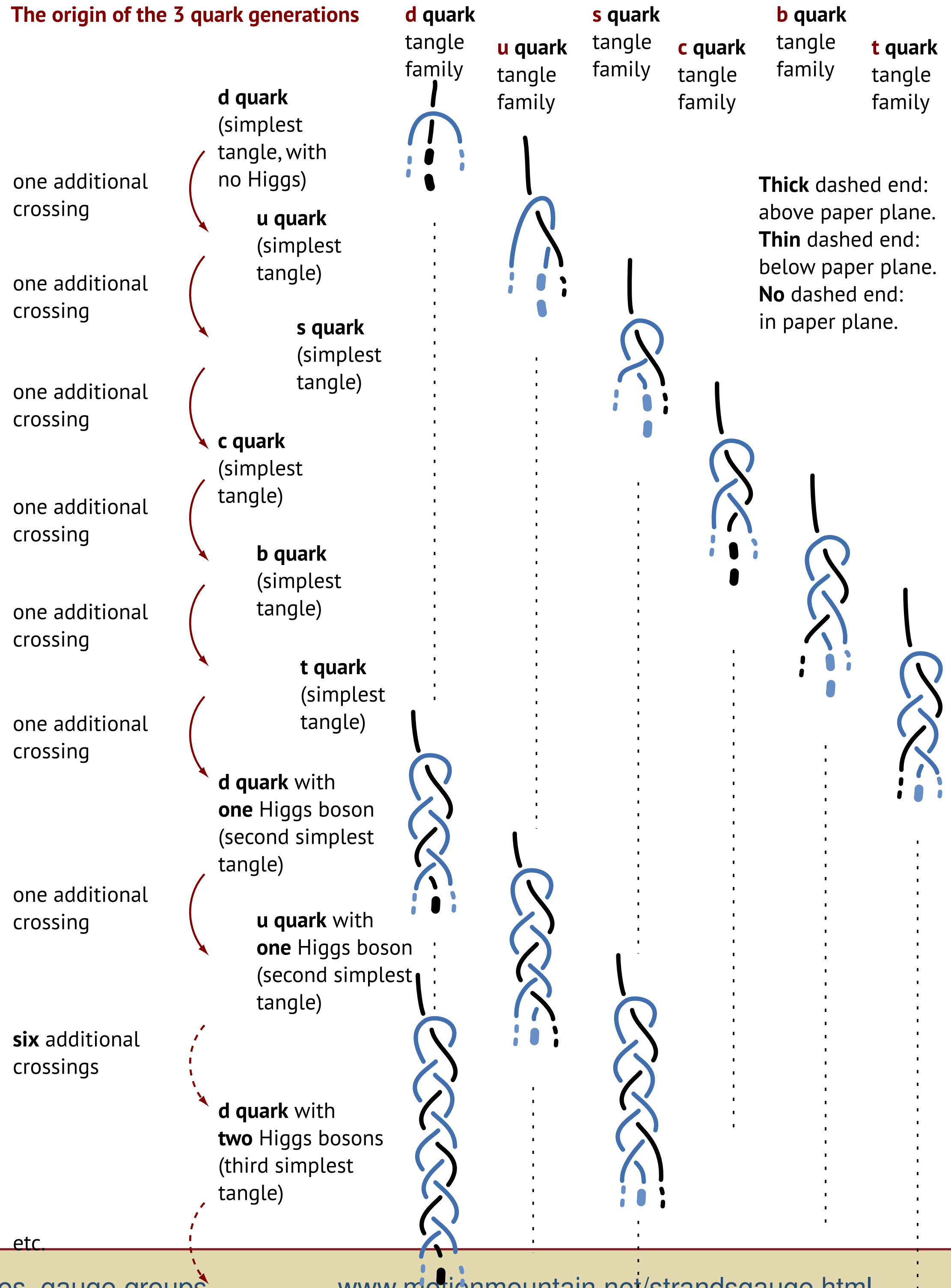
Strands imply the free **Dirac equation** $i\hbar\gamma^{\mu}\partial_{\mu}\psi = mc\psi$ and its propagator.
(Battey-Pratt and Racey 1980) **Dirac's equation is due to Dirac's trick.**
The **principle of least action** ("cosmic laziness") is the *principle of fewest crossing switches.*

6 Quarks

Quarks are infinite families of tangles.

Each family is due to Higgs boson interactions.

The three dimensions of space imply three quark generations.

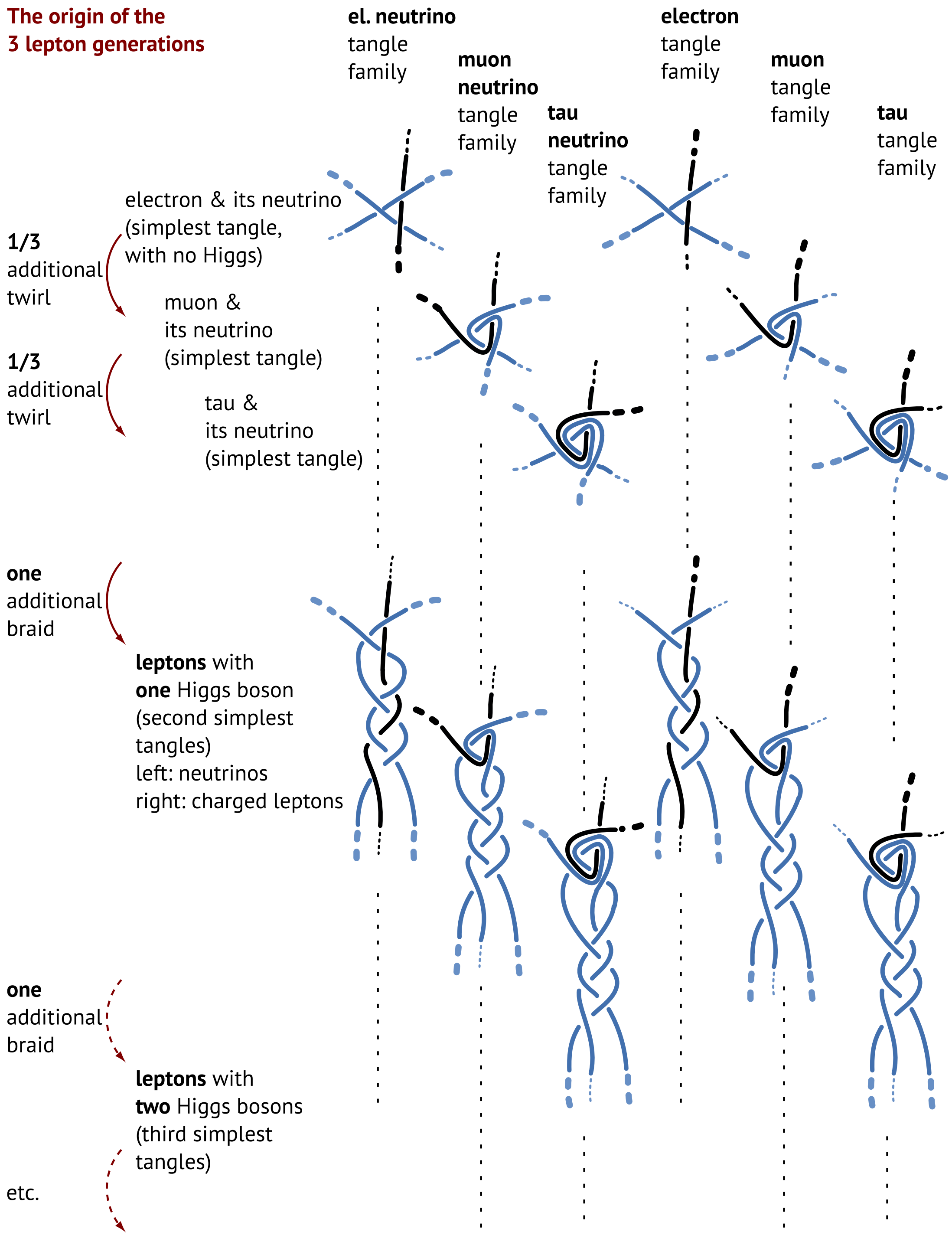


6 Leptons

Also leptons are infinite families of tangles.

Each family is due to Higgs boson interactions.

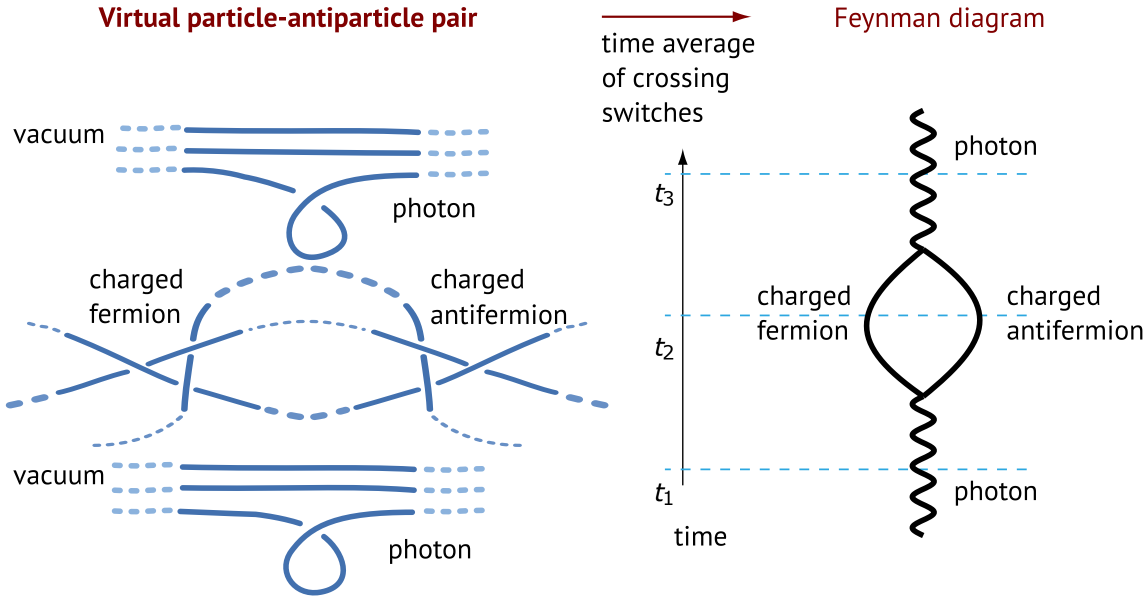
The three dimensions of space imply three lepton generations.

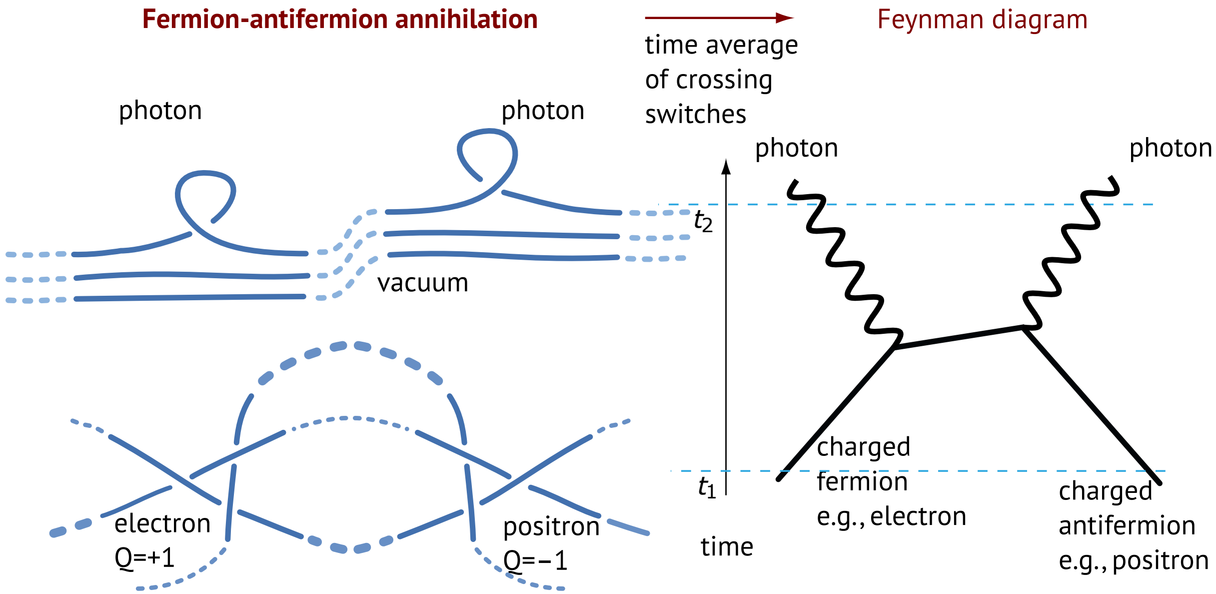


Electrons and Positrons

All effects of quantum electrodynamics arise.

This includes the running of masses and of charges.



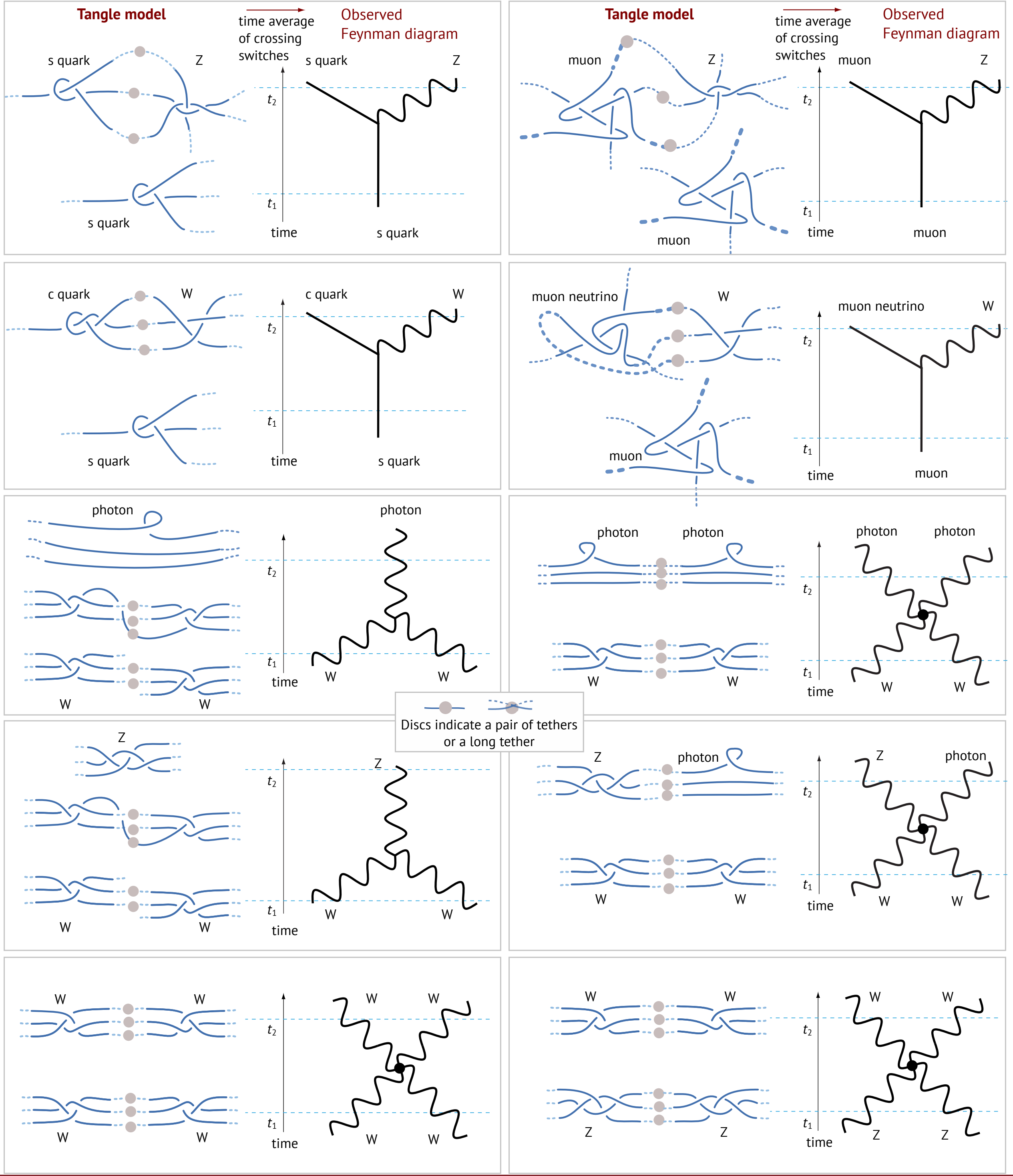


SM 1

The rational 3d particle tangles limit the possible interaction vertices.

Due to the tangle topology, only triple or quadruple vertices arise, but no fourfold fermion vertices.

Renormalizability is thus automatic in the tangle model.



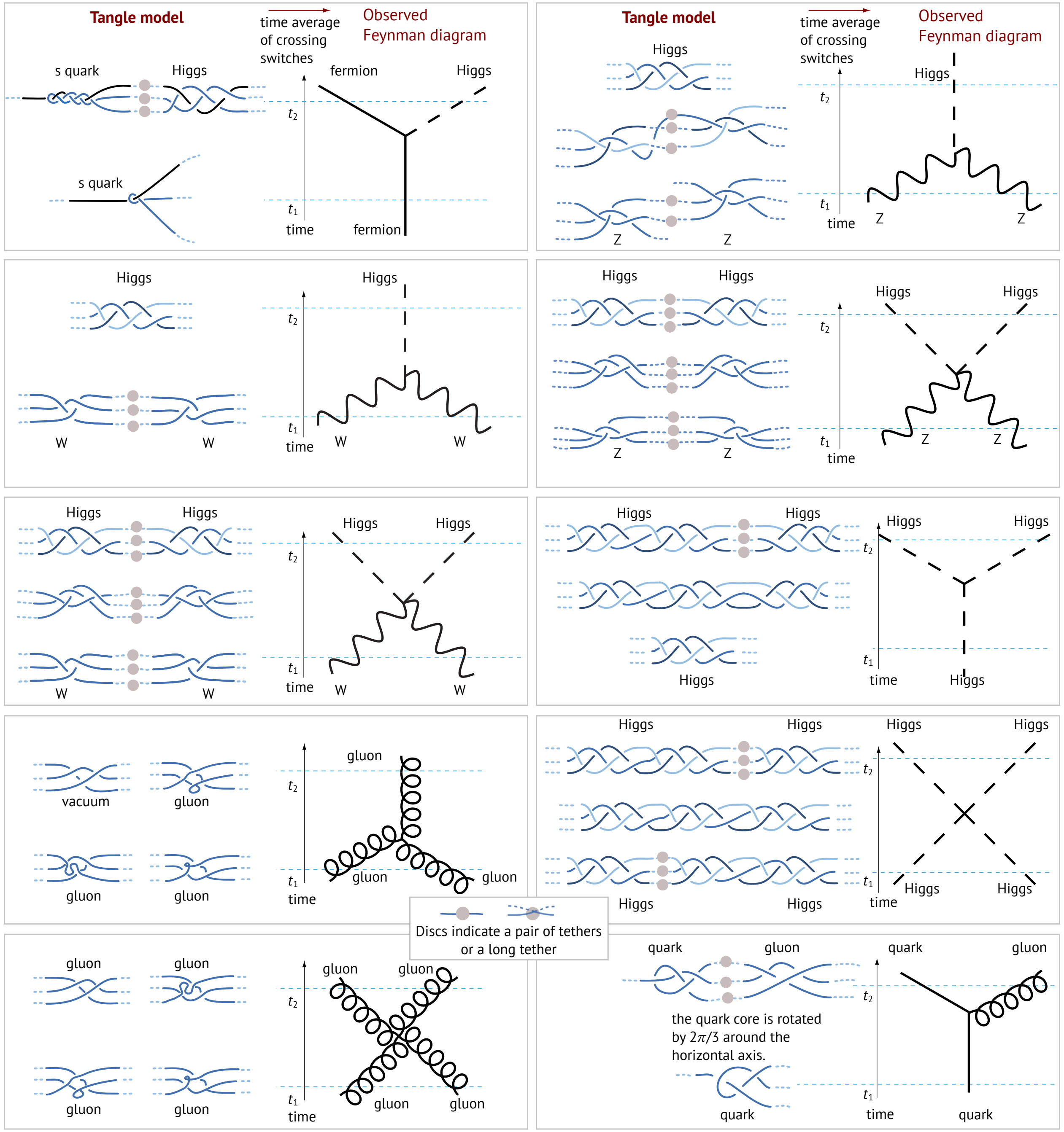
SM 2

The rational 3d particle tangles also yield Higgs self-interactions.

No vertex of the standard model is missing.

Due to the tangle topologies, no additional vertices arise.

The full standard model Lagrangian arises.



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Preprint: C. Schiller, *Testing a model for emergent spinor wave functions explaining gauge interactions and elementary particles*. **Pedagogical link.**

Additional preprints at **www.researchgate.net/profile/Christoph-Schiller-2/research**.

Other pedagogical material at **www.motionmountain.net/tangles**.

Animations at **www.motionmountain.net/videos.html#strands**.

Experimental and theoretical predictions at **www.motionmountain.net/predictions**.

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