Testing a model for emergent spinor wave functions explaining gauge interactions and elementary particles

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1 author:

Christoph Schiller
Motion Mountain Research - Germany and Italy

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Christoph Schiller *
Motion Mountain Research, 81827 Munich, Germany
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Abstract

A geometric model for wave functions, which also allows deducing space, general relativity and the standard model of particle physics, is tested against observations. The model is based on Dirac's proposal to describe spin 1/2 particles as tethered objects. Dirac's proposal is condensed into a fundamental principle that defines the quantum of action at the Planck scale. As a consequence, quantum particles are modelled as fluctuating rational tangles made of unobservable strands of Planck radius whose crossing switches are observable. Time-averaged densities of crossings are shown to have all the properties of wave functions and to follow the Dirac equation, as Battey-Pratt and Racey showed already in 1980. Strands provide the only known model for the emergence of wave functions, spinors, quantum theory, and quantum field theory.

Several consequences of the tangle model go beyond quantum field theory. Classifying the possible rational tangles yields the observed spectrum of elementary particles, their quantum numbers, and unique calculable mass values. This is the only known derivation of the particle spectrum. Classifying tangle deformations with the three Reidemeister moves yields the three observed gauge interactions, their symmetry groups, and unique, calculable coupling constants. This is the only known derivation of the interaction spectrum from the Planck scale. The tangle model also visualizes qubits, settles numerous issues of quantum field theory, highlights the contradictions in the Yang-Mills millennium problem, includes general relativity, and clarifies the relation to other approaches to relativistic quantum gravity. Overall, the tangle model yields more than 50 precision tests of quantum physics. All the tests agree with the observations. All tests predict a lack of effects beyond the standard model with massive Dirac neutrinos. Therefore, the fundamental principle of the tangle model proves that the standard model of particle physics is unique, simple, and elegant.

Keywords

Strand tangle model; origin of wave functions; origin of spinors; origin of quantum theory; origin of the standard model.

* cs@motionmountain.net

Contents

An appetizer: describing nature with high precision and little mathematics

A description of nature based on the Planck limits c, $\hbar$, $c^4/4G$ and $k \ln 2$ automatically contains special relativity, quantum theory, general relativity and thermodynamics. This result has been proven in many publications. They are summarized and cited in the following.

However, the Planck limits do *not* explain the appearance and the details of wave functions, elementary particles and gauge interactions. These aspects of nature can *only* be deduced from *tangles of strands* – and in particular from their three-dimensional topology and shapes. The arguments are given in the present article. The arguments are based on simple topological knot theory that has been textbook knowledge for almost 100 years.

The Planck limits also do *not* explain the values of the particle masses, the coupling constants, and the mixing angles. Instead, *tangles of strands of Planck radius* do explain these values. Explanations and ways to proceed have been presented in previous publications. They are summarized and cited later on. High-precision calculations are still the subject of research.

In short, all observations in nature – including the standard model of particle physics with massive Dirac neutrinos, quantum field theory, and general relativity – are described *exactly* by Planck limits and strands. A single fundamental principle describes all nature. Only the rotation curves of galaxies remain unclear. *Strands imply the lack of additional, unknown laws of nature.* This conclusion agrees with all observations in the past and is tested in every experiment. In particular, no new mathematics is needed to describe nature with precision. Therefore, almost no mathematics is needed in the following to deduce wave functions, particles and interactions.

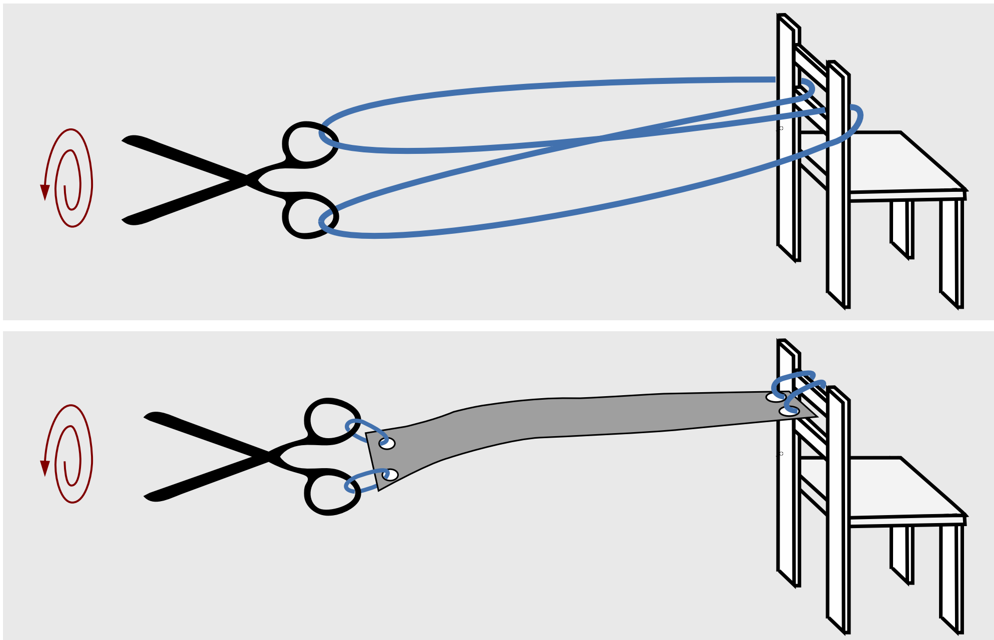

FIG. 1: In his lectures, Dirac demonstrated the properties of spin 1/2, a pure quantum effect, using a pair of scissors tied to a chair with strands or with a belt. The figure is inspired by the book by Penrose, who attended Dirac's lectures [1]. Any tethered object, such as a pair of scissors, returns to the original state only after a double rotation, but not after a single rotation. Dirac's trick is presented in detail below.

Part I: A geometric model for emergent quantum theory

The discovery of the quantum of action $\hbar$ led, via the observation of wave-particle duality, to the use of the *wave function* as the description of the state of a quantum particle or a quantum system. The wave function is described by one or several complex fields that follow an evolution equation, such as the Schrödinger equation or the Dirac equation. The present article argues, step by step, that wave functions, spinors and their evolution equations emerge from a more fundamental description, namely from *fluctuating tangles of unobservable strands with Planck radius.* The strand tangle model is based on the counterintuitive notion, due to Dirac, that every particle in nature is *tethered*, i.e., connected with unobservable strands to the cosmological horizon. Dirac demonstrated the idea for a matter particle with the arrangement illustrated in Figure 1. In this article, the scissors represent the particle and the chair the cosmological horizon – though in nature, particles and horizons consist of strands as well.

All the results given in the following are deduced from a single fundamental principle inspired by Figure 1: *crossing switches of unobservable strands with Planck radius define the quantum of action $\hbar$.* Part I of the present article uses the fundamental principle and Figure 1 to derive spin $1/2$ and all its properties. Part II derives wave functions, superpositions, entanglement, decoherence, and measurements from strands. Part III uses strands to derive the Schrödinger equation, antiparticles, spinors, the Dirac equation, and experimental predictions. All known results and observed about wave functions are recovered, without measurable deviations.

Part IV shows that several differences arise from quantum theory. Only these differences make the model worth exploring. The fundamental principle is shown to restrict the possible elementary particles, to derive the three particle generations, to derive the particle masses and quantum numbers, to restrict the possible gauge interactions, and to derive their gauge groups and coupling strengths. Strands imply the complete standard model of particle physics, including massive Dirac neutrinos, without any deviation. So far, strands provide the only available derivation of the standard model that does not predict unobserved phenomena. All differences from quantum theory deduced from strands agree with observations. The differences solve the open problems in quantum theory, quantum field theory, and the standard model of particle physics.

The presentation is self-contained. Each section ends with a summary paragraph that begins with *'In short'* and allows for quick reading. The appendices cover additional aspects of strands, from general relativity to qubits, preons, strings, the Yang-Mills millennium problem, and the limitations of axiomatic approaches to quantum field theory.

Strands allow the derivation of more than 50 high-precision experimental tests, including predictions about neutrino masses, supersymmetry, and dark matter. The predictions are highlighted and numbered as **Test *n***. All agree with all observations.

In short, the article tells how the fundamental principle of the strand tangle model leads to the emergence of wave functions, spinors, Dirac's equation, elementary particles, gauge interactions and the fundamental constants, including the coupling constants and particle masses. The Lagrangian of the standard model is derived. All open questions about quantum theory are answered. Experimental tests and predictions are deduced. They agree with all the data. No other model provides these results.

1 Nature provides a limit for every observable

In 1899, Planck discovered the quantum of action $\hbar = h/2\pi$ in light. In the years around 1910, Bohr used to present quantum theory [2] as the sum of all consequences of the smallest measurable action value $\hbar$. The limit agrees with all real experiments and thought experiments ever carried out.

In 1905, Einstein derived special relativity from the invariant maximum energy speed c observed in nature [3]. The *principle* of maximum speed, together with all its consequences, was confirmed in all real experiments and thought experiments ever carried out. The limit given by the quantum of action was found to be invariant.

In the years around 1926, applying the *principle* of the quantum of action to electrons led Schrödinger and many others to develop the concept of the wave function, to describe its evolution and to present its use for physical measurements. In 1928, Dirac incorporated both spin $1/2$ and the energy speed limit c. Dirac's equation for electrons and Planck's description of light led to quantum electrodynamics, which agrees with all experiments ever carried out.

Around the year 1973, Elizabeth Rauscher discovered that general relativity implies a maximum force [4]. Around the year 2000, the work by Gibbons and by the author showed that Einstein's field equations can be deduced from the *principle* of maximum force $c^4/4G$ or the *principle* of maximum power $c^5/4G$ [5–9]. If preferred, one can also start from the equivalent *principle* of maximum mass to length ratio $c^2/4G$ [10] or from the *principle* of maximum mass flow rate $c^3/4G$. All these limits are invariants and are related to black holes. The force and power limits, as well as general relativity, agree with all thought experiments and all real experiments ever carried out [5–46].

Also thermodynamics can be deduced from an invariant limit: thermodynamics follows from the *principle* of the smallest observable system entropy $k \ln 2$ [47–52]. The entropy limit, and with it thermodynamics, agrees with all thought experiments and with all real experiments ever performed. Together with the other limits, the entropy limit allows deriving Bekenstein's entropy bound and black hole entropy.

Around the year 2000, the mentioned results allowed summarizing special relativity as the consequence of maximum speed c, general relativity as the consequence of maximum force $c^4/4G$, quantum theory as the consequence of the smallest action $\hbar$, and thermodynamics as the consequence of the smallest system entropy $k \ln 2$. Ideally, the mentioned limits are called *corrected* Planck limits because, traditionally, the quantity c^4/G is called the Planck force. In nature however, an additional factor 4 is part of the force limit $c^4/4G$ [5, 9]. In general, the corrected Planck limits of nature arise when G is replaced by $4G$ in the conventional Planck units.

As a consequence of observations, *all corrected Planck units are limits of nature.* In particular, the corrected Planck limits include the minimum length $\sqrt{4G\hbar/c^3}$ and the minimum time $\sqrt{4G\hbar/c^5}$. Intervals smaller than these two values cannot be measured, cannot be observed, have no effect, and thus do not exist in nature [53–59]. For this reason, all Planck limits that are *lower* limits are also the lowest possible measurement errors. Nature does not allow vanishing measurement errors. Several Planck limits – the Planck energy, the Planck momentum and the Planck mass – are valid only for a single elementary particle because this condition is necessary to derive these particular limit values. In total, modern physics predicts that all searches for observations or effects *beyond* the corrected Planck limits will fail [60]. Indeed, no known observation contradicts the corrected Planck limits, despite intensive search efforts in special relativity [61], in quantum field theory and particle physics [62], in general relativity [9], in quantum gravity [63], and in thermodynamics [52].

Not all Planck limits are equal. The speed limit c is observable because some systems realize it, such as light and gravitational waves, and because other systems approach it quite closely, such as particles in accelerators or cosmic rays. The quantum of action $\hbar$ is observable because some systems realize it, such as trapped electrons, and because other systems approach it quite closely, such as in the photoelectric effect or the Frank-Hertz experiment. The force limit $c^4/4G$ is observable because some systems realize it, such as black holes or gravitational horizons, and because other systems approach it quite closely, such as colliding black holes. Nature also allows realizing *non-relativistic* quantum gravity limits, such as $4G\hbar$; this was shown by measuring the quantum states of cold neutrons in gravity [64–66].

In contrast, the Planck limits of *relativistic* quantum gravity – those containing c, $4G$ as well as $\hbar$ – such as the corrected Planck length, Planck time or, for single elementary particles, the Planck energy, are neither realized nor approached by any system in nature. Nature does not allow realizing the limits of relativistic quantum gravity. Experiments and observations about relativistic quantum gravity are at least several orders of magnitude away from the limit values. For example, measurements of the electron dipole moment are a factor 10^3 away from the corrected Planck length [67, 68]. Measurements of elementary particle energy in cosmic rays are more than a factor 10^7 away from the corrected Planck energy. Measured temperatures, such as in quark-gluon plasmas, are about a factor of 10^{20} away from the Planck temperature. Measurements of time intervals, such as the Higgs boson lifetime, are about a factor of 10^{20} away from the corrected Planck time. The Planck electric field limit is unattainable because of the Schwinger limit, which is over 10^{40} times lower. The reason for the unattainability of the relativistic quantum gravity limits is the existence of minimum measurement errors, in particular, the minimum error for length [53–59], and similarly, the minimum errors for time, area and volume.

In short, the corrected Planck limits imply that *no trans-Planckian effect* of any kind exists in nature. No observation has found a result *beyond* the corrected Planck limits. In addition,

▷ In experiments, nature allows reaching the *simple* limits c, $\hbar$ and $4G$ as well as limits due to combinations of *two* constants, such as $c^4/4G$, $\hbar/c$, $4G\hbar$.

▷ The minimum measurement errors *prevent* from reaching any combination of all *three* constants, such as $4G\hbar/c^3$ or $c^7/16G^2\hbar$, in experiments.

Any precise description of nature must reproduce these observations.

2 Nature's limits forbid fundamental equations of motion and Lagrangians

Experiments show that nature is made of particles and space. Therefore, the search for a theory of emergent quantum mechanics, which describes particles, is closely tied to the search for a theory of relativistic quantum gravity, which describes space. The first search is about an emergent description of particles and wave functions, whereas the second search is about an emergent description of space. Nature's limits guide both searches.

In nature, evolving systems are described by physical action. The action W can be defined as

$$W = F\,l\,t = \frac{F\,l^2}{v} \quad \text{or as} \quad W = E\,t = m\,c^2\,t = \frac{m}{l}\,c\,l^2 \quad , \tag{1}$$

where F is force, l is length, v is speed, E is energy and m is mass. The expressions can be used to insert the maximum speed c, the minimum action $\hbar$, and, for general relativity, either the maximum force $c^4/4G$ or the maximum mass per length ratio $c^2/4G$ of black holes. Both cases yield the same statement:

▷ Nature limits length intervals and length measurement errors:

$$l \geqslant \sqrt{\frac{4G\hbar}{c^3}} \approx 3 \cdot 10^{-35}\,\mathrm{m} \quad . \tag{2}$$

So far, no experiment disagrees. In other words, the domain of nature where maximum speed, maximum force, and the quantum of action play a role at the same time – the domain of relativistic quantum gravity – is characterized by a *minimum length*. The minimum length is given by twice the Planck length. Smaller lengths *cannot* be achieved, measured or observed. Likewise,

▷ Nature's corresponding lower limits for area, volume, and time cannot be exceeded.

Again, no experiment disagrees. Historically, scientists took two millennia to define space as a continuous set of points and to get used to working with real numbers as coordinates. Poking gentle fun at thinkers who challenged continuity and questioned the infinitely small, such as Zeno of Elea, is a part of modern science. In full contrast, relativistic quantum gravity implies:

▷ Nature has no point-like physical observables.

So far, in experiments, no point-like observable has been found. Point-like quantities cannot be detected, are not measured, nor do they play a role in nature. Point-like quantities – including point particles, δ functions (δ distributions), or singularities of any type – do not exist in nature. In other words, *every point-like concept is an approximation.*

The existence of the minimum length in nature has drastic consequences. Due to the minimum length and the minimum length error, space and time *cannot be continuous* and *cannot be discrete.* The minimum length implies that continuity, derivatives, differentials, discrete points and discrete instants of time do not exist in nature. As a result, all these concepts are only *approximate:* they are due to averaging of some random substrate. The intrinsic measurement errors in nature also imply that an *equation* can never be valid precisely, i.e., can never be tested without error or doubt. In relativistic quantum gravity, two quantities – the two sides of an equation – cannot be shown to be equal by any experiment.

▷ No unified theory of physics can make use of equations.

The conclusion is confirmed by a second argument. Any unified equation of motion or any unified Lagrangian would contain some fundamental fields. The origin of these fields would have to be defined. However, a unified theory must explain the origin of all quantities it uses. This is impossible in a description that uses Lagrangians.

▷ No unified theory of physics can be based on a Lagrangian.

All fields in nature – electric, magnetic, strong, weak, gravitational – must be emergent. In other words, all equations of motion and all Lagrangians are approximate and due to averaging. Because of the intrinsic measurement errors, any description of fundamental constituents must be *statistical* and must involve *many* constituents. The surprising conclusion that nature cannot have parts is explored in Appendices A and G.

Because of the minimum length and the lack of points, any unified description must deduce all known equations of physics without the use of further equations.

▷ Unification means: equations from no equation.

History shows that such a program can be realized. Planck's discovery that action W is limited by

$$W \geqslant \hbar \approx 10^{-34}\,\mathrm{Js} \tag{3}$$

led to quantum theory, the derivation of Schrödinger's equation and Dirac's equation. Rauscher's discovery that force F is limited by

$$F \geqslant \frac{c^4}{4G} \approx 3 \cdot 10^{43}\,\mathrm{N} \tag{4}$$

can be used to derive general relativity and its field equations, as shown by Gibbons [5] and in references [6–9]. The remaining step is to realize the program for the standard model of particle physics.

In short, because of the minimum length, fundamental physics cannot have equations. The only way left to describe nature and observations is to follow the usual approach of fundamental science. First, a fundamental principle is distilled. Then, the fundamental principle is used to logically deduce all possible consequences. These consequences can be equations even if the principle does not contain any. After that, each consequence is tested by comparing it with observations. If a comparison fails, the principle is falsified. If a comparison with observations is positive, the next test is performed. Comparison with observations is the *only* criterion for checking the validity of a principle. All other claimed criteria about fundamental physics are mere habits of thought. Of these habits, the yearning for fundamental equations is a particularly persistent one. Because of the minimum length, nature *does not fulfil* this yearning. Nature does not contain points. Nature's constituents differ from points.

3 Black holes, space and fermions require strands

A few lines are sufficient to deduce that both the search for emergent quantum mechanics and the search for relativistic quantum gravity require finding the *common constituents* of particles, wave functions, space, and black holes.

Black holes yield many details about the constituents of nature. The minimum length appears in the form of the minimum area $c^3/4G\hbar$ in the expression for their entropy S:

$$S = \frac{kc^3}{4G\hbar} A \quad . \tag{5}$$

Here, k is Boltzmann's constant, A is the surface of the black hole, and the factor 4 is due to the black hole limits. Even though the expression has never been confirmed by experiments, its order of magnitude is without doubt, as it follows from the limits of nature. Because black holes have entropy, they are composed of constituents that *fluctuate.* Because black holes have *finite* entropy, they are composed of a *finite* number of discrete constituents. Because black holes can be seen as limit cases for curved space *and* for densely packed particles, the constituents of black holes are also the constituents of both space and particles. Because the black hole entropy contains the minimum area, the common constituents of space and particles have an effective cross-section given by the minimum area. Because space and wave functions are *extended* and reach up to the cosmological horizon, this also holds for the common constituents. Therefore, particles and space are made of common constituents of Planck radius that fluctuate and reach the cosmological horizon. The common constituents thus are *filiform*.

The minimum length and the expression for black hole entropy eliminate many alternative types of constituents [60]. Black hole entropy eliminates filiform constituents of finite length as well as filiform constituents that are branched or knotted. The minimum length further eliminates all options for space and space-time based on discreteness or continuity. In particular, minimum length is in contrast with space lattices, with fractals, additional dimensions, non-commutative space, singularities, conformal symmetry, holography, higher dimensions, space-time foam, spatial lattices, supersymmetry, supergravity, conformal gravity, conformal field theory, anti-de Sitter space, de Sitter space, twistors, doubly special relativity, Wick rotation approaches, spin networks, and with any combination of continuous space with additional discrete or continuous mathematical structures. The minimum length and black entropy are incompatible with knots, bands, branched structures, networks, graphs, and ribbons. Of the many approaches to relativistic quantum gravity [69], only a few are not eliminated by the existence of a minimum length.

In the literature, fluctuating filiform constituents for matter particles were first explored by Dirac [70] and by Battey-Pratt and Racey [71] in the domain of quantum theory. Also photons have been visualized as helically deformed filaments [72]. In the 1990s, it was common lore that black hole entropy can be explained by filiform constituents, as told by Weber [73]. For a recent example, see Verlinde and Visser [74]. In general relativity, filiform constituents have been explored by Carlip [75] and independently by Botta-Cantcheff [76]. In relativistic quantum gravity, an interesting approach is the use of causal sets, as explored by Sorkin and by Bombelli [77]; filiform constituents can be seen as a specific realization of causal sets. A crossing of filiform constituents, defined below, also resembles a tetrahedron; such structures are used in some approaches to quantum gravity, such as the one by Oriti [78]. The behaviour of the spatial complement of filiform constituents, the motion of the space between them, was investigated in detail by Asselmeyer-Maluga [79, 80]; one then has a universe made of space only, albeit an intricately shaped one. The recent bit threads are discussed in Appendix B.

To date, none of the cited approaches has deduced a model for wave functions, for particles, or for gauge interactions. The only model that achieved these deductions is based on fluctuating filiform constituents of Planck radius that define the quantum of action with crossing switches. These filiform constituents are called *strands*. The strand segments leading to the cosmological horizons are called *tethers*. Fluctuating strands yield space, curvature, general relativity and black hole entropy [81, 82] and also yield quantum mechanics and the standard model [83–86]. It is worth exploring the background.

The main theories of physics – special relativity, general relativity, quantum theory, thermodynamics, and relativistic quantum gravity – arise from the invariant Planck limits. Can wave functions, spinors, the force spectrum, the particle spectrum, and the fundamental constants arise in an equally simple manner? Dirac provided the first important clue to the answer. From around 1929 onwards, as shown in Figure 1, Dirac used his *scissor trick* – also called the *string trick* or the *belt trick* – in his lectures [70]. Dirac's trick, illustrated in Figure 2, captures the basic properties of spin $1/2$: the original situation reappears after a rotation by 4π – keeping the observable particle, scissor or belt buckle *fixed* but *moving* the (assumed unobservable) belt or tethering strands around. In contrast, the original situation does *not* reappear after a rotation by 2π. Indeed, in experiments, after a rotation by 2π only, the sign of the wave function of a spin $1/2$ particle is observed to change, and it returns to the original sign only after a rotation by 4π. The only difference about a tethered system before and after a rotation by 2π is the sign of the tether crossings. This is illustrated in Figure 3. (In this context, a belt is equivalent to at least three tethers. The smallest number of crossing switches when rotating an object by one turn does not depend on the number of tethers.) If a system is *not* tethered, *no* difference is detectable between the situation before and after a rotation by 2π. Tethers, or strands, are the *only* possible visualization of spin $1/2$ in three dimensions. In simple terms: *Spin $1/2$ proves that in nature, everything is tethered and connected to everything else.*

In Dirac's belt trick – and the rest of this article – tethers and strands never intersect. For strands with Planck radius, the lack of intersections visualizes the minimum length. All strand crossings seen in diagrams are *apparent* or *skew* and are recognizable as 'crossings' only in a two-dimensional projection. In three dimensions, 'crossings' are defined and recognizable by the shortest distance between the two involved strand segments.

Above all, the belt trick implies that *crossing switches* are observable – even if the tethers themselves and their crossings are not. A crossing switch is the change of *sign of a crossing*, i.e., the change of which strand passes under the other. Specifically, Dirac's trick implies that a crossing switch leads to a *sign change* in the wave functions.

As Dirac explained [70], his trick also demonstrates that a spin value smaller than $\hbar/2$ is not possible. This result agrees with Bohr's statement: a smallest angular momentum value $\hbar/2$ implies a smallest observable action value given by a spin flip, which has the smallest possible

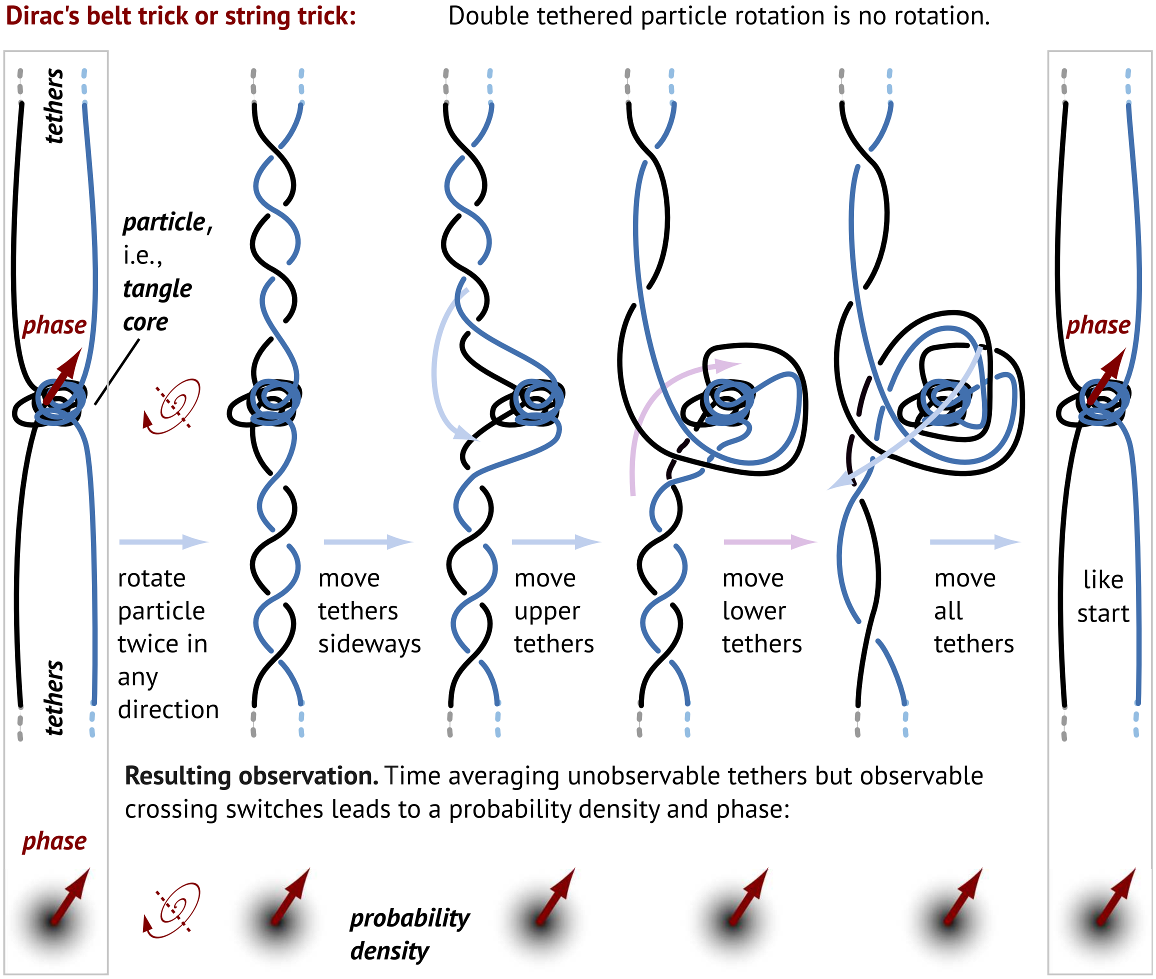

FIG. 2: Upper part: Dirac's *string trick* or *belt trick* shows that a *double* rotation by 4π of a tethered, indivisible particle – here an oriented tangle core – is equivalent to no rotation at all. Lower part: the particle *tangle* leads to a probability density and a phase once the ideas from Figure 11 and those explained in the text are used. The belt trick works as long as the tethers – *any number* of them – are allowed to fluctuate, untangle, and are assumed to be unobservable, whereas their crossing switches are assumed to be observable. A *single* rotation by only 2π does *not* allow untangling to occur. Indivisible localized cores with several tethers thus exhibit the properties that characterize spin $1/2$ particles. As a result, a tethered particle can *rotate continuously*. Careful observation also shows that Dirac's trick couples rotation and *displacement* of the tethered core. The coupling of rotation frequency and displacement determines the inertial mass, as discussed in Section 30. The twists in the tethers are virtual gravitons that determine the gravitational mass, as outlined in Section 35. Dirac's trick can be easily reproduced with real belts, strips of paper, or ropes. Links to animations illustrating the belt trick are given later on.

observable action value $\hbar$. In other terms, Dirac's trick implies that crossing changes of tethers are related to $\hbar$. This connection was most clearly stated by Kauffman in the 1980s [87, 88].

A variant of Dirac's belt trick is the *fermion trick*, illustrated in Figure 4. It describes the basic property of fermions: after a double particle exchange, two tethered fermions return to the original situation – but not after a single exchange. Again, crossing changes are observable through their effects on the sign of wave functions, even if the tethers themselves are unobservable. Again, no other visualization of fermion behaviour exists. The fermion trick also works if, in addition, the

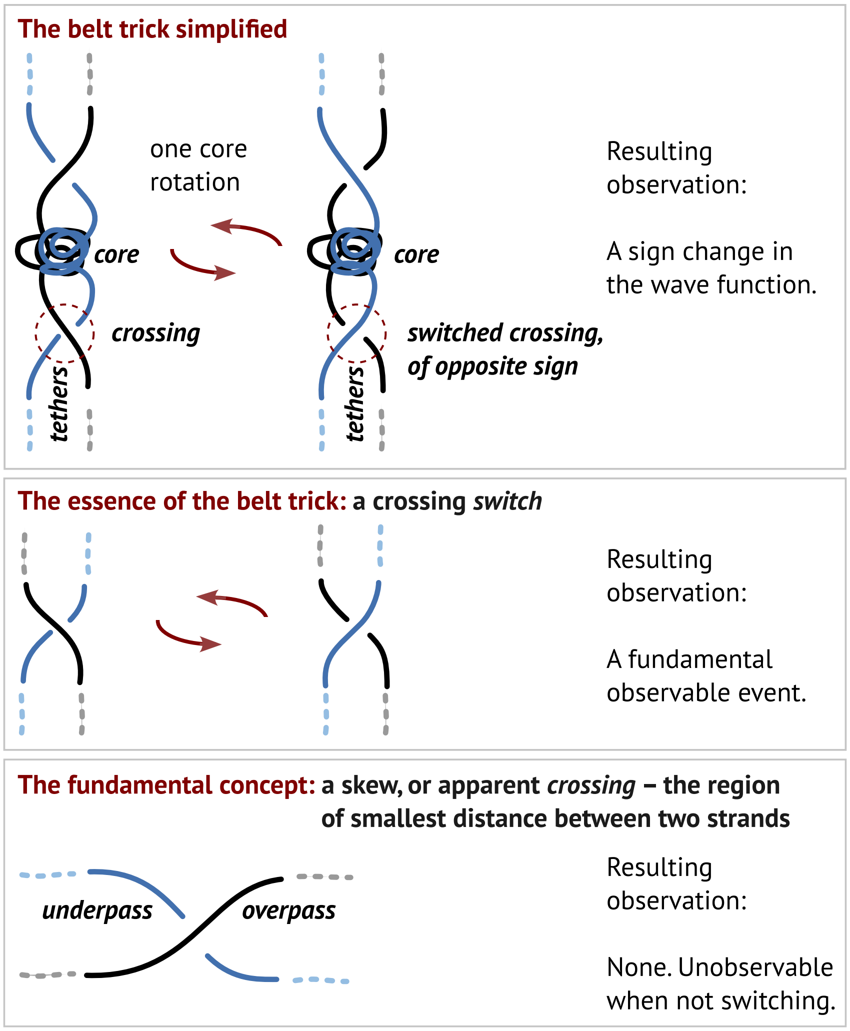

FIG. 3: Top: Dirac's belt trick is based on the idea that a *crossing switch* of tethers leads to an *observable* change in the sign of the wave function. The figure shows that a crossing switch is related to a tangle core rotation by 2π. Centre: Simplifying further, a crossing switch is the sign change of a crossing, a basic process in mathematical knot theory. It is defined in Part II. In the strand tangle model, this sign change is *observable* even though the tethers themselves are *unobservable*. The bottom diagram highlights that all crossings are apparent: each crossing consists of a strand segment passing over another one.

two fermions are connected by additional strands. (The two observable ends of a belt assumed to be unobservable also reproduce the fermion trick.) Also the fermion trick proves that in nature, everything is tethered and connected to everything else. Tethers thus explain the spin-statistics theorem: only spin $1/2$ particles are fermions, and vice versa [89]. The effects of tethers have been visualized in numerous internet videos. Examples are found in references [90–94].

Dirac's trick was the first hint that matter should be described with extended, filiform, *unobservable* constituents, for which *only crossing switches* are observable. More than fifty years later, in 1980, Battey-Pratt and Racey showed that tethers imply the full free Dirac equation [71]. Their argument is presented below. In simple terms, Battey-Pratt and Racey proved that *Dirac's trick implies Dirac's equation.* They wrote to Dirac about their discovery, but he never answered.

Taken together, the properties of strands and tethers suggest:

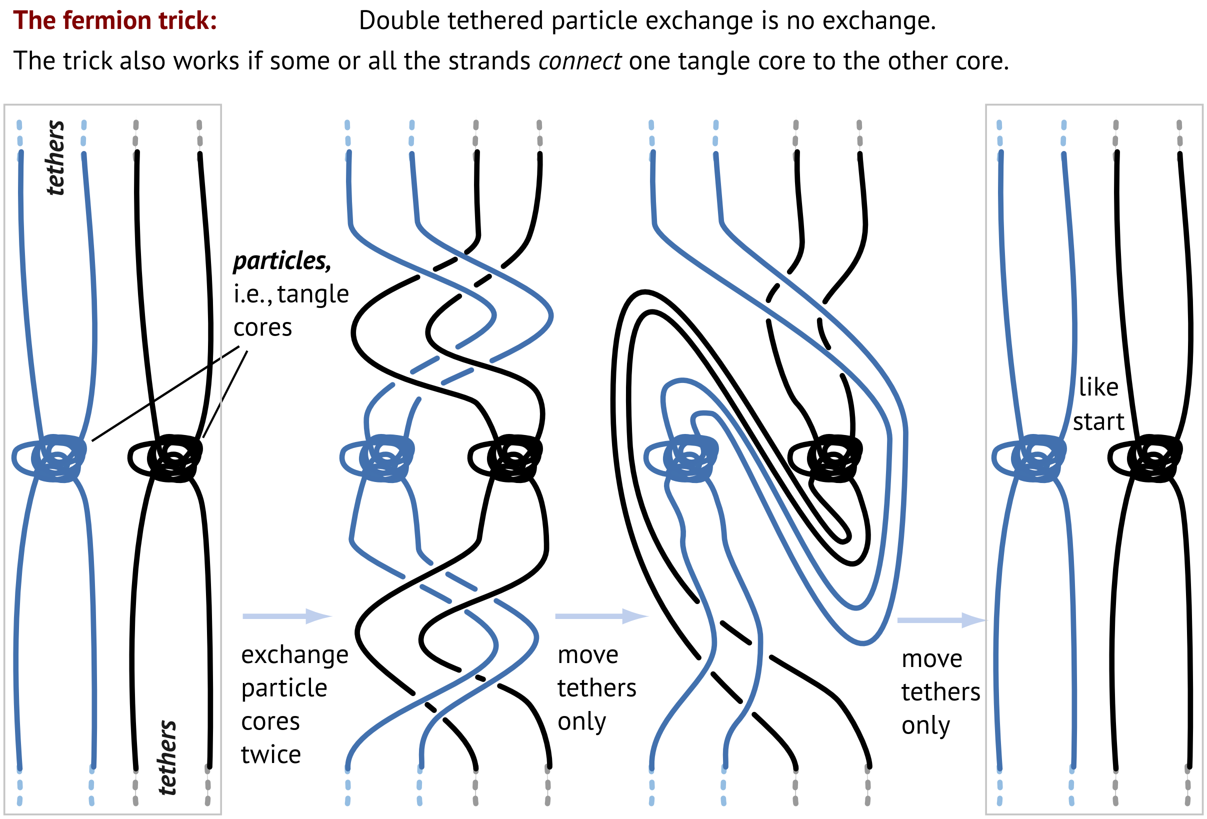

FIG. 4: The *fermion trick* shows that a *double* particle exchange of tethered particles is equivalent to no exchange at all. The fermion trick works if the tethers – *any number* of them, even if some connect the particles directly – are allowed to fluctuate and untangle. Tethers are assumed to be unobservable, whereas crossing switches are assumed to be observable. In contrast, a *single* particle exchange does *not* allow untangling. Indivisible tangle cores made of several tethers thus show all the properties that characterize fermions. Animations illustrating the fermion trick are presented later on. The fermion trick is easily reproduced with long strips of paper, ropes, or belts.

- ▷ *Every quantum effect* is due to observable crossing switches of unobservable strands of Planck radius.
- ▷ Strands are unobservable because of their Planck radius.
- ▷ The quantum of action $\hbar$ is due to a crossing switch.

These statements are valid for *every quantum system:* not for matter particles, but also for radiation particles, for black holes, and for space itself.

If everything in nature is composed of strands, what exactly are particles? The straightforward conjecture that particles might be modelled as *open knots*, i.e., as knots in infinitely long tethers, as illustrated in Figure 5, is unsuccessful. Despite intense attempts in this and similar directions [69, 95–109], open knots, prime knots, closed knots, bands, branched structures, loops, networks, or graphs do *not* fully explain the particle spectrum or the appearance of wave functions. Above all, none of these topological structures explains gauge interactions or particle reactions.

Between 2008 and 2014 it became clear that modelling elementary particles as *rational tangles* – i.e., as unknotted tangles – *does* explain wave functions, the particle spectrum, particle reactions, and particle interactions. The concept of a rational tangle – a three-dimensional braid – is illus-

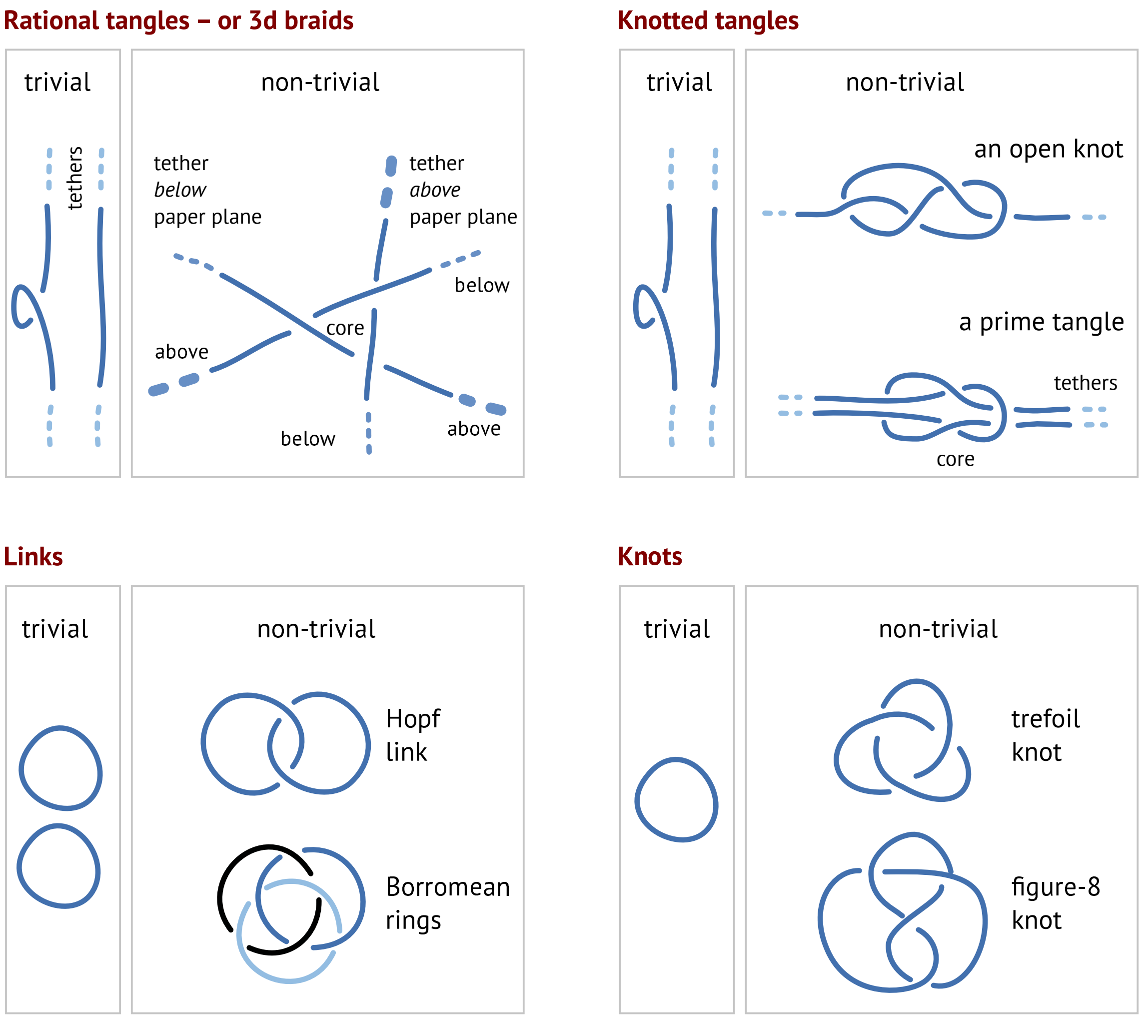

FIG. 5: The differences and overlaps among rational tangles – or 3d braids – knotted tangles, links and knots – as defined in mathematical knot theory – are illustrated. *Rational* tangles are formed by braiding their tethers (shown with dashed lines); in contrast, *knotted* tangles cannot be formed in this way. Equivalently, rational tangles, unlike knotted tangles, can be disentangled by moving their tethers in space. Rational tangles owe their name, for the case of two strands, to their close relation to rational numbers [110].

trated in Figure 5. As shown later on, the classification of rational tangles yields the elementary particle spectrum, with their observed charges and their other observed quantum numbers. The classification of strand deformations using the three Reidemeister moves yields the three known gauge symmetry [111, 112]. Also this result is presented in detail below. As mentioned, if strands are assumed to have Planck radius, gravity and general relativity also arise. Above all, with strands of Planck radius, unique values for the particle masses, coupling constants and mixing angles arise.

Together, the mentioned results allow deriving both gravity and particle physics from a single fundamental principle that is inspired by the crossing switch illustrated in Figure 3. In particular, strands yield the Lagrangian of general relativity and the Lagrangian of the standard model with massive Dirac neutrinos. The presentation of the strand tangle model in reference [83] was followed by extensive tests of the consequences for particle physics, including quantum electrodynamics and quantum chromodynamics [84–86], and by tests of the consequences for general relativity, black holes and quantum gravity [81, 82]. All the consequences agree with the observations. The present article is a *prequel* that describes basic *quantum mechanics* and *wave functions* as consequences of the strand tangle model.

In short, the results by Planck, Einstein, Dirac, Battey-Pratt and Racey, Rauscher, and Gibbons prove that three ingredients are *necessary* to describe all observations in nature: (corrected) Planck limits, unobservable tethers connecting everything, and observable crossing switches. Together, these ingredients yield general relativity and various quantum effects, such as spin $1/2$, fermions, the Dirac equation, and black hole entropy. The three ingredients are combined in the fundamental principle of the strand tangle model by using fluctuating strands with Planck radius, by defining $\hbar$ with crossing switches, and by limiting time intervals to twice the Planck time. In the present century it was shown that the strands and Planck limits in the fundamental principle are also *sufficient* to describe black hole thermodynamics, general relativity and the standard model of particle physics – and thus all observations in nature [81–86, 111, 112]. The rest of this article explains why and how crossing switches of strands are observable and how tangles of strands lead to wave functions, particles and interactions.

4 The search for emergent wave functions leads to strands

The search for an *emergent* description of quantum theory has been a research topic for almost a century. The efforts of de Broglie, Schrödinger, Bohm, Bell, Kochen, Specker and many other researchers in the twentieth century had a lasting impact that fuelled hope on the one hand and limited possibilities on the other. In this century, the books by Adler [113] and by Peña, Cetto and Valdés [114] have examined this subject for consistency. They concluded that an emergent description of quantum theory using a statistical basis is possible, but did not propose any specific model. Many other scholars have studied the topic over the past two decades [115–128]. Emergent quantum theory is also a topic of regular conferences. The collection presented in reference [129] is a recent example.

Despite all the efforts, no explicit model for emergent wave functions that agrees with observations has been proposed thus far. The first reason is due to the ancient statement that nature consists of particles and void. As a result, any model for emergent wave function must at the same time be a model for relativistic quantum gravity. The second reason for the lack of success appears to be that the options left by *the* mentioned investigations about emergent quantum theory are both difficult to identify and rather different from habits of thought. Indeed, the *strand tangle model* for wave functions is quite unusual, being based on the counterintuitive notion of *tethers*, that is, on *unobservable* connections between any quantum particle and the cosmological horizon. Tethers were introduced around 1929 by Dirac, in the way depicted in Figure 1, to explain spin $1/2$ [70]. Nevertheless, it took over fifty years before somebody took unobservable tethers seriously. In 1980, Battey-Pratt and Racey used tethers to visualize and derive Dirac's equation in their inspiring paper [71]. However, they did not explicitly explore wave functions. The strand tangle model extends the idea of tethers and suggests that elementary particles, despite being effectively point-like, are themselves made of tethers. Elementary particles are modelled as *tangles of strands* and wave functions as *crossing densities of strands.* Continuing the exploration requires a definition.

▷ A *strand* is defined as a smooth curved line surrounded by a cylinder with a tiny, Planck

> radius. More precisely, the line is a one-dimensional, open, continuous, everywhere infinitely differentiable subset of $\mathbb{R}^3$ (or of the spatial part of a curved Riemannian space) without self-intersection, unknotted, and without endpoints. In addition, the line is surrounded by a volume defined by a perpendicular disk of Planck radius $\sqrt{\hbar G/c^3}$ at each point of the line. The entire strand volume is not allowed to self-intersect, meaning that the curvature radius of the curved line is never smaller than the Planck length $\sqrt{\hbar G/c^3}$.

From a mathematical viewpoint, the definition of strands is the usual definition used in knot theory, for example, in the ropelength calculations of tight knots [130, 131]. From a physics viewpoint, strands *differ* in several ways from flexible ropes, cables, or cooked spaghetti. First of all,

▷ Strands randomly fluctuate in shape.

The origin of the shape fluctuations is the continuous pushing of strands against strands – including those that make up the vacuum, as explained in Appendix A. This pushing is a consequence of their impossibility to intersect. Due to the impossibility of measuring or preparing lengths smaller than the minimum length,

▷ Strands cannot be cut.

In other words, contrary to habits of thought, strands are *not* made of parts. Strands are *not* made of anything else. Strands are indivisible. In further contrast to everyday life,

▷ Strands have have no ends.

Strands reach up to the cosmological horizon, as visualized in Appendix A. But the main difference between strands and ropes in everyday life is due to their Planck radius. The radius of strands is so small that it is *negligible* in almost all situations – except in the domain of relativistic quantum gravity. In particular,

▷ Strands are unobservable and have *no* observable properties.

▷ Only changes of the tangling of strands, crossings switches, *are* observable.

The reasons and consequences will be clarified throughout this article. In particular, strands do not have mass, colour, energy, tension, momentum, charge, or any quantum number. Among others, strands have *no* fixed length, and *cannot* exert forces: strands cannot pull anything, and they do not offer any resistance when they are deformed.

The concept of a *tangle* is taken from topology [110, 132] and is defined intuitively in Figure 5 as a collection of strands tied together. *Rational tangles* are unknotted and similar to *braids*, though with tethers leaving to the cosmological horizon along specified directions in three-dimensional space. The term *tether* is used for those strand segments that connect the core to the cosmological horizon. The region of space in which the strands are tied up and tangled is called the tangle *core*. Thus, a tangle consists of a core and tethers. As will be shown below, tethers are responsible for the quantum of action $\hbar$, for spin $1/2$, for fermion behaviour, and all other quantum effects, including wave functions and their evolution equations. The classification of tangles and their deformations leads to the observed elementary particles and the observed gauge interactions. The tangling and untangling of rational tangles leads to particle reactions.

In physics, filiform particle constituents avoid all arguments against constituents *inside* elementary particles. Experimentally, all the known elementary particles lack particle-like constituents [62]. Also theory argues against particle-like constituents. Such constituents would have to be confined inside an extremely small volume, resulting in extremely large kinetic energy and thus in particle mass values that are much higher than those observed. In contrast, *extended filiform constituents* suggest that the region where they are tangled is the region where the wave function of the quantum particle is localized. As shown below, rational tangles of extended filiform constituents can be counted, exhibit fermion behaviour, and have spin $1/2$, parity, chirality as well as all other properties of quantum particles. However, tangles are also challenging: mechanisms must be found that allow *mass* and *gauge interactions* to emerge from tangling.

In short, already a long time ago, the counter-intuitive approach that quantum particles are *tethered* to the cosmological horizon with unobservable strands was used to explain spin $1/2$ and the Dirac equation. Strands are tiny, fluctuating and uncuttable connections spanning across nature. The idea of particles consisting *completely* of extended filiform constituents suggests that an emergent description of quantum theory may be possible. Proving the suggestion requires several tasks: specifying a strand tangle model for wave functions, deriving conventional quantum theory, explaining the origin of quantum particle mass values, explaining gauge interactions, and comparing the differences between tangles and conventional quantum theory with experiments.

5 Crossings of strands resemble wave functions

Spin $1/2$ proves that in nature, everything is connected to everything else. Dirac's trick shows that *crossings* of these connections are essential in the study of quantum particles. In this article, the term crossing is used in the sense of mathematical knot theory [133–135].

▷ A *strand crossing* is the region of the smallest distance between two skew strands.

In three dimensions, the strand segments are *always* at a distance, as illustrated in Figure 6. Equivalently, as is usual in knot theory, a crossing always implies a *skew* geometry of two strand segments. In other words, a *literal* 'crossing' appears only when the configuration is *projected* in two dimensions.

The *existence* of a crossing, which is simply due to the smallest distance between two strands, is *independent* of the observer. Crossings can occur between any two strand segments. In particular, crossings can occur in two segments of the same strand or of different strands. Crossings also arise between extremely distant strands. But such long-distance crossings have no physical importance because crossing *switches* – which define all physical observables – will not arise for them.

Crossings are consistent with the minimum length in nature. The definition of crossing assumes that in the region of the smallest distance, both strands have a radius of curvature that is larger than the Planck length. Due to the Planck-sized radius of strands, the shortest distance at a crossing is always larger than the smallest length.

Crossings have mathematical properties that resemble wave functions. The simplest wave function appears in the Schrödinger equation. It is defined by an amplitude R and a phase φ:

$$\psi(\mathbf{x},t) = R(\mathbf{x},t)\,\mathrm{e}^{i\varphi(\mathbf{x},t)} \quad . \tag{6}$$

The more involved Pauli spinors have three phases, whereas Dirac spinors have seven. Figure 6 shows that also strand crossings are described by amplitude and phase(s). The *amplitude* is large when the shortest distance between the two strands is small. Intuitively, the probability density is high whenever the strand distance is small. The (main) *phase* describes the orientation of the

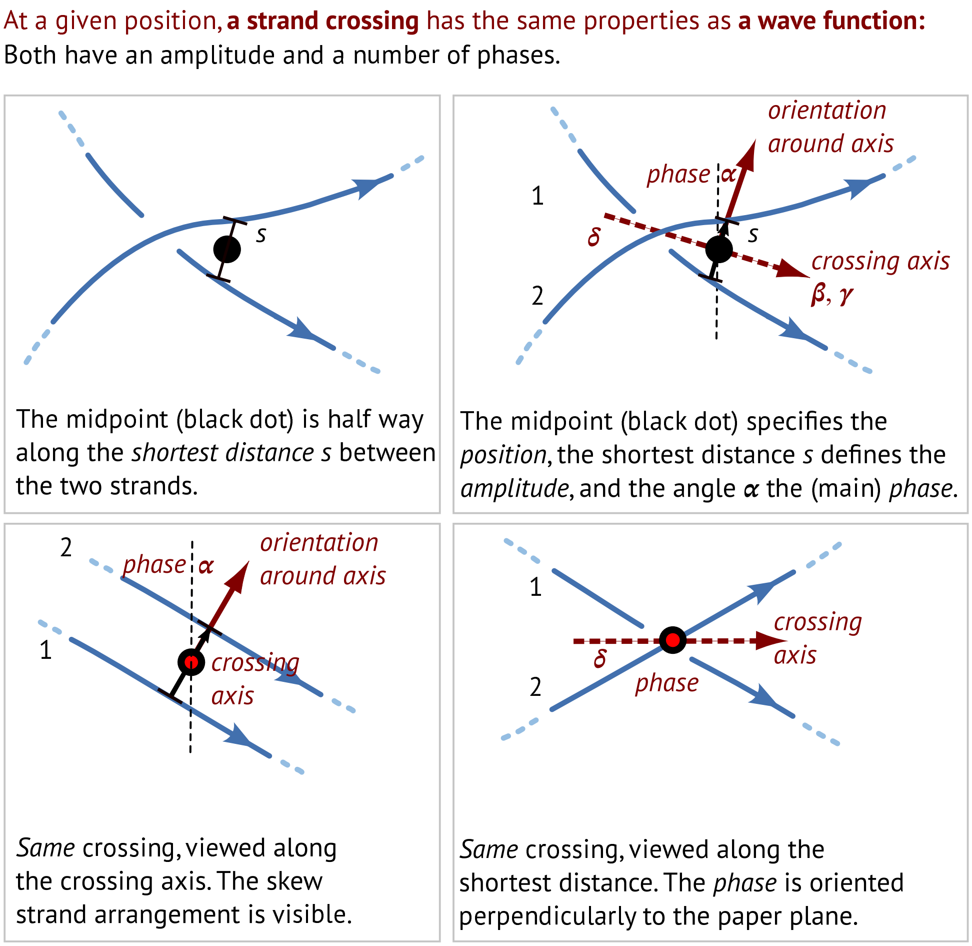

FIG. 6: A strand crossing consists of two skew strand segments separated by the shortest distance s. The shortest distance s is never shorter than the minimum length, due to strand impenetrability. A strand crossing allows defining amplitude and phase(s) around an axis. Therefore, a strand crossing at a point has the *same properties* as a wave function at a point. The *crossing axis* is defined, using the drawing in the centre, as the sum of the two tangent vectors at the two endpoints of s. (This requires defining a positive direction on each strand.) The (main) *phase* α describes the direction in which the shortest distance vector points around the axis, once a sequence is defined among strands. The dotted direction defining *vanishing* phase around the axis is a matter of choice, as usual for phase. Regardless of the choice of vanishing phase, phase differences are always defined uniquely. The angles or phases β, γ and δ play a role in spinors.

crossing around its axis. Both quantities are defined precisely in Part II, which will use crossings to define wave functions. The other angles illustrated in Figure 6 play a role in spinor wave functions, as shown below.

Apart from the ideas by Kauffman [87, 88], there seems to be no research literature on strand crossings in quantum theory. In particular, there seems to be no literature on the similarity between crossings and wave functions.

In short, the geometric properties of strand crossings – amplitude and phase(s) – closely resemble the mathematical properties of wave functions.

The fundamental, Planck-scale principle of the strand tangle model

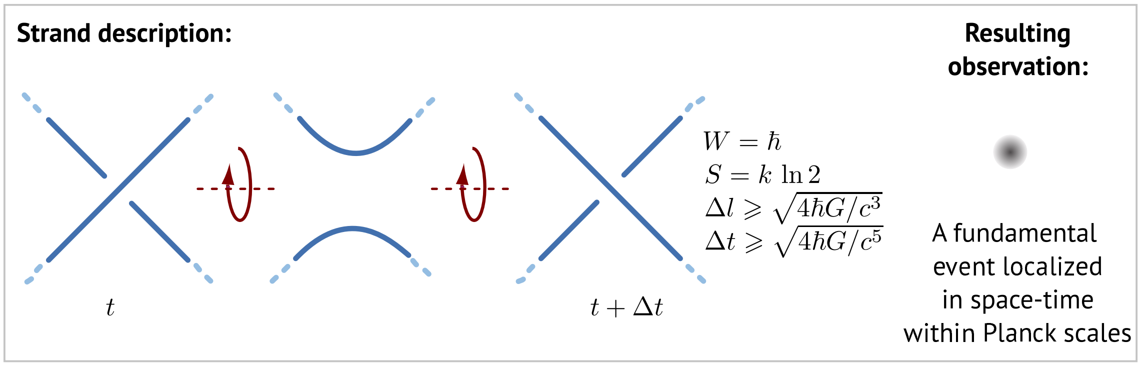

FIG. 7: The fundamental principle of the strand tangle model, deduced from Dirac's trick, describes the simplest observation possible in nature: *a fundamental event.* In the strand tangle model, a fundamental event arises at the Planck scale and is *almost* point-like. A fundamental event results from a *strand crossing switch* – the change of which strand passes above the other – at a location in three-dimensional space. The *location* of the crossing is defined by the centre of the shortest distance between the two strand segments. The *crossing switch* is a deformation of strands in space, illustrated by the circling arrows. The deformation results, for example, from rotating together the two tethers on the right-hand side against the two tethers on the left-hand side, around the west-east axis in the paper plane. The strands themselves, best imagined as ropes or cables with a Planck-size radius, are not observable, uncuttable and impenetrable. The crossing switch is the simplest observable process in nature; it defines $\hbar$ as the unit of the physical action W. The Planck length and Planck time arise from the most localized and the most rapid crossing switch possible, respectively.

6 Events are quanta of change due to crossing switches of strands

Traditionally, in quantum theory, the concept of event plays a negligible role. This changes in the strand tangle model. The fundamental principle, illustrated in Figure 7, states that a crossing switch defines every physical event.

▷ A crossing *switch*, a *fundamental physical event*, changes which strand passes over the other.

In mathematics, a crossing switch is the *change of sign* of a crossing. In the strand tangle model of nature, events are thus *discrete* and *countable*. Each event is a local *process*, defined with an error of the order of the Planck scale.

The fundamental principle states that every observed process in nature – motion, interaction, decay, particle transformation, observation, or measurement – is *composed* of crossing switches. In particular, every *observation* and every type of *change* is due to and is composed of crossing switches. Crossing switches are the irreducible 'building blocks' of change.

▷ Each crossing switch yields a *quantum of action* $\hbar$.

▷ Each crossing switch is a *quantum of change.*

These statements are the essence of Dirac's trick. Across physics, change is measured with action. Thus, the statements about crossing switches agree with the quantization of action. Crossing switches *visualize* the quantization of action.

Wave functions are unobservable. Also crossings are unobservable. But both can be used to define physical observables. In quantum theory, the simplest observable is the probability density. In the strand model, the simplest observable is a crossing switch. As will become clear shortly, crossing switches define the probability density – and all other observables. The final reason *why* crossing switches are observable – and that nothing else is – is given below, in Section 34, where it is shown that crossing switches couple to the electromagnetic field.

In the strand tangle model, every change and every process is composed of events. Strands provide an *event-based* approach to nature. Such an approach to quantum theory is also explored, among others, by Powers and Stojkovic [136], and by Giovannetti, Lloyd and Maccone [137]. Also many approaches to relativistic quantum gravity yield a similar description of nature [69, 81].

In short, in the strand tangle model, a crossing switch is a process and a fundamental event. Each crossing switch yields a quantum of action $\hbar$ and thus is an observable quantum of change. The remainder of this article will prove that crossing switches of strands yield and visualize every quantum process.

7 The fundamental principle of the strand tangle model describes all of nature

The strand tangle model states that matter, radiation, space, and horizons – thus all systems observed in nature – consist of strands that fluctuate [83, 84]. More precisely, the complete strand tangle model is based on one statement:

▷ **Crossing switches of strands with Planck radius determine the quantum of action $\hbar$ and the minimum time, as illustrated in Figure 7.**

This statement is the *fundamental principle* of the strand tangle model. Several statements follow.

▷ Although strands are themselves unobservable, *crossing switches are observable*, because of their relation to $\hbar$, c, k and $4G$.

▷ Crossing switches define all physical units and observables.

▷ Physical space is a *aggregate* of untangled strands. Horizons – including black hole horizons, Rindler horizons and the cosmological horizon – are *weaves* of strands, as summarized in Appendix A.

▷ Particles are *rational tangles* of strands, i.e., unknotted tangles of strands. Wave functions are due to tangle *crossings*. This is explored below.

▷ Probabilities and fields are due to crossing *switches* that occur in specific tangles. The details of these statements are explored in the following.

▷ Physical motion *minimizes* the number of observable crossing switches of unobservable fluctuating strands. This is the principle of least action. It is also explored below.

Every change observed in nature, from nuclear reactions to black hole mergers and life itself, is due to crossing switches of strands. As will become clear step by step, the fundamental principle implies both the standard model of particle physics and general relativity, including their Lagrangians [81–86].

The definition of the quantum of action in the fundamental principle covers and explains all quantum processes, from entanglement to the quark model. Doing so, the strand definition of the quantum of action eliminates the concept of *quantization*, in all the nuances that are found in the literature, from all domains of theoretical physics, including gravitation. Strands simplify quantum physics.

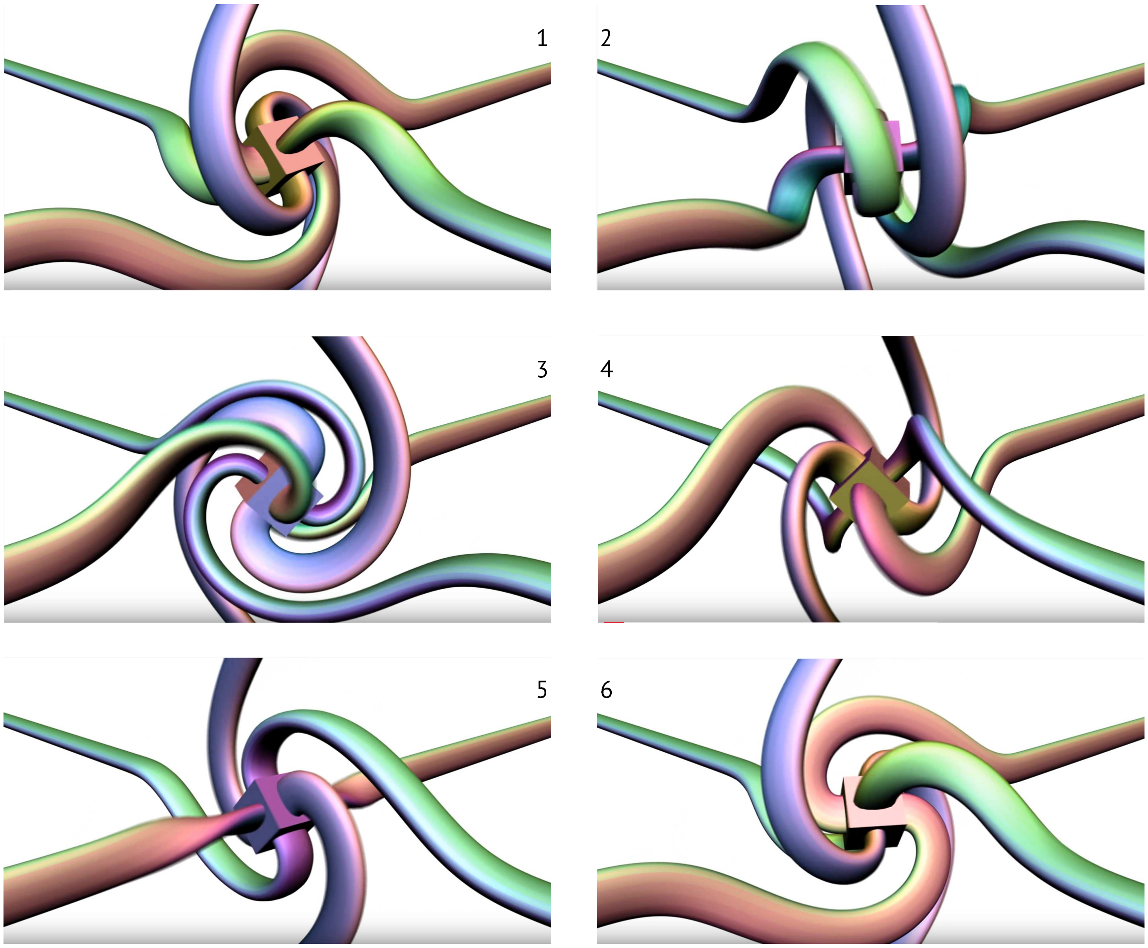

FIG. 8: The particle rotation for a (free and moving) lepton can be visualized with the animation produced by Jason Hise, available at [90]. The rotating (spinning) central cube symbolizes the tangle core, i.e., the (moving) tangled region where the particle is localized with the highest probability. The continuous rotation of a tethered particle is possible. (The figure is modified from reference [84].) The cube rotates twice before returning to the starting configuration. The factor of 2 appears in many expressions involving the rotation angle and the quantum phase of fermions.

As is usual in quantum mechanics, physical space is assumed to be flat throughout. The strands making up empty space are mostly not discussed in the following. In quantum physics, vacuum strands only play a role in vacuum fluctuations or in particle creation and annihilation.

In short, the fundamental principle of the strand tangle model states: crossing switches of fluctuating strands of Planck radius define the quantum of action and all other Planck units. Crossing switches are at the basis of physics, from particle physics to general relativity, and imply all laws of physics. In this article, the strand tangle model is tested against observations in the domain of *quantum mechanics.* The first test concerns the properties of matter particles: fermions.

8 Rotating and orbiting quantum particles emerge from tethers

As Dirac demonstrated in his lectures, tethers explain spin $1/2$ behaviour and fermion behaviour. These consequences of Dirac's trick are illustrated in Figure 2 and Figure 4. Both results apply

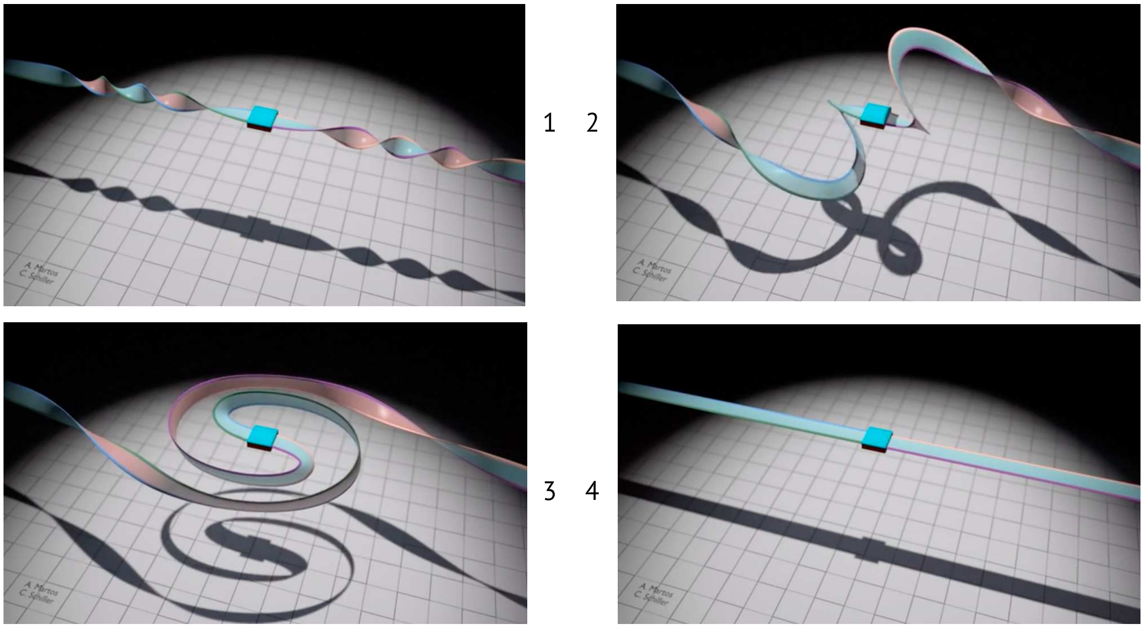

FIG. 9: The return to the original configuration after a double rotation can also be visualized with the animation produced by Antonio Martos, available at [91]. The rotating central belt symbolizes the tangle core. The continuous rotation of a tethered fermion is possible.

independently of the number of tethers, as long as the number is three or larger. But the two figures show more.

Dirac's belt trick implies that a tethered particle can *rotate continuously*. This is best observed in animations. A video showing the continuous spinning of a particle with six tethers was produced by Hise [90]; still images from the video are shown in Figure 8. A video of a spinning particle with four tethers (or two attached belts) was produced by Martos [91]; still images from that video are shown in Figure 9. Videos of rotating particles with dozens of tethers are also found on the internet [93, 94]. In other words, when unobservable tethers are allowed to fluctuate, particle rotation can continue forever, without any obstacles, despite the tethering. The relation between spin and rotation is not new and was regularly pointed out in the past [138]. In other words, spin can be visualized as the rotation of tangle cores.

A related statement can be made for systems consisting of *two* particles. The fermion trick implies that two tethered particles can *orbit each other continuously*. Martos published a further video showing the fermion behaviour of two tethered particles [92]; still images from the video are shown in Figure 10. For example, Figure 10 can be seen as visualizing an electron orbiting a proton in a hydrogen atom. In particular, the video shows that two tethered particles can *orbit each other forever* – if tethers are allowed to fluctuate. Again, the number of tethers is not limited. In other words, orbiting particles can be visualized as orbiting tethered tangle cores.

The mentioned properties imply that the tethers of two typical quantum particles do not disturb each other, as long as strands are unobservable, are infinitely flexible, and have no fixed length, no tension and no mass. The same is true for more than two particles. As a result, the many tethers filling empty space *usually* lead to *no* interaction between particles. Exceptions related to gauge interactions and virtual particle effects are discussed below.

In the strand tangle model, every quantum particle is a *tangle tethered to the cosmological*

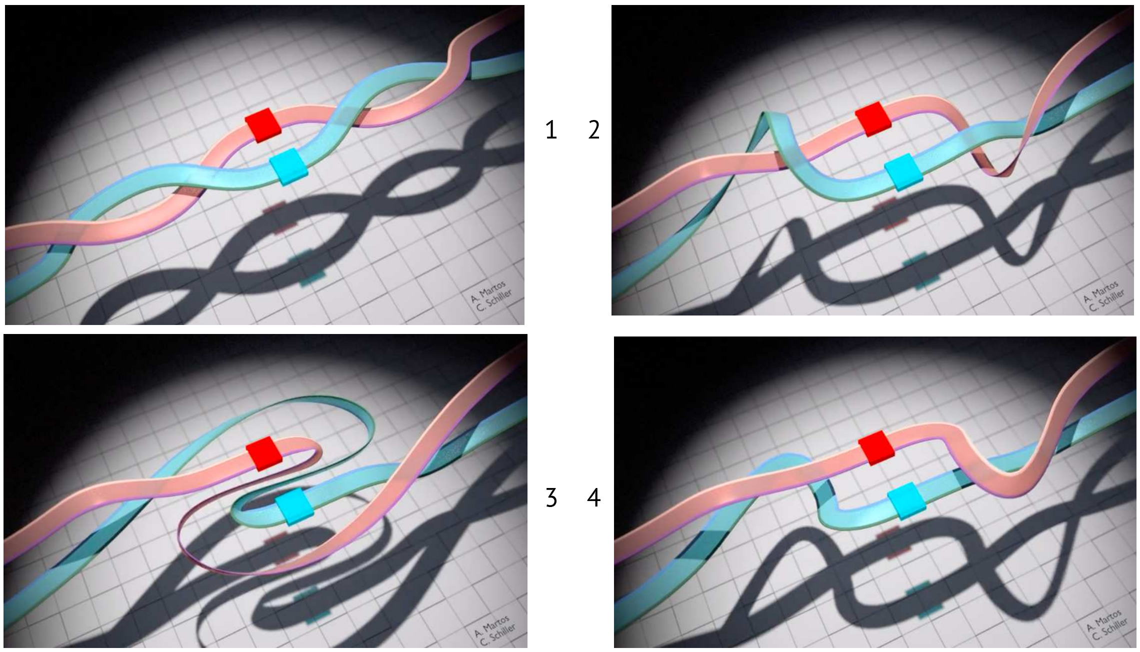

FIG. 10: Particle exchange can be visualized with the animation produced by Antonio Martos, available at [92]. The rotating central belts symbolize the two tangle cores, i.e., the regions where the two particles are localized with the highest probability. (In the strand tangle model, tethers and thus belts are unobservable.) The first image shows two particles whose positions were exchanged twice. The other images show that the shape changes of the belts (each consisting of two tethers) bring back the original, unexchanged situation. Therefore, the continuous exchange of the two tethered particles is possible. The animation thus illustrates, among others, the motion of an electron (red) continuously orbiting a proton (blue) in a hydrogen atom. In particular, the sub-figure 1 on the top left can be taken as the defining configuration for a composed system.

horizon in which the strands fluctuate continuously.

- ▷ Tangles with their tethers reproduce spin $1/2$ as tangle core rotation.
- ▷ Rotation and orientation of tethered cores reproduce particle spin, including the orientation of the spin axis in space, and thus the flag model of spin.
- ▷ Tethers thus show that spin is angular momentum.
- ▷ Tethered cores orbiting each other reproduce orbiting particles.
- ▷ Tethered tangle core exchange reproduces fermion behaviour.
- ▷ Tethers thus reproduce the spin-statistics theorem.
- ▷ No explanation of spin $1/2$ and fermion properties is possible without tethers.

These results are valid generally. In particular, the results are valid at all measurable energies.

In short, tethered tangles are essential for describing and understanding spin $1/2$, particle rotation, orbiting particles, particle exchange and fermions. This understanding is *impossible* without tethers. The result suggests that *every other* motion in the quantum domain – including translation, interference, scattering and interactions – can also be described with tethered tangle cores. This is indeed the case, as argued in the remainder of this article. However, before doing so, it is worth confirming the effects of tethers in the case of *composed* particles.

9 Tethers determine the spin of particles composed of fermions

In nature, when two spin $1/2$ fermions form a composite, the composite is observed to have either spin 0 or spin 1 and to be a boson. This behaviour can be reproduced with tethers.

In the strand tangle model, a system is *composed* when it forms a unique tangle core connected to the cosmological horizon by tethers. Examples showing composite systems most clearly are the second graph from the left in Figure 4 or sub-figure 1 in Figure 10. Composed systems usually have a different spin value than the particles they are made of. The two figures show that a system composed of two tangle cores – two belt buckles – returns to itself exactly after a rotation by 2π – the double exchange. The composite system does *not* return to itself after rotation by π, that is, after a simple exchange. These are the defining properties of *integer* spin. Thus, a composite system of two spin $1/2$ tangles behaves like a particle with integer spin.

The analogy can be made more precise by recalling that in the strand tangle model, each of the two particles in Figure 10 continuously spins along its axis. The simplest situation is that each particle continuously spins around an axis *along* the straight belts. If the rotation axes and rotation sense of the two fermions and that of the double exchange *agree,* the situation is described by $S = 1$ and z-component $S_z = +1$. If the rotation axes of the two fermions are the same but that of the double exchange is opposite, one has $S = 1$ and $S_z = -1$. If the rotation axes of the two fermions are the same, but that of the double exchange is perpendicular to them (e.g., in the direction of the line connecting the two cores), one has the situation $S = 1$ and $S_z = 0$.

The defining property of spin 0 is that a rotation by any angle keeps the system unchanged. If the rotation directions of the two fermions are *opposite* to each other, then the total system has $S = 0$ and $S_z = 0$, independently of the rotation axis of the double exchange. A composite for which two cores rotate in opposite directions has a total spin 0. Any tangle core that does not rotate when strands are randomly deformed has spin 0.

The exploration shows that a composite of two tethered spin $1/2$ particles cannot have any other spin value: no other behaviour under system rotation apart from the four mentioned cases is possible. In other words, composing two particles of spin $1/2$ can only yield a composite with spin 0 or spin 1. For tangle cores with integer spin, core rotation by 2π is equivalent to no rotation, in contrast to the case of a half-integer spin core. One notes that in these arguments, the two particles do not have to be identical or even elementary. Even if their tangle cores differ or if they are themselves composed, the results about spin composition still hold.

The above arguments can be extended to composites of *more* than two spin $1/2$ particles. The result is known: particles composed of odd numbers of spin $1/2$ particles are fermions; likewise, every composed fermion contains an odd number of spin $1/2$ particles. *All composite fermions* have a spin value that is an odd multiple of $1/2$. Tethers reproduce the behaviour of fermions under composition.

In nature, particles with integer spin behave differently from fermions: they are *bosons.* This is reproduced by the strand tangle model. When the positions of two identical tangle cores with integer spin are exchanged, no partial orbit of one core around the other core is required – in contrast to the case of half-inter spin cores. No double exchange is needed to restore the sign of the wave function. Indeed, in the strand tangle model, bosons, whether elementary or composed, can exchange positions without hindrance. For composites, exchange is achieved by the cores passing through each other. In other words, the tethers of such tangles explain why all spin 1 particles are bosons. In total, tethers yield the result that *all composite bosons* have a spin value that is an integer, and all particles with integer spin are bosons.

In short, tethers explain why all particles, whether elementary or composed, are either *fermions*, with a spin given by an odd multiple of $1/2$, or *bosons*, with an integer spin. Tethers imply the full spin-statistics theorem. Tethers thus explain the existence, position, spin, statistics, composition, rotation, and orbits of quantum particles. These results cannot be deduced without tethers – except if tethers are hidden in the Dirac equation. This is shown below. Given that tethers explain so many aspects of quantum behaviour, the next task is to use tethers to define wave functions.

Part II: Wave functions, superpositions, Hilbert spaces, and measurements

Tethers define the quantum behaviour of particles. The strand tangle model goes one step further than the tether idea by Dirac and the tethered particle model by Battey-Pratt and Racey. In the strand tangle model, quantum particles *themselves* are made of tethers, i.e., of strands.

▷ Quantum particles are tangles.

The starting point of the strand tangle model for quantum theory is the basic similarity of crossings and wave functions:

▷ Wave functions are short-time-averaged crossing densities of spinning tangles.

The short-time average is taken over a few Planck times. Two main ways lead from tangles to wave functions: Schrödinger's way and Feynman's way. They are illustrated in Figure 11 and Figure 13. The two ways differ in how they perform the time average, i.e., the blurring that leads from a given tangle to its wave function.

The first, Schrödinger's way, starts with a *loose* tangle and averages crossings over all possible strand shape fluctuations realized during the averaging time. *Each segment of strand* fluctuates continuously and changes shape continuously. Therefore, the crossings of a particle tangle continuously move around in space; some also appear or disappear. Averaging the amplitude and phase for the crossings at a point in space yields a local value for the amplitude and phase of the wave function. This averaging method thus yields the common *Schrödinger picture* of the wave function and quantum mechanics. For example, the spinning tangle for a free particle that is illustrated in Figure 8 leads, after averaging the shape fluctuations (not shown in that figure), to a rotating cloud with a rotating phase. Also Figure 2 gave an impression of the resulting cloud.

In Schrödinger's way, the inset at the top right of Figure 11 recalls that, at a given point in space, a crossing can be described by *one positive real number* that specifies the smallest strand distance, and by *(up to) four angles or phases.* Also a wave function at a point is described by one positive real number and several angles or phases. Three types of wave functions are important in this context: wave functions for the spin-less case, Pauli spinors, and relativistic Dirac spinors. They differ in the number of phases that they take into account.

The simplest type is the *spin-less wave function*, as used in the Schrödinger equation or the Klein-Gordon equation. It is described by a single complex field ψ that can be written as

$$\psi = R\,\mathrm{e}^{i\varphi} = \sqrt{\rho}\,\mathrm{e}^{i\alpha/2} \quad . \tag{7}$$

Here, R is the (positive) modulus or amplitude, $\rho = R^2$ is the probability density, φ is the phase,

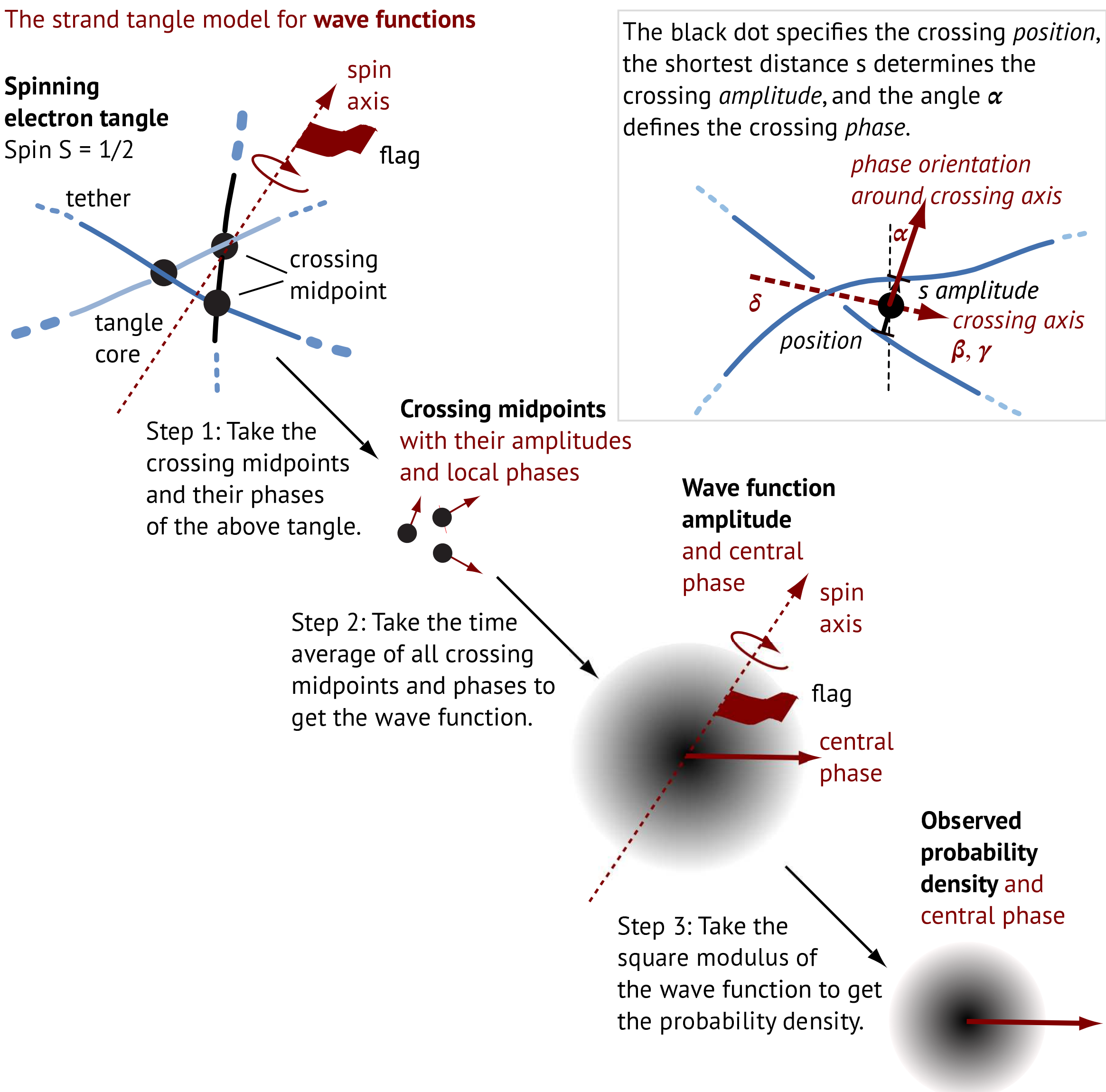

FIG. 11: In the strand tangle model, the *wave function* in the Schrödinger picture is the time-averaged *crossing density*, and the *probability density* is the time-averaged *crossing switch density*. The figure illustrates how a tangle – an electron in this case – defines crossings and local phases, how these fluctuating crossings lead to a wave function and a central phase, and how the observable *crossing switches* lead to a probability density. There is a simplification in the figure: in nature, the phase of the wave function *depends* on the position, as illustrated in Figure 12.

and $\alpha = 2\varphi$ is the tangle core rotation angle due to the belt trick. The mentioned similarities between crossings and wave functions suggest the following definition:

▷ The *amplitude* due to a crossing varies inversely with the shortest strand distance s. The *probability density* increases when the strand density increases. More precisely, the *amplitude* or *modulus* $R(\mathbf{x}, t)$ of the wave function at a point $\mathbf{x}$ is defined as

$$R(\mathbf{x}, t) = \left\langle \frac{1}{s^{3/2}} \right\rangle \frac{1}{\sqrt{n}} \ . \tag{8}$$

The average – denoted as $\langle\ \rangle$ – is taken over all possible strand shape fluctuations during a few Planck times. The crossing distance s is defined in Figure 11 as the shortest distance between two strand segments. The average of the crossing distance is normalized with the help of the number n, the so-called *(minimal) crossing number* of the tangle. The crossing number n is the smallest number of crossings that arises if a tangle is laid down on paper. The crossing number n is a topological invariant; for example, $n = 3$ for the electron tangle presented in Figure 24. In particular, the crossing number n is observer-invariant; it is a constant factor introduced in the modulus to normalize the wave function.

As a result of the definition, strand fluctuations imply that the amplitude $R(\mathbf{x}, t)$ is a continuous and differentiable real function of space and time, as expected. In the same way, the inset at the top right of Figure 11 leads to the definition of the quantum phase.

▷ The *(quantum) phase* is due to the average orientation of the shortest distance s at a point in space. More precisely, the quantum phase $\varphi(\mathbf{x}, t)$ of a spin-less wave function at point $\mathbf{x}$ is *half* the time-averaged local strand *crossing phase* α of the particle tangle – the rotation around the crossing axis – at that point:

$$\varphi(\mathbf{x}, t) = \langle \alpha(\mathbf{x}, t)/2 \rangle \quad . \tag{9}$$

The factor of $1/2$ is due to the belt trick. Again, the average is denoted by $\langle\ \rangle$ and is taken over a few Planck times and thus over strand shape fluctuations.

▷ The crossing *axis* is defined with the two unit tangent vectors of the strands at the endpoints of the shortest distance s, as illustrated in the inset of Figure 11. In particular, the *axis* of a crossing is given by the *sum* of the two tangent vectors. The axis is always perpendicular to the shortest distance vector.

▷ The *sign* of a crossing is the direction in which the right hand turns when the thumb and index follow the two strands in the direction of their axes: clockwise hand rotation corresponds to a positive sign.

Thus, the *phase* of the crossing specifies the orientation of the shortest distance around the crossing axis. In the spin-less case used in the Schrödinger equation, the phase angle α is measured against a predefined direction, as indicated by the vertical dotted line in Figure 11. The other angles describing the crossing are ignored in the case of the Schrödinger equation. As usual, there is freedom in the definition of the direction that corresponds to the vanishing phase of a particle tangle. In Figure 11, the freedom is the ability to choose the direction of the black dotted line. In the case of tangles yielding gauge fields instead of wave functions, the freedom to choose the orientation of the zero phase is related to the freedom of gauge choice.

The averaging procedure leading from crossings to probability densities is visualized in Figure 11, using three steps. The first step consists of reducing the tangle to its crossing midpoints, crossing amplitudes, and respective crossing phases. The second step consists of averaging crossing amplitudes and phases over the strand shape fluctuations occurring during a few Planck times. Strands and their crossings imply that the quantum phase varies from point to point, a feature not shown in Figure 11, but shown in Figure 12. The third step consists of averaging crossing *switches*. It leads to the probability density that describes a quantum state.

A productive way to visualize spin-less wave functions in the Schrödinger picture is to use *colour* for the phase and to use colour *saturation* for the amplitude, as shown in Figure 12. This is

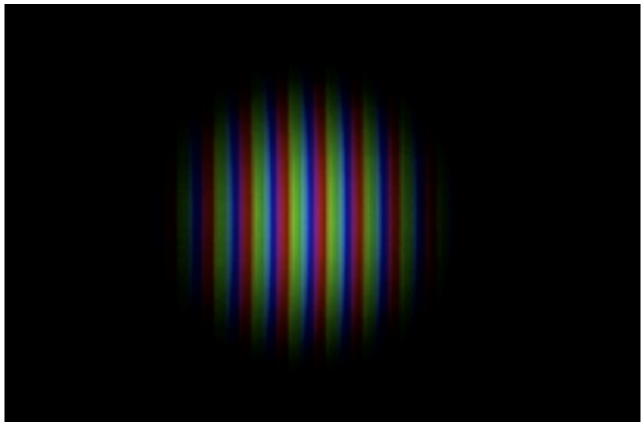

FIG. 12: In a moving wave packet, the phase depends on the position, as seen in this illustration by Bernd Thaller, from reference [139], which illustrates phase with colour and amplitude with saturation.

done in the fascinating quantum theory books by Thaller [140, 141] and in the beautiful animations on his site [139].

The second type of wave function, the non-relativistic *Pauli spinor*, also takes the angles β and γ of Figure 11 into account. They describe the orientation of the crossing in space and are thus related to the spin orientation. Such a *non-relativistic spinor*, as used in the Pauli equation, has two complex components and can be written [142, 143] as

$$\Psi = \sqrt{\rho}\, \mathrm{e}^{i\alpha/2} \begin{pmatrix} \cos(\beta/2)\, \mathrm{e}^{i\gamma/2} \\ i \sin(\beta/2)\, \mathrm{e}^{-i\gamma/2} \end{pmatrix} \quad , \tag{10}$$

Again, ρ denotes the probability density. In all situations without interactions, the tangle core can be approximated and imagined to be a rigid spinning body. The three angles α, β and γ are the Euler angles that describe the orientation of the rigid rigid body in three dimensions. The factors $1/2$ are due to the belt trick. In other words, a Pauli spinor, usually considered a matrix of two complex numbers, can be described by one positive real number and three phases. This corresponds to the description of a Pauli spinor with a flagpole and a flag. The additional sign that is required in the flag model is due to the tangling of the tethers.

The third and last type of wave function, the *relativistic Dirac spinor* with four complex components, can be described by one positive real number, four phases and three real parameters. Dirac spinors also take the angle δ of Figure 11 into account. The angle describes the relative importance of particles and antiparticles. Relativistic spinors are explored later on.

All definitions of wave functions with strands use a general relation.

▷ All *continuous quantities* arise through *time averages* of strand shape fluctuations. The averaging time is a few Planck times.

▷ Quantum theory emerges through strand shape fluctuation averaging.

Thus, the averaging time is much shorter than any other time interval that plays a role in observations or measurements. Because strands and strand shapes are not observable, there is no 'speed' limit for their shape fluctuations. Even the metric of empty space emerges through strand shape fluctuation averaging.

In short, the strand tangle model strongly suggests that *wave functions are blurred (oriented) crossing densities* of spinning tangles. Equivalently,

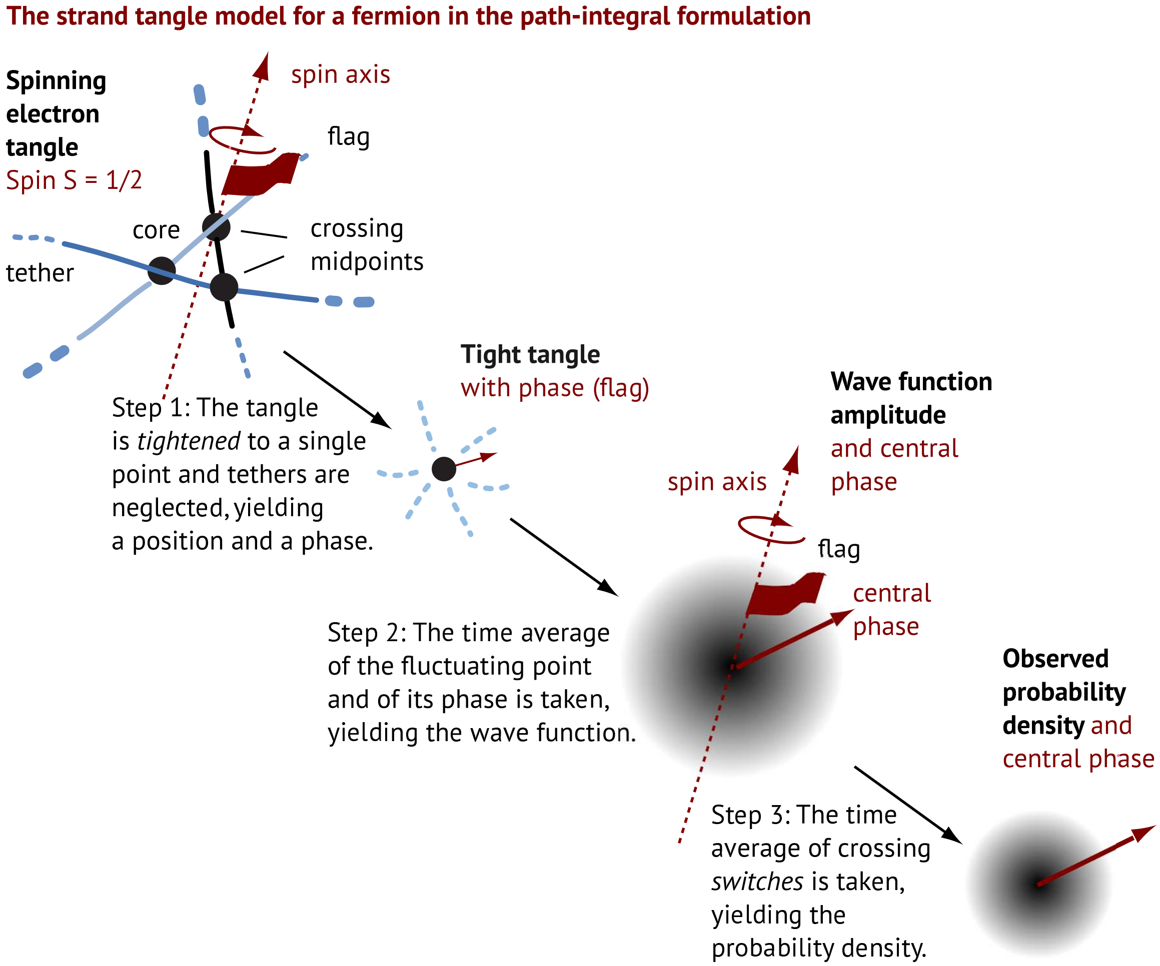

FIG. 13: The wave function and probability density in Feynman's *path integral formulation* are due to time-averaged fluctuating (almost) point-like particles. The figure illustrates how a tangle that is "pulled tight" defines the position and the local phase. The fluctuations of the point-like tangle core then lead to a wave function with phase. In particular, *an advancing particle is an advancing rotating arrow.* An alternative image is that of a rotating flag. The arrow or the flag represents the phase. Again, the modulus of the wave function leads to a probability density. Again, the figure is simplified: in reality, the phase of the wave function of a localized particle *depends* on the position, as shown in Figure 12.

▷ Tangles are the fluctuating skeletons of wave functions.

The tangle model can be described as follows: a spin-less particle is described by its time-averaged density of crossings generated by its fluctuating particle tangle. The *local amplitude* describes the time-averaged *local density* of strand crossings and is a positive real number. The *local phase* describes the time-averaged *local orientation* of the crossings. Consequently, a spin-less particle appears to be described by a *complex-valued field* that describes its crossing density and crossing orientation. The *probability density* is given by the *crossing switch density*. In other words, *probability densities are blurred switch densities*. The next sections will *prove* that crossing densities have all the known properties of wave functions.

10 Path integrals and rotating arrows emerge from tangles

Feynman's path integral approach is presented in his beautiful book QED [144]. He described the motion of a quantum particle as an advancing and rotating arrow. Wave functions arise when the effects of all possible paths are superposed. In particular, the phase and amplitude for all paths arriving at a point must be added.

In the strand tangle model, Feynman's path integral approach leads to the other way, to derive and visualize wave functions. It is based on *tight* tangles and is illustrated in Figure 13. Again, the derivation consists of three steps. The first step is to neglect the radius of the unobservable strands and to imagine that the tangle core is *tightened* to a point-like region by 'pulling' the tethers. This yields a point particle with a phase arrow attached to it. For example, the spinning tangle for a free particle illustrated in Figure 8 can be imagined to be reduced in size to a point with an arrow attached to it, representing the phase of the chiral core. In the strand tangle model, the continuous rotation of the core visualizes Feynman's rotating arrow. The second step is to imagine that the *tight tangle core*, which is continuously spinning, randomly changes its position, taking different paths. Averaging over all possible paths of the tight tangle core yields the wave function. The third step is, again, the deductions of the crossing switch density.

In the strand tangle model, the effects of all possible paths are added through the fluctuations of the tangle motion. With the definition of tangle addition given above, path addition occurs in the exact manner of Feynman's path integral formulation of quantum theory. Point-like contractions of electron tangles and photon tangles with unobservable tethers reproduce all the chapters of Feynman's book. The two possible rotation directions of advancing cores distinguish right-handed from left-handed particles. The two mirror versions of a chiral core distinguish particles and antiparticles.

Feynman's way of deriving wave functions from strands was implied and used already by Battey-Pratt and Racey [71]. Mathematically, it yields the same results for amplitude and phase that arise in Schrödinger's way. However, the phase definition with a tight tangle corresponds more directly to the idea of the rotating arrow along a path that was used by Feynman. In addition, the appearance of half angles in the definition of the wave functions becomes more intuitive. In any case, the description of the quantum state using a *tight* tangle fluctuating as a whole is equivalent to the description with a *loose* tangle where each strand segment is fluctuating. Both ways lead to a wave function described by an amplitude and one or several phases, that is, to one or several continuous complex functions of space and time.

How can one be sure that the tight tangle path averaging process results in the correct values of the wave function $\psi(\mathbf{x}, t)$? The answer is the same as the one used in the original path integral formulation. Because an advancing rotating tight tangle reproduces the fermion propagator, its path integral reproduces the wave function and, as will appear below, its evolution equation. It will become clear that the wave function of a free particle is a *rotating and diffusing cloud* of crossings.

In short, the strand tangle model of quantum particles reproduces the *path integral formulation* of quantum mechanics when particle tangle cores are approximated as *tight* chiral tangles of strands with vanishing radius, always taking into account that tethers are unobservable. Tight tangles that fluctuate over space yield, when averaged over time, the usual wave function.

11 Wave function superpositions are described by tangles

Crossing densities can only be models for wave functions if they form a *Hilbert space*. A Hilbert space is a vector space with an inner product and several technicalities. To show that crossing densities from a vector space, *linear combinations* of two wave functions – called *superpositions* in physics – need to be defined. This requires the definition of two operations: scalar multiplication and addition. The first, simple way is to define the operations for wave functions, as in quantum mechanics:

▷ The *scalar multiplication* $a\psi$ and the *addition* $\psi_1 + \psi_2$ of wave functions ψ_i are defined by applying the respective operations on the complex numbers at each point in space, i.e., on the local values of the wave function. In the strand tangle model, this implies performing the operations on the corresponding crossing densities.

This first definition is sufficient to show that crossing densities form a vector space. The same approach can be used to define an inner product and then show that the crossing densities form a Hilbert space, as expected. Also the time evolution of the wave functions follows. This is explained from page 47 onwards. One can then continue directly with Part IV of this article, which explores the differences from quantum theory, starting on page 59.

However, a second way to define a vector space for wave functions, equivalent to the first, is more intuitive and striking. Addition and multiplication can be defined for the *tangle* describing a quantum system – the fluctuating skeleton of its wave function. The short-time average – the blurring – can be taken *after* the operations on the underlying tangle are performed. This is done now, by using the defining *axioms* for vector spaces, for inner products, and Hilbert spaces.

▷ The *scalar multiplication* $a\psi$ of a state ψ by a complex number $a = r\mathrm{e}^{i\delta}$, with $r \leqslant 1 \in \mathbb{R}^+$ is formed by taking the underlying tangle, rotating its tangle core (i.e., each local crossing) by the angle 2δ, and then 'pushing' a *fraction* $1 - r$ of the tangle to the cosmological horizon, thus keeping the fraction r of the original tangle at finite distances. Thus, scalar multiplication with $r \leqslant 1$ is a process of *tangle core rotation and thinning*. Time averaging then results in the wave function $a\psi = r\mathrm{e}^{i\delta}\psi$.

A simple case of scalar multiplication for a tangle representing a state ψ is illustrated in Figure 14. The figure illustrates the idea of *thinning* a tangle core. Pulling a tangle core apart in such a way that no crossings arise in between corresponds to dividing the tangle into two fractions. The relative size of the two fractions is determined by (the square root of) the relative volume integrals of the probability densities. The mentioned process of tangle core thinning is visualized by the untangled strand segments in the so-called *addition region*.

The tangle version of scalar multiplication is effectively *unique*. Indeed, even though there is a choice about which specific fraction r of tangle crossings is kept and which specific fraction $1 - r$ of crossings is sent away, this choice is only apparent. The resulting crossing density, defined as the average over fluctuations, is independent of this choice because the tangle topology, which specifies the particle, remains intact.

The tangle version of scalar multiplication is *associative:* the relation $a(b\psi) = (ab)\psi$ holds by construction. Also, the scalar multiplication of strands behaves as expected for 1 and 0. Finally, strand multiplication by -1 is defined as the rotation of the full tangle core by 2π, as required by the belt trick. In other words, scalar multiplication by complex numbers can indeed be modelled

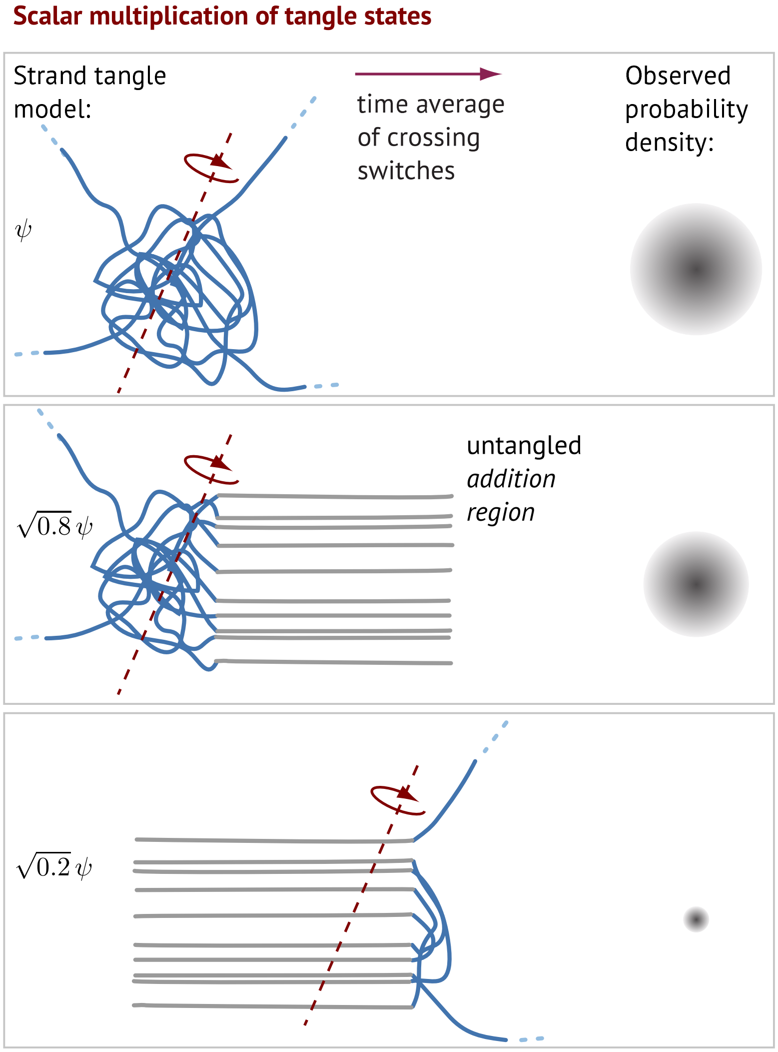

FIG. 14: The scalar multiplication of a localized tangle by two different *real* numbers smaller than 1 is illustrated. The resulting thinning of the corresponding wave function ψ is also visualized.

as tangle core rotation and thinning.

Also addition can be defined for tangles.

▷ The *addition* of two tangles $a_1\psi_1$ and $a_2\psi_2$, for which $|a_1|^2 + |a_2|^2 = 1$ and for which ψ_1 and ψ_2 have the same topology, is defined by directly *connecting* the crossings not pushed far away during the scalar multiplication by a_1 and a_2. The connection of tangles must be performed in such a way as to maintain the topology of the original tangles; in particular, the connection occurring in the spatial *addition region* must not introduce any crossings. Time averaging then leads to the superposition $a_1\psi_1 + a_2\psi_2$.

An example of superposition for the case of two quantum states at different positions in space is shown in Figure 15. No strand is cut or re-glued during addition – although imagining doing so might help for visualizing the operation. The *addition region* with untangled strands shown in the figure will be of importance later when entanglement is explored.

The definition of linear combination requires the final tangle $a_1\psi_1 + a_2\psi_2$ to have the same

Linear combination of tangle states

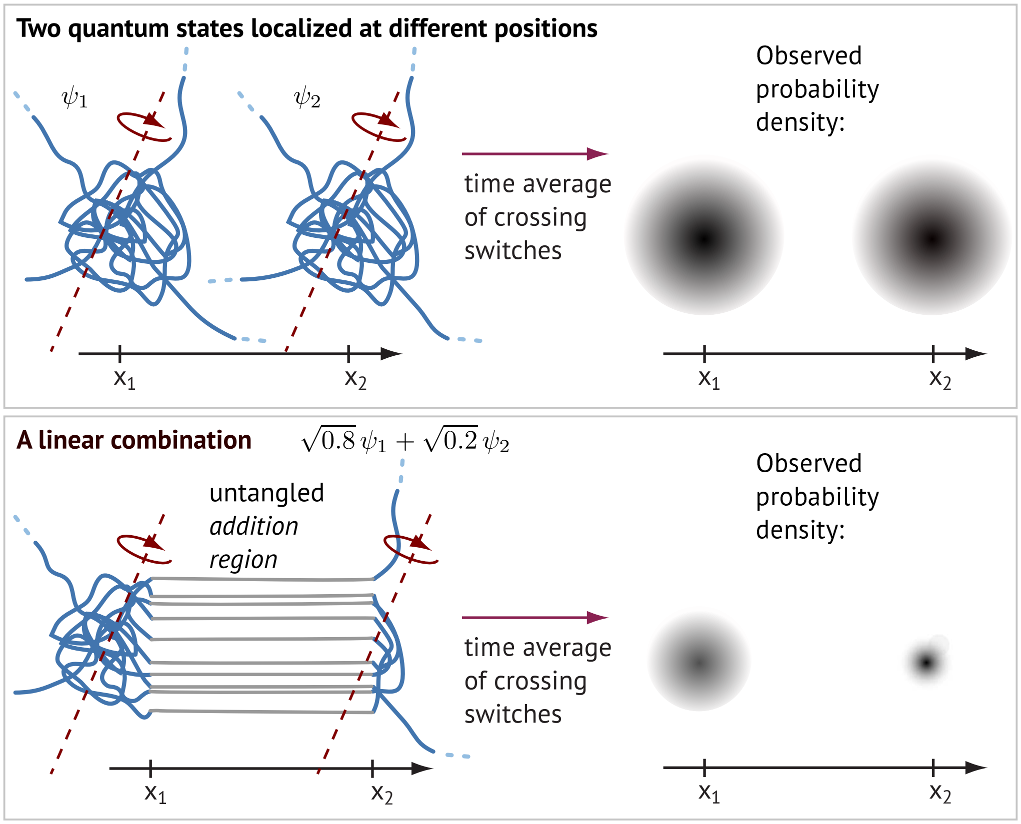

FIG. 15: A linear combination of two states representing a particle localized at two different positions visualizes the superposition of wave functions in the strand tangle model.

topology (and the same norm) as each of the two (normed) tangles ψ_1 and ψ_2 to be combined. Physically, this means that only states for the same type of particle can be added; this also means that particle number is preserved. Tangles thus automatically implement the corresponding *superselection rules* of quantum theory. This property is welcome because, in conventional quantum mechanics, the superselection rules need to be added by hand. In contrast, the strand tangle model contains them automatically.

One notes that the sum of two tangles is *unique*, for the same reasons given in the case of scalar multiplication. The tangle addition is commutative and associative, and there is a zero state, or identity element, given by the trivial, i.e., the untangled tangle. Tangle addition also implies distributivity with respect to the addition of states and with respect to multiplication by scalars.

In short, the definitions for the addition and for the scalar multiplication of crossing densities of tangles – using tangle thinning and tangle connections – prove that such crossing densities, as expected from wave functions, form a *vector space*.

12 Tangles imply Hilbert spaces

To form a Hilbert space, crossing densities deduced from tangles must allow the definition of an inner (or scalar) product. This implies reproducing the properties from quantum mechanics:

- ▷ The *inner (or scalar) product* between two states $\psi_1(\mathbf{x},t)$ and $\psi_2(\mathbf{x},t)$ is defined as $\langle\psi_1|\psi_2\rangle = \int\overline{\psi}_1(\mathbf{x},t)\psi_2(\mathbf{x},t)\,\mathrm{d}\mathbf{x}$.
- ▷ The *norm* (or *modulus*) of a state is $\|\psi\| = \sqrt{\langle\psi|\psi\rangle}$.
- ▷ The *probability density* ρ is $\rho(x,t) = \langle\psi|\psi\rangle = \|\psi\|^2$.

In the strand tangle model, the *conjugate tangle* $\overline{\psi}$ is formed from the tangle ψ by exchanging the sign of each crossing, i.e., by exchanging underpasses and overpasses. Conjugation arises by switching crossings. As a consequence,

- ▷ The *inner product* $\langle\psi_1|\psi_2\rangle$ is a complex number whose phase is given by the average crossing rotation between the two states and whose magnitude is the spatial average ratio of switched crossings in the tangle $\overline{\psi}_1$ that also appear in the tangle ψ_2.

This inner product has all the required properties: it is Hermitian, sesquilinear, and positive definite. The required technicalities about the completeness of the norm are also realized by tangles of strands. As a consequence, wave functions defined with strand tangles form a *Hilbert space*. Being related to crossing switches, the inner product is a physical observable, in contrast to the states themselves. This property is as expected.

The inner product of wave functions allows defining the *norm* of a wave function. In conventional quantum mechanics, the norm of a wave function is the square root of the integral of $\overline{\psi}\psi$, taken over full space. (For $\psi = r\mathrm{e}^{i\delta}$, the product $\overline{\psi}\psi$ is equal to r^2.) In the strand tangle model, the norm of the wave function is defined in the same manner. The resulting integral has the value 1 for every quantum state defined with tangles. Thus, one-particle wave functions are normalized in the strand tangle model.

Anticipating the results below, the investigation of electrodynamics [85] shows that the (appropriately signed) minimum crossing number of a tangle is the electric charge of a particle in units of $e/3$. In the tangle model, the probability density thus reproduces the charge density. All movements, all shape fluctuations and all strand exchanges of strand tangles maintain *charge conservation*.

As mentioned previously, the probability density is the crossing *switch* density. Because the definitions of the probability density and the inner product involve crossing switches, both quantities are physical *observables*. This is as expected from the fundamental principle of the strand tangle model.

Thus, wave functions and Hilbert spaces arise as consequences of $\hbar$. This result retraces the history of quantum theory. Schrödinger derived his evolution equation from de Broglie's matter waves. More precisely, he derived his equation from the expression $\lambda = \hbar/mv$ for the wavelength of quantum particles. Because the Schrödinger equation is linear, its solutions also yield and describe superpositions, interference, and entanglement. The solutions thus form a vector space. Exploring the solutions of the Schrödinger equation shows that they form a Hilbert space.

Non-relativistic quantum mechanics of free electrons is based on only two quantities: the quantum of action $\hbar$ and the mass m of the electron. These are the only parameters in the Schrödinger equation for free electrons. In particular, all quantum effects disappear when $\hbar$ is set to zero. In other words, quantum theory, in all its mathematical aspects and all its counter-intuitive details, follows from the non-zero value of $\hbar$. Nothing else is necessary to derive the full mathematical structure of quantum theory. The simplicity of the historical basis for quantum theory explains why assigning $\hbar$ to a crossing switch allows deducing quantum theory, including wave functions, Hilbert spaces, and, as shown below, the Schrödinger equation and entanglement.

Once states and their Hilbert spaces are defined, *operators* on that Hilbert space can be defined using strands. In quantum theory, *unitary* operators preserve inner products. In the strand tangle model, unitary operators, such as the time evolution operator, deform tangles such that the corresponding wave function retains its norm, that is, in such a way that the corresponding tangle *retains* both its topology and the shape of its core. In quantum theory, self-adjoint or *Hermitian* operators are important because they conserve probabilities, have a real spectrum, and describe physical observables. In the strand tangle model, Hermitian operators leave the tangle topology invariant but *change* the shape of the tangle core. Both conventional quantum theory and strands lead to the operator equation

$$px - xp = i\hbar \quad . \tag{11}$$

This well-known *canonical commutation relation* was first deduced by Born and Jordan in 1925 [145]. In 1987, Kauffman suggested that the commutation relation is due to a crossing switch [87, 88]. However, at that time, no one took up this suggestion. The strand model does. Further aspects of operators are explored in Section 18.

In short, it was proven that if strand tangles are used to define wave functions as crossing densities, then they reproduce superpositions, form Hilbert spaces, and allow deducing unitary and Hermitian operators. The resulting probability densities, defined as crossing switch densities, behave as expected from conventional quantum theory and observations. The next checks are whether tangles of strands reproduce the free motion of particles and whether crossing densities reproduce interference.

13 Tethered particles can pass each other

In everyday life, we can imagine two wooden balls, each connected to the border of space by six mutually perpendicular steel tethers under tension. Two such wooden balls cannot move past each other, because at a certain relative position, their steel tethers will touch and thus will prevent the free movement of the two wooden balls past each other.

In the strand tangle model, the situation for two tethered fermions differs markedly. Because tethers have no observable properties, they have no tension, are extremely flexible, have no fixed length, produce no forces, and have no mass. Therefore, tethers do *not* hinder each other in any way, nor do they have any effect on the motion of the tangled cores to which they are attached. The touching of distant, unobservable, extremely flexible, and extremely extendible tethers does not affect the motion of the particle cores to which they are connected.

In short, for all practical purposes, the touching of tethers far away from particle cores can be ignored. Fluctuating tethers do *not* hinder the free motion of non-interacting particles across space. This result is useful for the description of interference with tangles.

14 Strands lead to quantum interference of fermions

The observation of *interference* of quantum particles – such as electrons, neutrons, atoms and molecules – is due to the superposition of quantum states with different phases at one position in space. Interference results from the linear combination of wave functions. Such superpositions

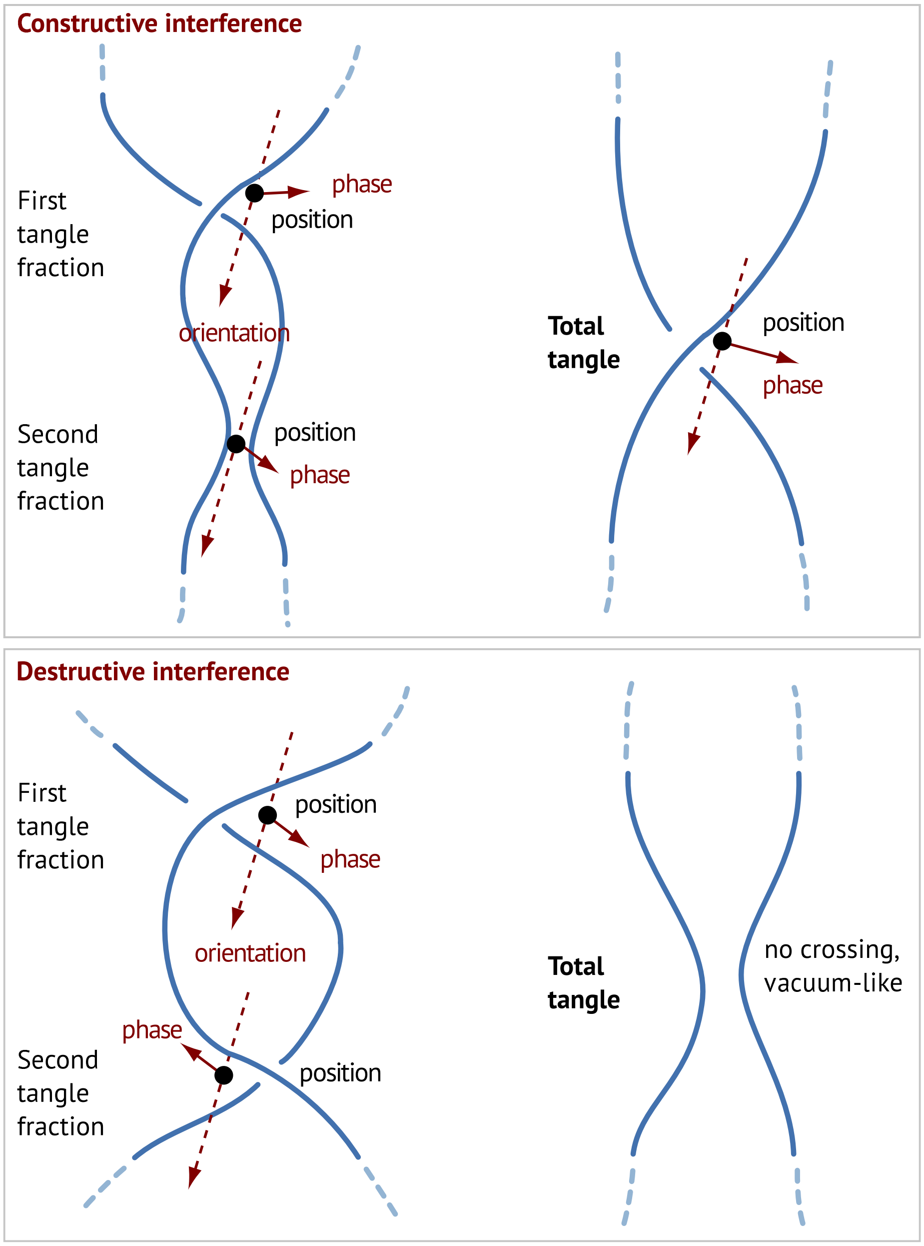

FIG. 16: The strand explanation at the basis of interference: two crossings connected by strands superpose constructively (top) and destructively (bottom).

are a central feature of quantum physics. In particular, they are central to the description of wave-particle duality.

As illustrated in the upper half of Figure 16, the strand tangle model reproduces interference. Using the definition of superpositions given above, an equally weighted sum of a tangle and the same tangle with a phase rotated by $\pi/2$ (thus with a core rotated by π) results in a tangle whose phase is rotated by the intermediate angle, thus with a phase rotated by $\pi/4$. Strands thus describe interference in the same way as wave functions do.

The most interesting case is *destructive* interference, or *extinction*. In experiments, the multiplication of a wave function ψ by -1 yields the negative of the wave function, i.e., its additive

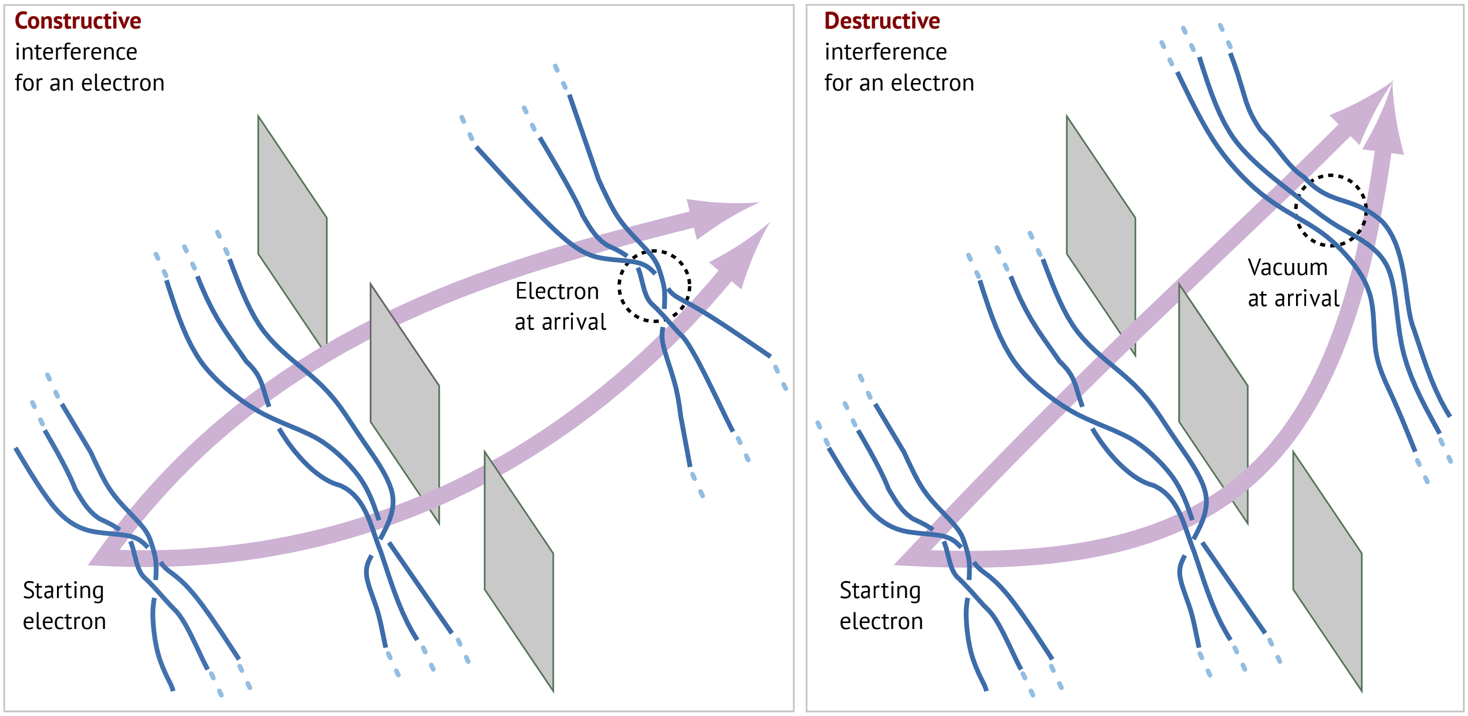

FIG. 17: The tangle explanation of interference is illustrated for a fermion tangle passing a double slit. Depending on the phase difference arising in the two paths, the fermion tangle shows constructive interference (left) or destructive interference (right). The tangles of the particles making up the screen can be ignored, as explained in Section 13.

inverse $-\psi$. The local sum of a wave function and its exact negative is zero. This is the explanation of extinction in conventional quantum theory.

In the strand tangle model, the negative of a fermion tangle has a core rotated by 2π. Using the strand definition of linear combinations, the sum of a fermion quantum state with its *exact* negative requires rotating half of the core by 2π and then connecting it to the other, unrotated half, without crossings between them. This is impossible while keeping the same tangle core. Equivalently, for many tangle cores, this operation yields untangled strands at that position. This is shown in the lower half of Figure 16. Therefore, the result has a *vanishing* crossing density in the spatial region where this operation is attempted, that is, in the spatial region where a state is added to its exact negative. Vanishing crossing density is the defining characteristic of the vacuum. Strand tangles thus explain extinction as a consequence of particle tangle superpositions.

As a consequence, tangle superposition allows describing and visualizing the double-slit experiment. Depending on the two paths taken to the point of superposition, the phases either add up or cancel out. This behaviour is visualized in Figure 17. As explained in the preceding section, the influence of the tethers of the particles making up the screen with the slits can be ignored.

In short, fluctuating strand tangles describe and explain both the constructive and the destructive quantum interference of matter particles. This result leads to exploring the case of photons.

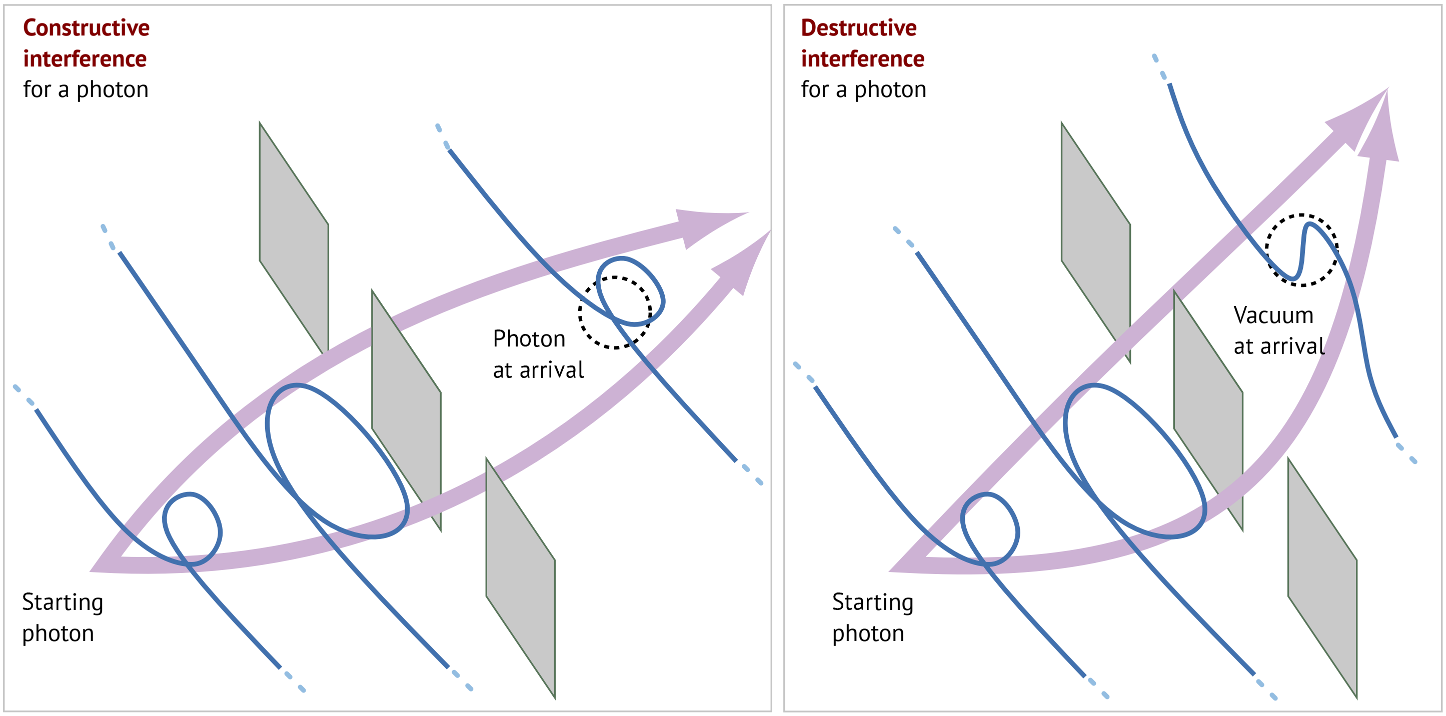

FIG. 18: The tangle explanation of photon interference is illustrated. Depending on the relative phase between the two paths, a photon passing a double slit shows constructive interference (left) or destructive interference (right).

15 An intermezzo: strands lead to quantum interference of photons

In the strand tangle model, a photon is a single strand with a looped *twist*, as illustrated in Figure 18. The relation of photons tangles to electrodynamics is given in Figure 19. The relation of photons to the other elementary bosons in nature is shown below, in Figure 33, As expected and explained in detail below, a photon is a propagating first Reidemeister move.

The twist is the tangle core of a photon; the size of the twist is the wavelength. For a photon, only crossing switches are observable; the tethers are unobservable. When a photon advances, its core advances and rotates. The phase of a photon is determined by the pointing direction of its twisted core. (This is in slight contrast with fermions, for which the phase is given by *half* the pointing direction of the core.) As illustrated in Figure 19, also the photon, like the electron, can therefore be seen as an advancing rotating arrow. Thus, the strand tangle model simply adds a twist and two tethers to the conventional description of the photon as a rotating arrow.

The tangle model shows that a photon whose core has rotated by 2π is equivalent to a photon with an unrotated core: the tethers can fluctuate from one case to the other. This is the – much simpler – *spin 1* version of the belt trick and is illustrated in the top left of Figure 19. An almost insignificant 'boson trick' corresponding to the 'fermion trick' of Figure 4 also exists: after a single core exchange, two nearby photons yield the same observables as before. Photons thus have spin 1 and show boson behaviour. The photon tangle is topologically trivial; as explained below, this implies a vanishing mass. The description of electric and magnetic fields with densities of twists is possible. In particular, electric charge, its motion slower than light, and its conservation appear

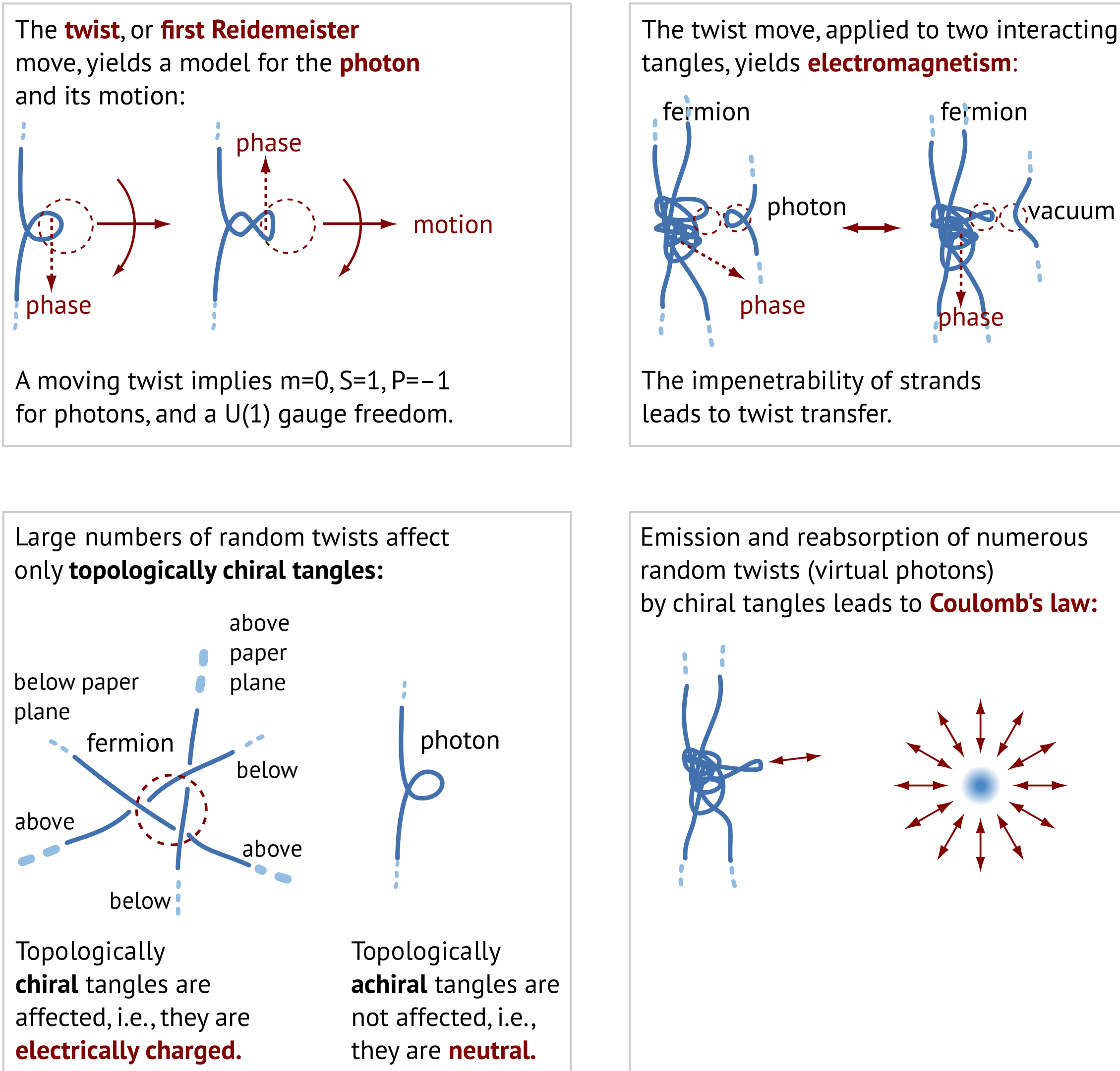

FIG. 19: The strand tangle model for electromagnetism is illustrated, as explored in detail in reference [85]. The motion of a photon with its rotating phase, the absorption of a photon, the origin of electric charge from tangle chirality, and the origin of Coulomb's law are visualized.

naturally; also, Figure 19 shows that strands imply Coulomb's law. *Mathematical theorems* based on these properties by Heras [146–150] and by Burns [151] then *prove* that Maxwell's equations hold, including the Lagrangian of classical electrodynamics. In particular, strands imply minimal coupling. The way of visualizing Maxwell's equations and quantum electrodynamics with the help of twisted strands has been explored elsewhere [85].

Figure 18 visualizes the interference of photons in the case of a double-slit experiment. The 'negative' of a photon state is a strand whose looped twist points in the opposite direction. The addition of half a twist with its other negative half yields a strand without crossing. This situation visualizes extinction and is shown on the right-hand side of the figure. Thus, in the case of extinction, photon states of opposite phases add up to vacuum strands.

In short, strand tangles describe and explain the quantum interference of photons, once one recalls that only crossing switches are observable and that crossing switches can only appear where

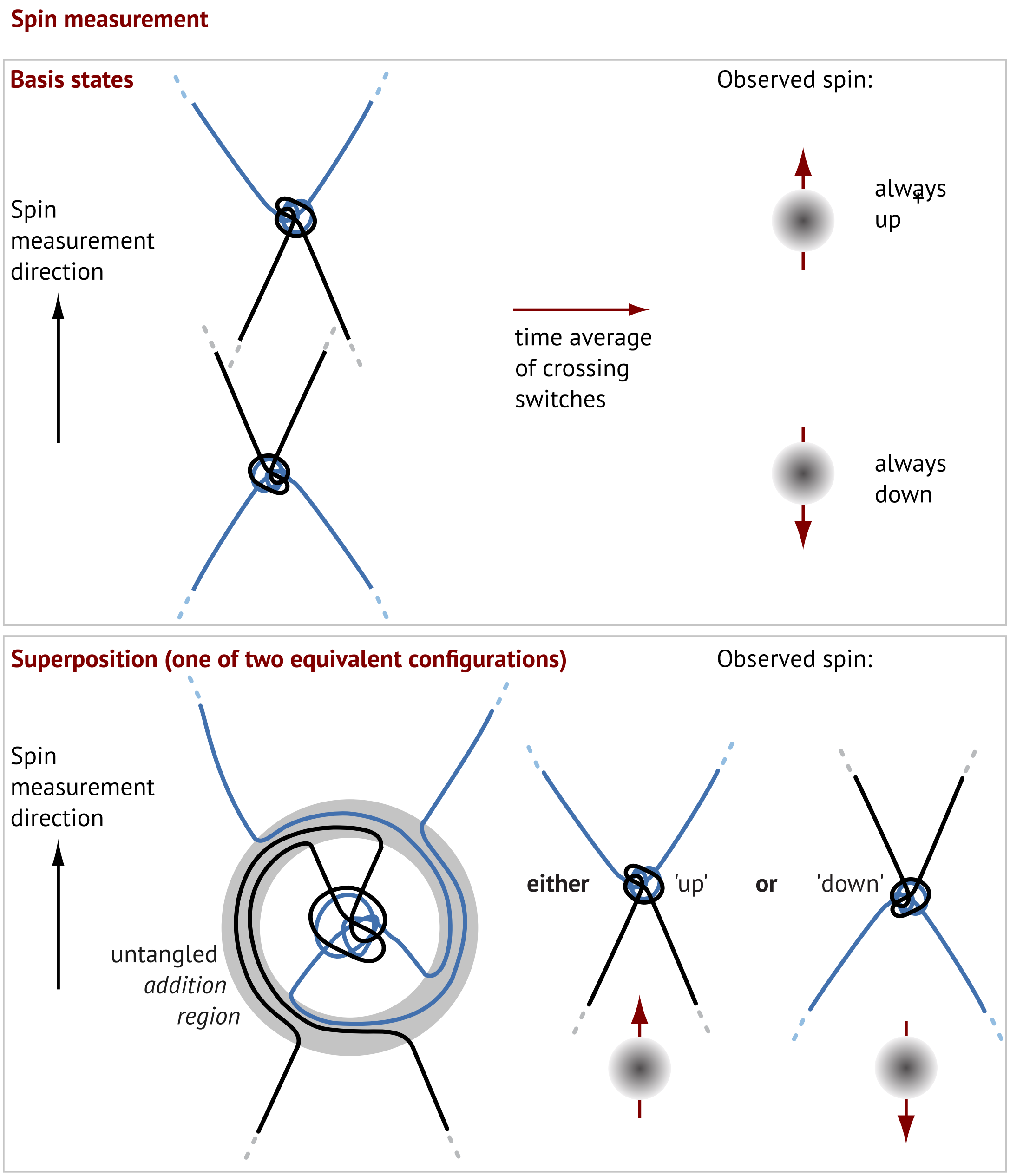

FIG. 20: The measurement of a spin superposition makes the addition region disappear either outwards or inwards.

crossings exist. The strand tangle model of the photon, including the implied U(1) gauge symmetry, is confirmed in more details below. A literature search shows that similar descriptions of single photons with a curved strand, such as the 'corkscrew model' of the photon [72], are part of physics lore but have never been published.

16 Measurements, Born's rule, wave function collapse and decoherence

Experiments show that every measurement of an initial quantum state ψ has two effects. First, every measurement yields a real *eigenvalue* a of the operator of the variable to be measured. Secondly, every measurement changes, projects, or collapses, the initial quantum state into the corresponding *eigenstate* ψ_a (in the most common situation without degeneracy). The collapse occurs with a probability given by $|\langle\psi_a|\psi\rangle|^2$, the squared inner product between the quantum state and the eigenstate. This is known as *Born's rule*.

In nature, every measurement apparatus is a device that stores and displays measurement results. This is possible because every measurement apparatus is a device with a memory, and thus is a *classical* device. All devices with memory contain at least one *bath*, i.e., a subsystem described by a temperature [152]. Thus, every measurement apparatus *couples* a bath to the system that it measures. The detailed coupling depends on and defines the observable to be measured by the apparatus. Every coupling of a quantum system to a bath results in *decoherence*. Decoherence leads to wave function collapse and probabilities. Therefore, collapse and probabilities are necessary and automatic consequences of decoherence in conventional quantum theory [153, 154].

The strand tangle model describes the measurement process in precisely the same way as conventional quantum theory. In addition, strands *visualize* the process.

- ▷ A *measurement* is modelled as the specific deformation, induced by the bath of measurement apparatus, that *deforms* the strands of a quantum system into the resulting eigenstate.
- ▷ This deformation of the particle tangle by the bath tangles is the *collapse* of the wave function.
- ▷ The storage of the result in the measurement apparatus, in its memory, involves the tethers of the quantum system and the tethers in the bath of the apparatus.

An example of measurement for the case of a spin superposition is illustrated in Figure 20. Tangles visualize the measurement process:

- ▷ When a measurement is performed on a superposition of two spin states, *the untangled 'addition region' is made to shrink or expand into disappearance by the bath.*

When this occurs, one of the underlying eigenstates 'gobbles up' the other eigenstate: the wave function *collapses*. This process is triggered by the strands in the bath inside the measurement apparatus (not shown in the figure). The strands of the apparatus make the addition region disappear either towards the outside or the inside. The choice is determined by the details of the state of the bath coupled to the system in the measurement. Thus, the bath fluctuations determine the outcome of measurements. Furthermore, the probability of measuring a particular eigenstate will depend on the (weighted) volume that the eigenstate takes up in the superposition because the bath will choose that eigenstate more often that has the higher weighted volume.

In the strand tangle model, when the bath in the measurement apparatus selects an outcome, all the bath tangles change the original quantum state being measured into the eigenstate of the outcome. This change occurs by deforming all the strands of the quantum state being measured into the strand configuration of the eigenstate. In other words, a large number of bath tangles causes the quantum state to collapse by pushing the strands of the quantum system. Therefore, in any measurement, the initial quantum state and the final quantum eigenstate differ only in their strand shapes, and not in their topologies. This is observed.

For example, when the position of a charged particle is measured, an initially delocalized, spread-out quantum state is localized at the measured position. When the particle charge triggers the particle detector, the tangle crossings of the particle interact with the crossings of the charges inside the detector. This interaction localizes the quantum state of the particle, and thus the tangle core, at the spot where the detector is triggered. This happens because whenever charges interact via virtual photon exchange, their cores are automatically localized.

Thus, the strand description of measurement is a specific realization of wave function collapse induced by the environment. This approach has been championed by Zeh and many after him. The details of contextual collapse are still being refined [155]. In simple words, strands state that there is a stochastic aspect in the environment of every quantum system. This description appears to resolve the issues raised by Adler [156].

The strand visualization of the wave function collapse triggered by the bath also implies that the collapse *takes time*. The collapse time is the time taken by the bath to trigger the collapse. The strand visualization of the wave function collapse also clarifies that the collapse is *not limited by any speed limit*, as no energy and no information are transported, and thus no signal is transmitted. Indeed, the collapse occurs because the strands of the system are deformed by the strands from the bath. If one disregards the bath, collapse *seems* superluminal. If the bath is taken into account, the limit for signal speed is respected.

In short, it was shown that the strand tangle model describes measurements in the same way as conventional quantum theory. A measurement apparatus – which by definition includes a bath – interacting with a quantum system has no other choice than to enforce Born's rule through the tangles of its bath. In particular, strands visualize the collapse of the wave function as a shape deformation from a superposition tangle to an eigenstate tangle triggered by the large number of tangles in the bath. This description agrees with the usual description of collapse due to environmental decoherence.

17 Strands imply a finite decoherence time

In nature, the decoherence process takes time, a fact that has been confirmed by numerous experiments [157–162]. The value of the decoherence time is due to the interaction with the bath in the apparatus or the environment. Generally speaking, the denser and the more energetic the microscopic degrees of freedom of the involved bath, the faster the decoherence.

The decoherence time can be estimated in various ways. A simple estimate arises for baths composed of many scattering particles. In this case, the decoherence time typically is of the order of the time between two scattering events due to the bath. The resulting spatial localisation is of the order of the (thermal) de Broglie wavelength of a typical bath particle. More precisely, the decoherence time is given by the particle flux and the interaction cross-section, as explained by Joos and Zeh [153] and by Tegmark [163]. In most practical situations, the resulting decoherence time is extremely short and results in localisation in an extremely small domain.

A second estimate of the decoherence time uses the relaxation time and temperature of the bath [154, 164]. Extremely small values for the decoherence time result in all everyday situations. Long decoherence times are possible if interactions with baths are minimized. This requires a careful experimental setup in laboratories. Such setups are regularly realized in experiments.

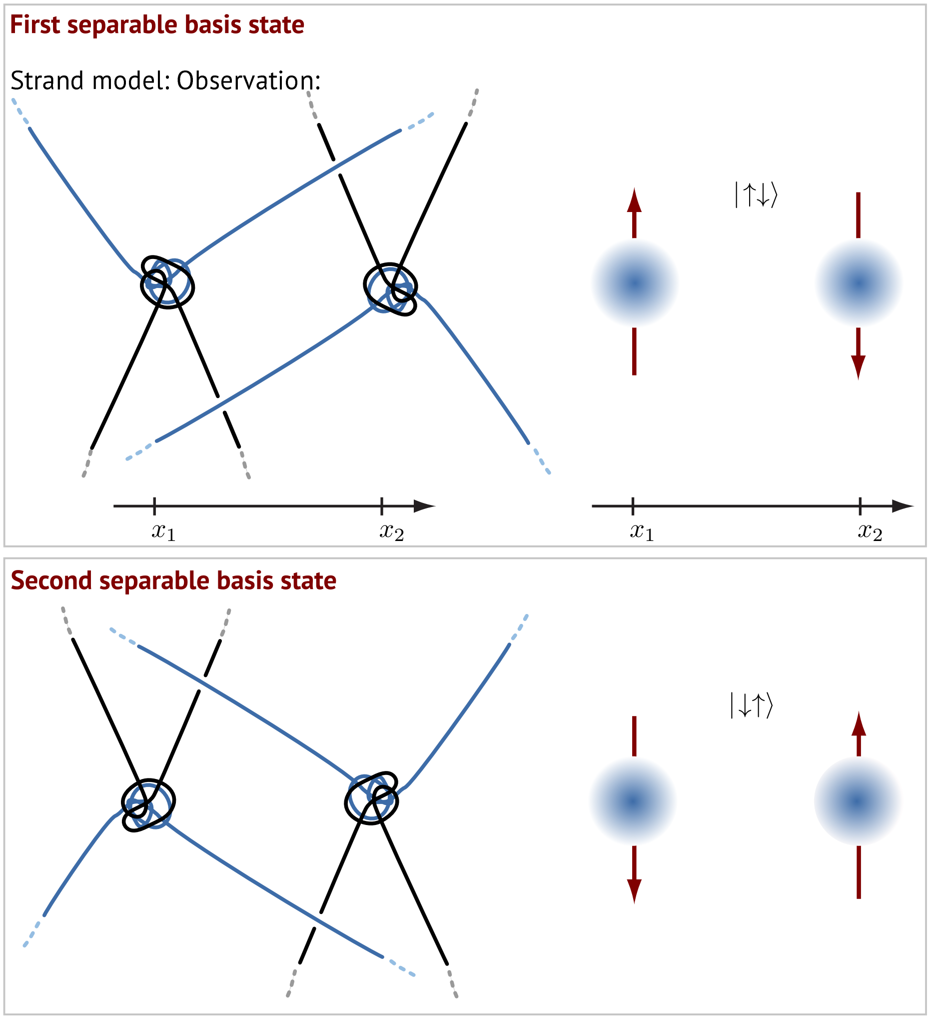

FIG. 21: Two examples of two distant particles with spin in *separable*, *unentangled* and *incoherent* states are illustrated, both in the strand tangle model and in the corresponding observed probability densities.

Also in the strand tangle model, the effects of scattering processes occur. Also in the strand tangle model, the relaxation time at the origin of decoherence arises due to microscopic processes in the bath. Also in the strand tangle model, the decoherence time is an interaction time. It is the time that the bath strands take to project the particle tangle onto the eigenfunction of the measured observable value.

The combination of decoherence and the smallest length that characterizes the strand model of quantum mechanics is also found in the Montevideo interpretation of quantum mechanics proposed by Gambini and Pullin [165, 166]. Investigations into the overlaps will be useful.

In short, strands reproduce decoherence in all its effects: decoherence takes time, destroys macroscopic superpositions, and is a result of fluctuations in the strands present in the environment or the measurement apparatus, more specifically, in their baths.

Entangled state $\sqrt{90\,\%}\,|\uparrow\downarrow\rangle + \sqrt{10\,\%}\,|\downarrow\uparrow\rangle$

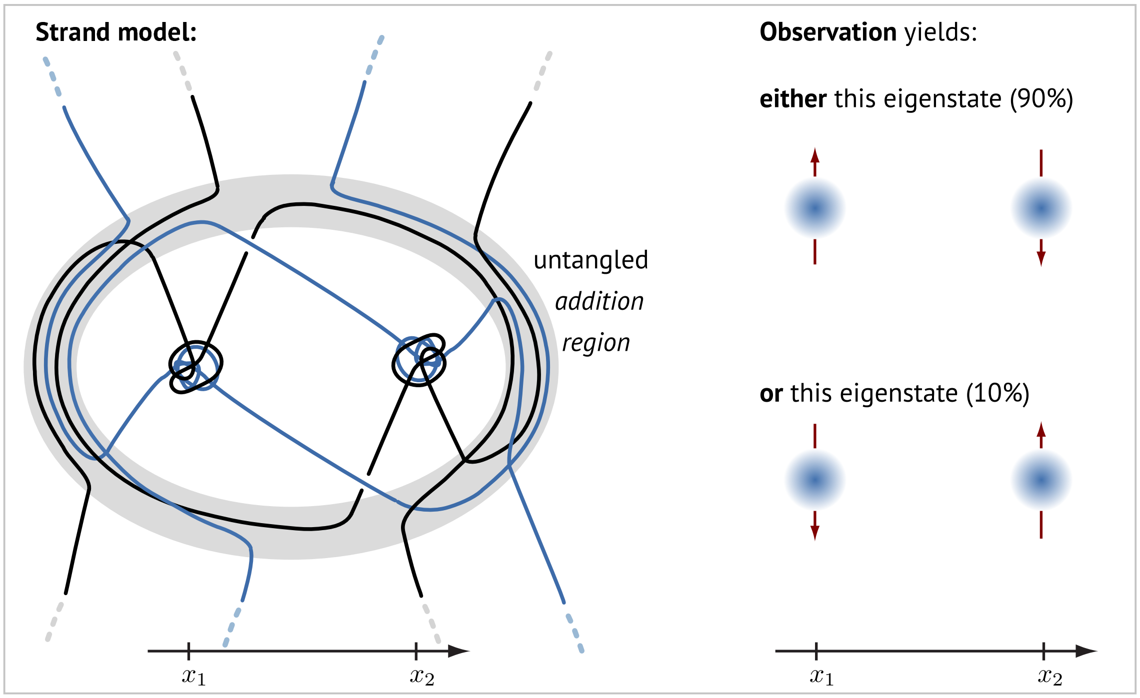

FIG. 22: An *entangled*, *inseparable* and *coherent* spin state of two distant particles is illustrated. The *addition region* around both particles prevents either particle from being seen as a separate system that is independent of the rest of its environment.

18 Quantum entanglement is due to topological entanglement

In nature, two or more particles can be *entangled*. Entangled states are coherent N-particle states that are not separable, that is, they cannot be written as the product of single-particle states. Entangled states are a fascinating and central aspect of quantum physics. Entangled states do not exist in classical physics but are observed and explored in many quantum experiments.

The strand tangle model allows describing N-particle systems in 3 spatial dimensions. In conventional quantum theory, an N-particle wave function is usually described by a single-valued function in $3N$ dimensions. However, the strand tangle model naturally defines N wave function values at each point in space: each particle has its own tangle, and each tangle yields, via short-term averaging, its own complex value(s) at each point in space. Due to this separation of particle tangles and states, strands can describe the state of N particles in 3 spatial dimensions – despite the recurring prejudice that this is impossible.

Creation operators are operators that add a particle to the vacuum. In the strand tangle model, a creation operator transforms untangled vacuum strands into a particle tangle. Similarly, *annihilation operators* untangle the strands of a particle tangle, yielding vacuum. Thus, both operators automatically ensure an *integer* number of particles. Both operators are also *almost* local, realizing the observational constraints and theoretical demands. In other words, the tangle model eliminates the need for superselection rules. Again it becomes clear, as mentioned at the beginning, that strands also eliminate the need for second quantization – and for any other type of quantization.

The discussion of quantum fields is expanded in Appendix C.

In particular, the strand tangle model of many-particle states allows defining entangled states for two particles.

▷ An *entangled state* is a non-separable superposition of several particle states. Mathematically, an entangled state is *not* a product of separate particle states. In the strand tangle model, a state is *non-separable* whenever the tethers of the particles remain topologically entangled even if the tangle cores are pulled apart. More precisely, as illustrated in Figure 22, two states are entangled if their strand addition region surrounds *both* tangle cores.

A well-known example of entanglement is the spin entanglement of two identical, but distant fermions in a spin 0 state. The example was introduced and discussed in detail by Bohm [167]. It was simplified by Mermin [168], and experimentally tested, for example, by Fry [169].

In the strand tangle model, two distant fermions in a *separable* state are modelled as two distant, separate tangles of identical topology. Figure 21 illustrates two such separable basis states in the strand tangle model. In this case, they are two states with total spin 0, given by $|\uparrow\downarrow\rangle$ and by $|\downarrow\uparrow\rangle$. Such states (with their linked tethers) are typical outcomes of spin measurement experiments. However, other states are also of interest. Using the definition of tangle addition, a superposition such as $\sqrt{90\,\%}\,|\uparrow\downarrow\rangle + \sqrt{10\,\%}\,|\downarrow\uparrow\rangle$ of the two spin-0 basis states looks as illustrated in Figure 22. Such a state is not a product state, and therefore it is not separable, but *entangled.* In the strand tangle model, the entanglement of the state is visible in the addition region. When the spin orientation of one of the particles is measured, the untangled 'addition region' disappears. The result of the measurement will either be the state favoured by the inside of the addition region or the state favoured by the outside. Because the tethers of the two particles are linked, after the measurement, independently of the outcome, the spins of the two particles will always point in opposite directions. This happens independently of particle distance. Despite this extremely rapid and seemingly superluminal collapse, no energy travels faster than light. After the measurement, the state is separable. Thus, the strand tangle model reproduces the behaviour of entangled spin $1/2$ states as it is observed in experiments.

The similarity of quantum entanglement and topological entanglement has been noted for a long time [170–177]. Tethers provide the basis for this analogy. For example, suitably connecting the tethers at spatial infinity should recover the results of the classification of multi-particle entanglement given in reference [174]. Likewise, the description of entanglement with strands reproduces the mentioned demonstration by Mermin [168].

In short, using the definition of wave functions as tangle crossing densities, it was shown that strand tangles reproduce quantum entanglement through topological entanglement of tethers. Entanglement thus follows from the fundamental principle. This leads to a topic of basic importance.

19 Strands are not hidden variables

At first sight, the strand tangle model seems to introduce hidden variables into quantum theory. One is tempted to argue that the fluctuating shapes of strands play the role of hidden variables. In apparent contrast, non-contextual hidden variables are impossible in quantum theory, as shown most famously by the Kochen–Specker theorem, which is valid for sufficiently high Hilbert space dimensions [178]. In real-life systems, the conditions of the theorem are always satisfied.

Despite the first impression, the strand tangle model does *not* contain hidden variables. First, strands and their shapes are neither observable nor measurable in any way. In particular, strand shapes are *not physical observables* and thus neither physical nor hidden variables. Strands have *no* measurable properties: they have no tension, no momentum, no energy, no spin, no charge, no quantum numbers, etc. Therefore, it is impossible to define a measurable *position* or *shape* for strands or strand segments. All strand figures in this article are unphysical, because they show strands in certain positions and shapes, and because they suggest that strands can be counted. The figures become physical only once they are extended in two ways. First, the figures must be extended with an indeterminacy of time. Second, the figures must be extended with an indeterminacy of position for every strand segment. In nature, only crossing switches are observable, because only crossing switches couple to electromagnetic fields. This topic is explored in Section 34.

Secondly, strand shapes evolve in a manner dictated by the influence of the environment, which consists of all other strands in nature, including those of empty space itself. Therefore, the evolution of strand shapes and crossing switches is *contextual*.

For the two reasons just given, the strand tangle model does not contradict the Kochen-Specker theorem. The strand tangle model provides *no* observables beyond those of quantum theory. As expected from any model that reproduces decoherence, the strand tangle model leads to a *contextual* and *probabilistic* description of nature. The tangle model thus reproduces the approach to quantum theory as a theory of extrinsic properties, as explained by Kochen [179].

The results of the strand tangle model agree with all research on probabilities in quantum mechanics. This includes the investigations of Colbeck and Renner [180–182], who showed the lack of alternatives to quantum theory, and in particular, the lack of 'more complete' theories that are in agreement with observations and the freedom of choice. The results of the strand tangle model also appear to agree with the Pusey–Barrett–Rudolph theorem, which states that wave functions are more than just information about a quantum state [183].

In short, it was shown that the strand tangle model does *not* make use of non-contextual hidden variables. Nevertheless, in the strand tangle model, quantum theory *emerges* from fluctuating tangle shapes. The emergence does not change the usual probabilistic description of quantum phenomena and does not go beyond it. Quantum theory remains as fascinating as ever.

20 The probability density is limited

The fundamental principle of the strand tangle model, with its (corrected) Planck limits and its definition of wave functions as crossing densities, implies the corrected Planck limit for the probability amplitude. Equivalently,

Test 1: Strands limit the value of probability density to

$$||\psi(\mathbf{x},t)||^2 \leqslant \left(\frac{c^3}{4G\hbar}\right)^{3/2} \approx 3\cdot 10^{103}\,\mathrm{m}^{-3} \quad . \tag{12}$$

The strand limit value is the inverse of the smallest possible volume in nature. Finding an exception to the limit would falsify the strand tangle model. In particular, the density limit contradicts the existence of Dirac's δ distribution. So far, no observation contradicts the (corrected) Planck limit for probability density and the corresponding Planck limit for the modulus of wave functions.

The Planck limits for wave function amplitude and probability density do not appear to have been discussed in the research literature. The main reason for this is that the limits are extremely large, and as argued in Section 2, cannot be achieved nor approached in any experiment.

There is no corresponding Planck limit for the phase or phase difference. A phase angle is a length ratio, and length ratios are not limited by Planck limits.

In short, strands imply that the probability density is limited by the inverse of the smallest volume in nature. Strands further imply that this limit cannot be approached in any experiment. This result concludes Part II of this article, which showed that crossing densities of particle tangles are wave functions and visualize interference, measurement, decoherence and entanglement. In simple language, *wave functions are blurred tangles, and tangles are fluctuating skeletons of wave functions*. This description allows exploring their evolution in time.

Part III: Dynamics of states

The present Part III explores the *time dependence* of wave functions. The basic idea follows naturally from the exploration so far:

▷ A moving particle is described as a fluctuating, spinning tangle core advancing in space.

The tangle model combines the descriptions of a particle as a rotating phase, a rotating arrow, or a rotating flag that are due to Schrödinger, Feynman, Hestenes, Penrose and many others.

Why do advancing particle tangles rotate? This is the only situation in this article in which it is necessary to recall that the vacuum itself is made of strands as well. The vacuum strands, in combination with the asymmetric, chiral tangle core, lead to a rotation of the core around the particle path, similar to a (tethered) propeller moving through a fluid. This propeller motion is the origin of the rotating phase arrow for moving quantum particles. A propeller rotates in a fluid because it is chiral. Also the tangle cores of fermions are chiral. The rotation speed of a propeller advancing through a fluid depends on the geometrical shape of the propeller, in particular on the angles of its fins. In the strand tangle model, the rotation speed of a core depends on its average shape, which in turn depends on its topological structure.

In short, in the strand tangle model, a quantum particle moving through a vacuum behaves like a tethered propeller moving through a fluid. Dirac's belt trick couples tangle core propagation and core rotation. The continuously rotating core of the tangle corresponds to the spinning propeller.

21 The Schrödinger equation emerges from tangles

The fundamental principle of Figure 7 states that crossing switches define the quantum of action $\hbar$. As shown now, the fundamental principle also explains how the crossing density of a free particle tangle evolves in time.

Experiments show that free quantum particles propagate, spin and diffuse. For the strand tangle model, a qualitative visualization of the motion of a free quantum particle is given in Figure 23. In the tangle model, a localized particle with constant speed is described by a localized tangle core that *advances*. While advancing, the tangle core *rotates* and *precesses*. The rotation and precession are due to the belt trick deforming and moving the tethers *around* the advancing particle core during the belt trick. The belt trick of Figure 2 already showed that the rotation and the

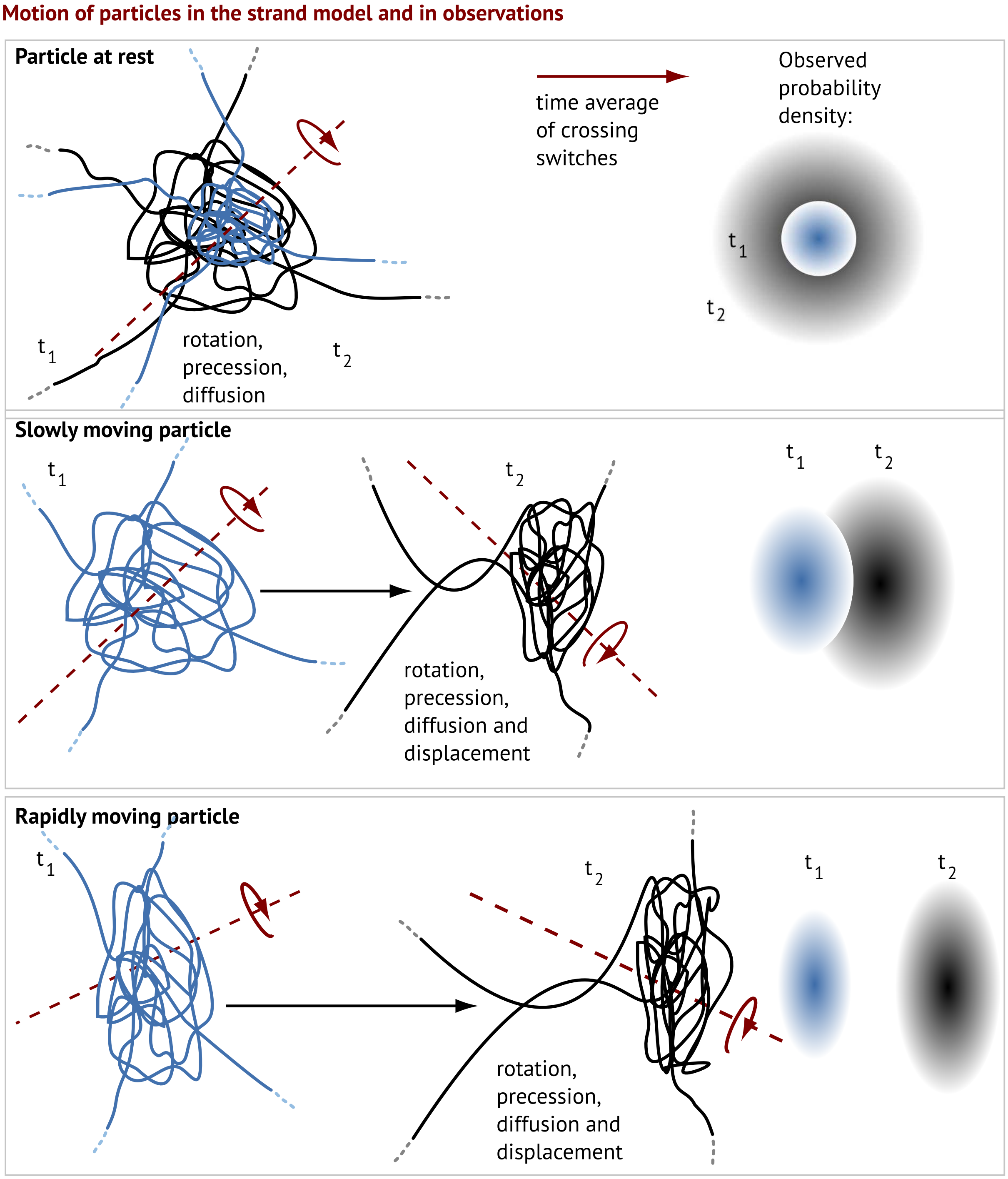

FIG. 23: Tangles of free particles – at rest (top), in slow motion (middle) and fast motion (bottom), vacuum strands not shown – illustrate how relativistic speeds lead to core flattening.

displacement of the core are related. On average, after every second core rotation, the fluctuations of the tethers lead to a rearrangement of the tethers. The probability of this process will depend on the complexity of the core, the average shape of the core, and the number of tethers.

While advancing, due to its fluctuations, the tangle core also *spreads out in space.* The spreading yields the *diffusion* of the probability density. The diffusion of a tangle is a consequence of the impenetrability of the strands: the various fluctuating strand segments continuously push against each other, thus spreading out the core over time.

Every tangle core rotation leads to crossing switches. The fundamental principle states that each crossing switch yields a quantum of action $\hbar$. A rapid core rotation results in many crossing switches per time, whereas a slow rotation results in a few crossing switches per time. The quantity defined by action per time of a free particle is commonly called *(kinetic) energy*. In other terms,

▷ Particles with *high* (kinetic) energy have *rapidly* rotating tangle cores; particles with *low* energy have *slowly* rotating tangles.

Using the fundamental principle that relates crossing switches to $\hbar$, the kinetic energy E of a tangle is automatically related to the angular frequency ω of its core rotation by the relation

$$E = \hbar\omega \quad . \tag{13}$$

Equivalently, the local phase of the wave function ψ changes during the rotation. This implies that

$$\omega = i\partial_t \quad , \tag{14}$$

a relation that will be used shortly. The change of phase with *time* needs to be combined with the change of phase over the *distance* covered when the particle is advancing.

▷ Rapidly moving tangles show many crossing switches per distance; slowly moving tangles show few crossing switches per distance.

The fundamental principle implies that the natural observable for this situation is action per distance:

▷ The *(linear) momentum* of a moving tangle is the number of crossing switches per distance.

Due to the fundamental principle, the momentum p of a tangle is related to the wave number $k = 2\pi/\lambda$ of the core rotation by

$$p = \hbar k \quad . \tag{15}$$

Equivalently, the phase of the tangle core rotates during propagation. This implies

$$k = -i\partial_x \quad . \tag{16}$$

Again, the appearance of the imaginary unit i describes the rotation of the phase during propagation. This second relation completes the description of the phase of propagating wave functions.

An advancing particle tangle implies a continuously repeated belt trick. The greater the momentum of the particle, the more its rotation axis must align with its direction of motion. This effect is illustrated in Figure 23. In the non-relativistic case, this alignment leads to a *quadratic* increase in the number of crossing switches with momentum p: one factor p is due to the increase in the speed of rotation, and the other factor is due to the increase in the alignment. This yields

$$E = \frac{p^2}{2m} \quad \text{and} \quad \omega = \frac{\hbar}{2m}k^2 \quad . \tag{17}$$

The constant m is a proportionality factor that depends only on the details of the tangle core structure; it is called the *mass* of the particle. This dispersion relation agrees with the measurements, as long as $p/m \ll c$. In the tangle image, the dispersion relation is valid as long as the speeds are so low that the fluctuating core retains its spherical shape on average. The complexity of the belt trick implies that tangle cores that are spherical on average – massive particles – must move more slowly than light. In contrast, in relativity, the fluctuating core is a deformed sphere with an ellipsoidal shape. Figure 23 illustrates that situation.

At this stage of the exploration, Schrödinger's argument from 1926 can be repeated. Substituting the differential relations into the dispersion relation, the evolution equation for the wave function ψ is

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\partial_{xx}\psi \quad . \tag{18}$$

This is the Schrödinger equation for a free particle in one spatial dimension.

In short, in the strand tangle model, the tangle core of a matter particle is localized. The belt trick implies that core displacement and core rotation are related, thus allowing the definition of an inertial mass value. As a consequence, the wave function, or crossing density, of a non-relativistic spin-less particle obeys the Schrödinger equation. The equation describes how a free quantum particle advances, how its phase rotates, and how its probability density diffuses. In contrast to usual quantum mechanics, the mass value is *not a free parameter* but uniquely determined by the tangle structure. Several tasks remain: confirming the indeterminacy relation, deducing the relativistic description, including spin, including interactions, and calculating mass values.

22 Indeterminacy is a consequence of the fundamental principle

The Schrödinger equation implies Heisenberg's *indeterminacy relation*, which is also called *uncertainty relation*. It has been confirmed in every experiment to date.

Also in the strand tangle model, the indeterminacy relation can be deduced from the Schrödinger equation. In addition, the indeterminacy relation is a *direct consequence* of the fundamental principle illustrated in Figure 7. The figure shows that the smallest indeterminacy of every action measurement is half a crossing switch. When a strand configuration corresponds to the middle case of Figure 7, it is not clear to which of the two outer configurations it belongs. The other two configurations are separated by $\hbar$. Therefore, for any tangle configuration,

$$\Delta W \leqslant \hbar/2 \text{ and thus } \Delta x \Delta p \leqslant \hbar/2 \quad . \tag{19}$$

For the same reason, the indeterminacy of the measurement of any two observables whose product is an action value is given by half a crossing switch.

In short, in the strand tangle model, the fundamental principle implies that quantum particles obey the usual indeterminacy relation.

23 The Klein-Gordon equation emerges from tangles

In 1980, Battey-Pratt and Racey [71] showed that tethered, relativistic and spin-less particles are described by the Klein-Gordon equation. Their approach can be used in the strand tangle model. The spin-less situation effectively assumes a spin orientation that is constant in time and space. The only difference to the derivation of the spin-less Schrödinger equation is the relativistic behaviour.

Time dilation, combined with the belt trick, leads to a relation that includes the core speed v and half the core spinning frequency ω:

$$\nabla^2\psi = \frac{\omega^2 v^2}{c^4(1-v^2/c^2)}\psi \quad . \tag{20}$$

Inserting the relativistic expression for the observed angular rotation frequency of the tangle phase $\omega/\sqrt{1-v^2/c^2}$ (the core rotates with twice that frequency) yields

$$\nabla^2\psi - \frac{1}{c^2}\partial_{tt}\psi = \frac{\omega^2}{c^2}\psi = \frac{m^2c^2}{\hbar^2}\psi \quad , \tag{21}$$

where mass m is again introduced as a constant that relates the translation speed and the angular frequency of the belt trick. This is the well-known *Klein-Gordon equation.* In other words, as Battey-Pratt and Racey showed, the Klein-Gordon equation follows from Figure 11. The result confirms that in the strand tangle model, the core spinning frequency $\omega = mc^2/\hbar$ due to the belt trick reproduces what Schrödinger called the *Zitterbewegung*.

The differences between the ideas of Battey-Pratt and Racey and the strand tangle model are small. First, the fluctuations inherent in the strand tangle model – and the definition of the quantum phase with crossings – explain that the phase varies all over three-dimensional space, and is not only defined at the location of the rotating core. Secondly, the crossing density also yields an amplitude that is a function of position. In this way, the strand tangle model solves the issues mentioned by Battey-Pratt and Racey in their paper. In other words, the crossing density of a fluctuating strand tangle yields a complete description of the state of a quantum particle with a wave function $\psi(x,t)$ that fully behaves as expected from special relativity.

In short, the strand tangle model *reproduces* the result by Battey-Pratt and Racey: for relativistic spin-less matter particles, tethers imply the Klein-Gordon equation when wave functions are defined by crossing densities. The next task is to add spin.

24 Deducing Pauli spinors and the Pauli equation from tangles

An important requirement for any model of wave functions is that it includes *spin*, and in particular the variation of spin orientation over space and time. The results about tangles derived so far can be used to realize this requirement. This section treats the non-relativistic case.

To extend the wave function by including the orientation of the rotation axis, it is most practical to use the *Euler angles* α, β and γ [142, 143]. The Euler angles allow a description of the crossing density as

$$\Psi(x,t) = \sqrt{\rho}\,\mathrm{e}^{i\alpha/2}\begin{pmatrix}\cos(\beta/2)\mathrm{e}^{i\gamma/2}\\ i\sin(\beta/2)\mathrm{e}^{-i\gamma/2}\end{pmatrix} \quad . \tag{22}$$

Similar to the spin-less case, the crossing density is the square root of the probability density $\rho(x,t)$. Again, the angle $\alpha(x,t)/2$ describes the phase, i.e., half the rotation *around* the axis. The newly added orientation of the spin axis is described by a two-component matrix that uses the two angles $\beta(x,t)$ and $\gamma(x,t)$ shown in Figure 11 and 6. Due to the half angles, the two-component matrix is a *spinor*, as Paul Ehrenfest called it, in analogy to the terms 'vector' and 'tensor'. For the case that $\beta = \gamma = 0$ at all times and all positions, the wave function ψ used in the Schrödinger equation is recovered.

As described by Payne [184], by Penrose and Rindler [1], and by Steane [185], a Pauli spinor can be visualized as a flag on a pole and a sign. The length of the flagpole is described by a positive real number $\sqrt{\rho}$. The length of the flagpole is the *amplitude* of the wave function. The direction of the flag around the pole is described by the first angle. The flag angle around the pole is the *phase* of the wave function; it corresponds to the rotating arrow used by Feynman in his book on QED [144]. These two parameters are the same as those for the spin-zero case. The direction of the flagpole is described by two additional angles. These two angles describe the spin orientation and do not appear in the case of spin-zero particles. Finally, the sign of the spinor indicates whether the flag has been rotated by an even or uneven multiple of 2π.

The strand tangle model for a fermion naturally reproduces spinors and their flag visualization. As before, it is assumed that the tangle core rotates rigidly. As illustrated in Figure 6 and 11, every fluctuating tangle core has a local crossing density, a local average phase, but also a local average orientation of the crossing axis. The crossing density of the tangle core – the 'size' or 'density' of the core – reproduces the positive real number $\sqrt{\rho}$. The orientation of the tangle core – the flag – is described by three angles. Because of the belt trick, the expression for the axis orientation naturally contains *half* angles. The twistedness of the tethers reproduces the sign of the spinor.

The final ingredient is a description of the orientation of particle spin during particle motion. For a propagating wave function, the spin orientation is described by the wave vector $\boldsymbol{k} = -i\boldsymbol{\nabla}$ multiplied by the spin operator $\boldsymbol{\sigma}$. Here, the *spin operator* $\boldsymbol{\sigma}$, for a spin $1/2$ matter particle, is defined as the vector built from the three specific matrices

$$\boldsymbol{\sigma} = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) \quad . \tag{23}$$

These three matrices are called the *Pauli matrices*. The description of the spinning motion and the spin axis can be inserted into the non-relativistic dispersion relation $\hbar\omega = E = p^2/2m = \hbar^2 k^2/2m$. This yields the wave equation

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}(\boldsymbol{\sigma}\boldsymbol{\nabla})^2\Psi \quad . \tag{24}$$

This is *Pauli's equation* for the evolution of a free, non-relativistic quantum particle with spin $1/2$.

Anticipating the inclusion of electrodynamics [85], it is also possible to include the electric and the magnetic potentials, using the minimal coupling to the electromagnetic field deduced below. Minimal coupling implies substituting $i\hbar\partial_t$ with $i\hbar\partial_t - qV$ and substituting $-i\hbar\boldsymbol{\nabla}$ with $-i\hbar\boldsymbol{\nabla} - q\boldsymbol{A}$. This introduces the electric charge q and electromagnetic potentials V and $\boldsymbol{A}$. A bit of algebra involving the spin operator then leads to the Pauli equation for a charged particle

$$(i\hbar\partial_t - qV)\,\Psi = \frac{1}{2m}(-i\hbar\boldsymbol{\nabla} - q\boldsymbol{A})^2\Psi - \frac{q\hbar}{2m}\boldsymbol{\sigma}\boldsymbol{B}\Psi \quad , \tag{25}$$

where the magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ appears explicitly. This equation is famous for describing the motion of non-relativistic silver atoms, which have spin $1/2$, in the Stern-Gerlach experiment. The magnetic field splits a beam of silver atoms into two beams. This effect is due to the new, last term on the right-hand side, which does not appear in the Schrödinger equation. The term is a pure spin effect and implies a g-factor of 2. Depending on the spin orientation, the sign of the last term is either positive or negative. Thus, it acts as a spin-dependent potential. The two options for

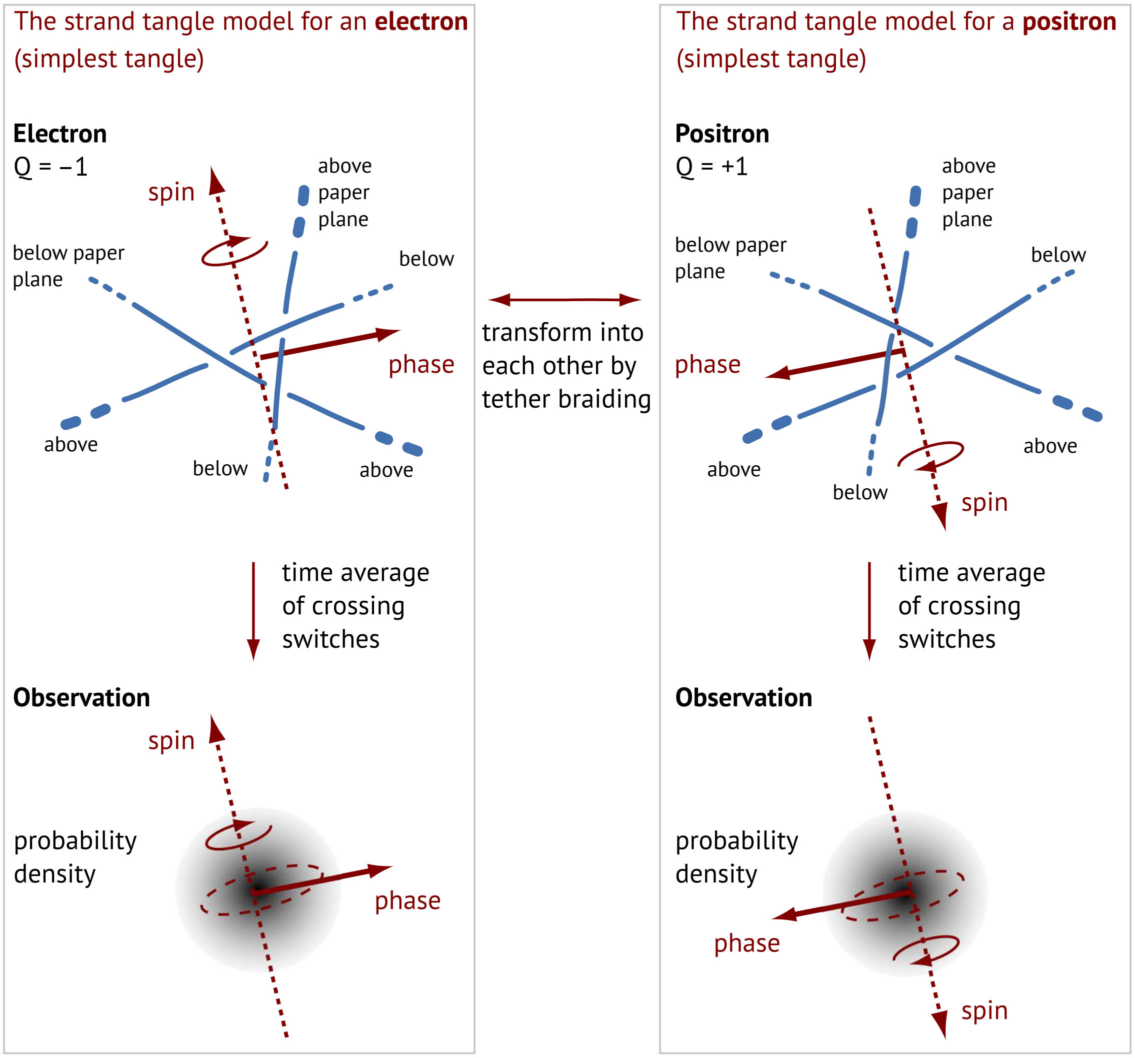

FIG. 24: The simplest tangle for an electron (left) can be *continuously deformed* into the simplest tangle for a positron (right). One way to proceed is to untwist two neighbouring tethers, shift the third strand, and then properly twist the other two tethers of the first two strands. Another way to perform the deformation is to bring all tethers into one plane, then rotate three tethers together against the other three, and then bend all six tethers again out of the paper plane.

spin orientation then produce the upper and lower beams of silver atoms that are observed in the Stern–Gerlach experiment.

In short, a non-relativistic tangle core that rotates continuously and rigidly visualizes the spinors and their flag model. It also *reproduces* the Pauli equation for non-relativistic particles with spin $1/2$. The next step is to explore the relativistic case.

25 Rational tangles describe antiparticles

The explanatory power of the strand tangle model is impressively confirmed for the case of antiparticles. In the tangle model, *antiparticles* are mirror tangles that rotate in the opposite direction. Because particle tangles are also chiral, the strand tangle model correctly models the opposite handedness of particles and antiparticles, their opposite charge and their equal mass.

Figure 24 illustrates the situation using the rational tangles for the electron and the positron. Their tangles will be introduced later on, in Section 31, which also confirms their quantum numbers. The figure allows deducing that the two tangles can be continuously transformed into each other by moving and rearranging the tethers in space. This possibility explains how *rational*, i.e., unknotted tangles can reproduce Dirac spinors, for which the particle–antiparticle content can vary from one position to the next. Such a continuously varying anti-particle content is observed in experiments and is an essential aspect of the Dirac equation. For example, the transformation of particles into antiparticles is at the basis of Klein's paradox [186]. The ability to reproduce this transformation is a further indication that the strand tangle model of particles agrees with reality.

A *continuous* transformation between an electron and a positron can only be reproduced using *rational* tangles. As explained in Figure 5, a tangle is called *rational* if it is unknotted and if it arises purely through the braiding of the tethers. Topologically, the transition between particles and antiparticles is a change in tether braiding. Open knots, prime tangles, and all other knotted tangles do not allow a smooth transition between particles and antiparticles. Only rational tangles, the simplest of all tangles, are natural candidates for describing elementary particles.

In short, only rational tangles of strands allow visualizing the continuous transition between particles and antiparticles. Rational tangles do so by using mirror tangles and tether deformations.

26 Tangles lead to Dirac spinors and the free Dirac equation

The free Dirac equation, describing spin as well as relativistic effects, *follows* from strand tangles. This section summarizes a published quantitative argument and adds two qualitative arguments.

The first, *quantitative* argument is based on the work by Battey-Pratt and Racey from 1980 [71]. They showed that for the relativistic motion of a tethered massive particle, the average size of the belt trick is contracted by the Lorentz factor. When the Lorentz contraction is taken into account, the Dirac equation appears.

In the strand tangle model, the approach of Battey-Pratt and Racey is taken over, with the only difference that the quantum particle is also made of strands. Thus, a particle is a *tangle*. Each particle tangle defines a 4-component *Dirac spinor* $\psi(x)$ as follows:

- ▷ Averaged over a few Planck times, the density of the strand crossings yields the *amplitude* of the wave function, and its square the probability density.
- ▷ Averaged over a few Planck times, the position of the *centre* of the core yields the *maximum* of the probability density.
- ▷ Averaged over a few Planck times, the orientation of the core yields the *spin orientation*.
- ▷ At each position x, the upper two components of the spinor $\psi(x)$ are defined by the local average of finding, at that position, the (tight) tangle, with a given orientation and *phase*. These components determine the orientation of the flag of the particle spinor.
- ▷ At each position x, the *lower* two components of the spinor $\psi(x)$ are defined by the local average of finding, at that position, the (tight) *mirror* tangle, i.e., the antiparticle, with a given orientation and phase. These components determine the orientation of the flag of the antiparticle spinor.

In the strand tangle model, a free fermion advancing through space is described by a constantly rotating rational tangle core moving through space along a straight line. Particles are tangles and thus tethered tangle cores; as such, they behave as assumed by Battey-Pratt and Racey [71]. Therefore, using their argument, rational tangles of strands follow the Dirac equation.

Using the fundamental principle of the strand tangle model, the conclusion of Battey-Pratt and Racey [71] can be rephrased in the following concise way:

▷ The free Dirac equation is the *differential* version of Dirac's trick on a fluctuating rational tangle.

A more precise formulation is as follows:

▷ The belt trick implies the γ^μ matrices and their Clifford algebra, i.e., their geometric algebra properties [187–189].

▷ The first two components of the γ^μ matrices describe the rational tangle core, i.e., the particle, whereas the last two components of the γ^μ matrices describe the rational mirror tangle, i.e., the antiparticle.

The gamma matrices arise from tethers. This is best seen by comparing their Dirac representation with the Pauli representation of SU(2). Both sets of matrices are due to strands. (Reference [111] shows this for SU(2).) It is worth noting that strands imply, together with Part IV of this article, that generalizing Clifford algebras or adding more dimensions does not help in understanding particle physics.

As mentioned, rotating advancing tangles reproduce Feynman's description of a quantum particle as an *advancing* and *rotating* arrow [144] once the tangle is imagined to have a negligible core size. Likewise, a rotating tangle visualizes the description given by Hestenes [187–189] of the free Dirac equation. A rotating tangle also visualizes the flag description of spinors [1, 185, 190] if one idealizes the chiral tangle core as a flag.

A second, *qualitative* argument helps to understand the appearance of the free Dirac equation from strands. The free Dirac equation

$$i\hbar\gamma^\mu\partial_\mu\psi = mc\psi \tag{26}$$

is due to five basic properties of nature:

1. The action limit given by $\hbar$, which yields wave functions ψ.
2. The speed limit for massive particles is given by c, which yields Lorentz transformations and Lorentz invariance.
3. The spin $1/2$ properties in Minkowski space-time.
4. Particle–antiparticle symmetry, where this and the previous point are described by Dirac's γ^μ matrices.
5. A particle mass value m that connects phase rotation frequency and wavelength using the imaginary unit i.

These five properties are necessary and sufficient to yield the free Dirac equation. (The connection between the γ^μ matrices and the geometry of spin was first worked out about a century ago by Fock and Iwanenko [191].) The strand tangle model reproduces these five properties in the following way:

1. All physical observables are due to crossing switches, which imply a minimum observable action $\hbar$ and thus the existence of wave functions.
2. Tethers constrain the tangle cores to advance less than one Planck length per Planck time, which is slower than c (see Figure 2).
3. Tethers reproduce the spin $1/2$ properties for rotation, exchange and boosts, including fermion behaviour.
4. Tethers yield the γ^μ matrices, with tangles and mirror tangles corresponding to particles and antiparticles.
5. Through the belt trick, tethers connect tangle core rotation and tangle core displacement and lead to a finite mass value m.

In other words, both in nature and in the strand tangle model, the inability to observe action values below $\hbar$ leads to wave functions and probability densities. Both in nature and in the strand tangle model, the inability to observe speed values larger than c leads to Lorentz invariance and the relativistic energy-momentum relation. Both in nature and in the strand tangle model, a finite mass value, the spin $1/2$ properties, and the γ^μ matrices arise. This implies the Dirac equation for free particles. For conventional quantum theory, this argument was made by Simulik [192–194]. In the strand tangle model, all of these properties are due to the tethers and the fundamental principle.

The strand tangle model also explains quantum motion in a third, *qualitative* way. In nature, all motion, also quantum motion, can be described with the principle of least action: motion minimizes action. The Dirac Lagrangian specifies how to determine and how to minimize the value of the action of a relativistic fermion. In the strand tangle model, *action* denotes the number of crossing switches. The principle of least action then becomes the *principle of fewest crossing switches.* In the strand tangle model, motion indeed minimizes crossing switches: strand deformations without crossing switches are simpler and preferred. After spatial averaging, crossing switch minimization for spinning and advancing fermion tangles leads to the free Dirac Lagrangian and the free Dirac equation. Still, it is an open challenge to deduce Schwinger's quantum action principle directly from the strand tangle model.

In short, the ideas of Battey-Pratt and Racey imply that rational tangles represent relativistic fermions and follow the free Dirac equation. The simplicity of the fundamental principle and the principle of least action support the conclusion.

27 A second quantitative derivation of the Dirac equation

In the last decades, research has produced several derivations of the Dirac equation. They can be reproduced with the help of strands. To see this, it is useful to rewrite a general Dirac spinor, with its four complex components, in the way given by Loinger and Sparzani [195], as

$$\Psi = \sqrt{\rho}\,\mathrm{e}^{i\delta}\,L(\boldsymbol{v})\,R(\alpha/2,\beta/2,\gamma/2) \quad . \tag{27}$$

Here, $\sqrt{\rho}$ is the amplitude, δ is the fraction of particle and antiparticle. R is a matrix with three real parameters describing the orientation and phase of the spin (or flag), and L is a matrix with three real parameters describing the boost transformation. All quantities depend on space and time. Most parts of a Dirac spinor wave function can be visualized using a relativistic spinning top. In particular, Loinger showed in the 1960s that the amplitude and six further parameters follow from spinning tops [195]. Only δ is not reproduced.

Once Dirac spinors are defined, the simplest derivation of the Dirac equation might well be the one given by Lerner in 1996 [196]. His derivation is based on only two conditions: conservation of the spin current and Lorentz covariance. Lerner showed that together, these two conditions completely and uniquely imply the Dirac equation.

In the strand tangle model, a Dirac spinor is described by the geometry of its rotating tangle core, which can be imagined, because the tethers are unobservable, as a spinning top. The geometry of a spinning top suggests describing a Dirac spinor using the following quantities derived from a spinning tangle core: *one* strand density, *three* angles specifying the rotation or spin axis and the phase of the core (its flag direction), *three* parameters that describe the contracted shape of the tangle core and thus specify the boost direction and magnitude, and *one* angle that specifies the relative weight of particle and antiparticle.

For the strand tangle model, density, rotation, and (flag) orientation of the tangle core were defined and visualized in Part II using crossing densities, in particular in Figure 11. In the relativistic case, the spherical core is changed to an ellipsoid, as illustrated in Figure 23. The matrix L just mentioned thus describes and yields the ellipsoidal shape of a tangle core boosted in a general direction. The relation between rotation frequency and displacement reproduces the mass of the particle. In addition to the visualization that uses spinning tops, a spinning *rational* strand tangle also explains the last phase δ. This phase describes the relative fraction of particle and antiparticle, i.e., of core and mirror core. The extreme cases of the phase are visualized in Figure 24. Thus, the geometry of strand tangles and their crossing densities yields eight real parameters that correspond to the eight real parameters that describe Dirac spinors. These eight real parameters describing tangle cores correspond to the four complex numbers used by Dirac when he wrote down his equation [195, 197, 198].

Thus, spinning rational strand tangles naturally reproduce Dirac spinors. This is possible because free quantum particles behave as tethered spinning tops. In other words,

▷ Free fermions are rotating tangle cores.

Equivalently, a free fermion (and antifermion) can be imagined as a spinning flag (pair).

In the strand tangle model, an advancing particle in a vacuum is visualized either as a tethered propeller or, more precisely, as a tethered spinning top. The definition of spin with tethers implies that particles move in a vacuum at a constant speed, that is, tangle cores move at a constant speed and keep rotating with a constant rotation frequency. In other words, strand tangles imply that *spin current is conserved* over space and time. (Indeed, there is no friction or any other mechanism in the tangle model that can destroy spin current conservation. The strands that make up the vacuum yield a constant effect on free particle tangles: their rotation speed stays constant.) Thus, the first condition used by Lerner is fulfilled by tangles. In addition, the definition of spin using tethers implies the *Lorentz covariance* of spin, i.e., the proper behaviour under rotations and boosts. This property was already confirmed by Battey-Pratt and Racey [71] in 1980. In particular, Lorentz covariance arises because under boosts, tangle cores change shape (in a way that resembles the change of a sphere to an ellipsoid), and the belt trick therefore changes accordingly. In total, both conditions used by Lerner are reproduced by the strand tangle model. Therefore, *strand tangles follow the free Dirac equation.*

The quantitative derivation of the Dirac equation just given, like the three arguments in the previous section, implies:

Test 2: Strands predict the lack of measurable deviations from the free Dirac equation.

Deviations from the free Dirac equation have been searched in detail. So far, for all measurable energies and length scales, none has been found, not even in thought experiments [113].

In short, using the result by Lerner, the relativistic invariance and the conservation of spin current by the belt trick imply the free Dirac equation. *Dirac's equation is due to Dirac's trick.* As a consequence, the strand tangle model with its automatic appearance of antiparticles allows visualizing all relativistic quantum effects. They include the Zitterbewegung, which is due to core rotation. Thus, the strand tangle model agrees with the free Dirac equation and with all measurements so far. Strands yield several additional predictions.

28 Strands imply the Dirac equation despite the lack of trans-Planckian effects

The derivation of the free Dirac equation from tangles of strands implies the lack of *any observable* deviation from relativistic quantum theory.

Test 3: Strands predict that not only the Dirac Lagrangian for free particles but also the Lagrangian for particles *with gauge interactions* is valid at *all* measurable energy scales.

To date, no deviations have been observed. In the case of electromagnetism, this prediction includes Klein's paradox. The prediction implies that the quantum properties of the gauge fields are taken into account.

In apparent contrast, the strand tangle model also predicts the absence of any trans-Planckian effect in nature.

Test 4: Strands imply that *no elementary* particle with energy beyond or equal to the corrected Planck energy $\sqrt{\hbar c^5/4G} \approx 6 \cdot 10^{18}\,\mathrm{GeV}/c^2 \approx 1\,\mathrm{GJ}$ will be observed. A similar upper limit also applies to the linear momentum and the mass of every elementary particle.

Test 5: Strands imply that *no* system or particle will ever exceed or achieve the limits for frequency, acceleration, density, pressure, area, volume, temperature, length or time given by the corrected Planck limits of relativistic quantum gravity.

As explored in Section 1, in contrast to the everyday Planck limits, the limits of relativistic quantum gravity have not been approached by several orders of magnitude. As a consequence, the Planck limits due to relativistic quantum gravity are not in contrast with the Dirac equation in any observation, even though the corrected Planck energy poses an effective limit to boosts.

In short, strands predict both the lack of measurable deviations from the Dirac equation and the lack of any measurable trans-Planckian effect, in agreement with all observations, and despite the apparent contradiction between the two statements.

29 Strands predict the lack of other models for wave functions

Strands imply that wave functions are blurred tangles and that tangles are fluctuating skeletons of wave functions. This statement was deduced and tested in the previous sections. However, step, by step, without noticing, strands made a stronger statement.

Test 6: The strand tangle model predicts that *no alternative* microscopic model of wave functions – that describes gauge interactions – is possible. If any inequivalent alternative would be found, the tangle model would be falsified.

The prediction is due to the explanation of spin $1/2$, of fermion behaviour, and of the free Dirac equation. Describing the motion of the space between strands, as proposed by Asselmeyer-Maluga [79, 80], would be an *equivalent* microscopic model. Strands claim to be the *unique* model for emergent wave functions. This uniqueness claim of the strand model is phrased provocatively for two reasons.

First, the uniqueness claim aims to motivate the search for proof or for a counterexample. On the one hand, the lack of alternative descriptions so far should not stop the search in the future. On the other hand, the uniqueness of strands was already implicit in Section 3, when it was suggested that all of nature can be described by unobservable tethers, observable crossing switches, and Planck limits. The uniqueness claim implies that it should be possible to prove that each step taken in this article is logically unavoidable, without any alternative. Some aspects of strand uniqueness are discussed in Appendices D, E, F, and G.

Secondly, the uniqueness claim contains a precise experimental prediction. If any other model for wave functions differs in its consequences from the strand tangle model, observations will falsify the other model and confirm the strand tangle model. Equivalently, there are no measurable deviations from the strand model. This prediction aims to motivate precision experiments of every possible type.

As mentioned in the introduction, the literature on emergent quantum theory is vast. Exploring the other approaches, including those by Adler [199], 't Hooft [200], Elze [201], de la Peña et al. [202], Blasone et al. [203], Grössing [204, 205], Acosta et al. [124], Hollowood [206] and Torromé [207], two conclusions appear to arise. First, several proposals appear to be compatible with the strand tangle model. Secondly, so far, no approach contradicts the uniqueness claim. These conclusions should be tested in future research.

In short, the strand tangle model predicts that it is the only possible model for emerging quantum theory. However, despite the agreement between strands and observations, several differences arise. These differences are explored next.

Part IV: Differences to conventional quantum theory

Thus far, the strand tangle model deduced from the fundamental principle has simply *reproduced* conventional quantum theory. If this were the only result of the strand tangle model, the model would be unnecessary because Occam's razor would speak against it. However, the strand tangle model and quantum theory *differ*. The differences are due to the possibility of *classifying* both particle tangles and their core deformations. It turns out that *elementary* particles are the *simplest possible* tangles of strands:

▷ Elementary particles are *rational* tangles of one, two or three strands.

The concept of rational tangle was introduced in Figure 5. Rational tangles arise by moving tethers around in space. Rational tangles are three-dimensional generalizations of braids. Therefore, they do not contain knotted regions but do contain tangled cores. It will appear that modelling elementary particles as rational tangles *explains* and *fixes* their spectrum, their interactions, their spins, charges and other quantum numbers, their masses, and all their other particle properties.

In addition, it will turn out that the interactions of particle tangles can be classified. All observed gauge interactions arise in this way. It will appear that rational tangles *derive* and *fix* the gauge interactions, their Lie groups, and their coupling constants.

The derivation of particles and interactions provided by strands is *hard to vary*, thus realizing Deutsch's requirement for a good science explanation [208]. Tangles of strands have an explanatory power that continuous wave functions lack. This explanatory power does not arise in any other proposal in the research literature and is the real reason for exploring the strand tangle model.

In short, the strand tangle model promises to restrict quantum theory to a specific set of allowed elementary particles and to a specific set of allowed interactions. The strand tangle model can be tested by comparing the allowed particle properties, including mass values, and the allowed interaction properties, including the gauge groups, with experiments. This comparison is performed in the following.

30 Particle mass is due to chiral tangles

According to experiment – and to textbook quantum theory – when a particle advances, the quantum phase rotates.

▷ The (inertial) mass m describes the coupling between translation and phase rotation.

A *larger* mass value implies, for a given momentum or energy value, a *slower* translation.

In the strand tangle model, particle translation and rotation are modelled by the translation and rotation of the tangle core. Figure 2 illustrates *why* core translation and rotation are coupled. When the core moves through the vacuum, the vacuum strands and the tangle core touch and move each other because of their impenetrability. This results in a core motion that resembles that of a tethered asymmetrical, i.e., chiral body – a propeller or spinning top – moving (with a low Reynolds number) through a fluid. When an asymmetrical body (without or with tethers) moves through a (viscous) fluid, it starts to rotate. For example, this occurs when a chiral pebble falls through water. This rotation is due to the asymmetrical shape of the body [209–212]. Almost all of the tangle cores of the elementary particles deduced below are chiral. This also applies to the d-quark once Higgs braids are taken into account. The same occurs for the Higgs boson itself.

Tethers do not prevent the rotation of asymmetrical bodies, as shown in Part I. Strands imply that tangle cores rotate when they move through the vacuum. Tethers yield a coupling between translation and rotation. Thus, the strand tangle model implies

Test 7: Tethered asymmetric, or chiral, rational tangle cores, such as localized cores composed of two, three, or more strands, are predicted to be *massive particles*. A tangle is *localized* or *localizable* if the core collapses to a tight tangle when the tethers are imagined to be ropes that are 'pulled outwards'.

Test 8: The mass value for tangles made of a few strands is *not arbitrary*, but is uniquely determined by the tangle structure and by the resulting average core *shape*.

Test 9: Particle masses are thus *calculable* – when the tangle topology is known.

Test 10: The *more complex* the tangle core – for the same number of tethers – the slower the translation per rotation, and thus the *larger* the predicted mass value.

Test 11: *Unlocalized* tangles – which disappear if their ends are pulled outwards, such as a loop in a single strand – are predicted to be massless, even if they are chiral.

These predictions about particle mass sequences agree with the fermion tangles of Figure 25 and with the boson tangles of Figure 33. Above all, these predictions agree with observations. In particular, the predictions agree with the quark model and with the observed mass sequences of mesons and baryons [86]. The mass of neutrinos also arises. But there is more.

In the strand tangle model, the highest possible energy value for an elementary particle, the corrected Planck energy, is one crossing switch per corrected Planck time. The spontaneous belt trick of a tangle core is less probable by far.

Test 12: The complex motion of the belt trick implies a low probability for the involved strand movements. Strands therefore predict that the mass m of *elementary* particles – thus of particles made of one, two or three strands – is much smaller than the corrected Planck mass:

$$m \ll \sqrt{\hbar c^5/4G} = 6.1 \cdot 10^{27}\,\mathrm{eV}/c^2 \quad . \tag{28}$$

Thus, the strand tangle model solves the *mass hierarchy problem* in particle physics. This mass inequality also agrees with the *maximon* concept introduced long ago by Markov [213]. Above all, the mass inequality agrees with experiments. The inequality is also valid for the product γm of mass and Lorentz factor for elementary particles. Again, this is in agreement with observations.

In short, the strand tangle model solves the hierarchy problem and predicts that elementary particle mass values are unique and can be calculated. Calculations of mass values require three steps: tangles must be classified, the assignment of tangles to the known elementary particles must be clarified, and a precise calculation method for the probability of the belt trick for a given tangle must be developed.

31 Classifying tangles leads to the spectrum of elementary fermions

This and the following sections summarize how classifying rational strand tangles leads to the observed spectrum of elementary fermions and bosons, with their observed quantum numbers, as told in references [83–85]. As shown above, only rational tangles describe the appearance of antiparticles and realize the Dirac equation. In the strand tangle model, the rational tangles of *elementary particles* can consist of only one, two or three strands. More than three strands imply *composite* particles, such as protons, or even more complex systems, such as atoms or solids.

The proof of the composition statement can be deduced from the overview of elementary fermion tangles in Figure 25 and of the elementary boson tangles in Figure 33. No *simpler* rational tangles are missing. Any *more complex* rational tangle, with *more strands* than those in the two figures, falls into one of the following two cases:

- The more complex tangle can be a tangle already found in the two figures. For example, a photon tangle that is extended with an additional strand can yield, depending on the details, a graviton tangle, a quark tangle or an (unbroken) W_i tangle. Similarly, a strand added to a quark tangle can lead to a lepton tangle. These cases thus lead from one elementary particle to another.

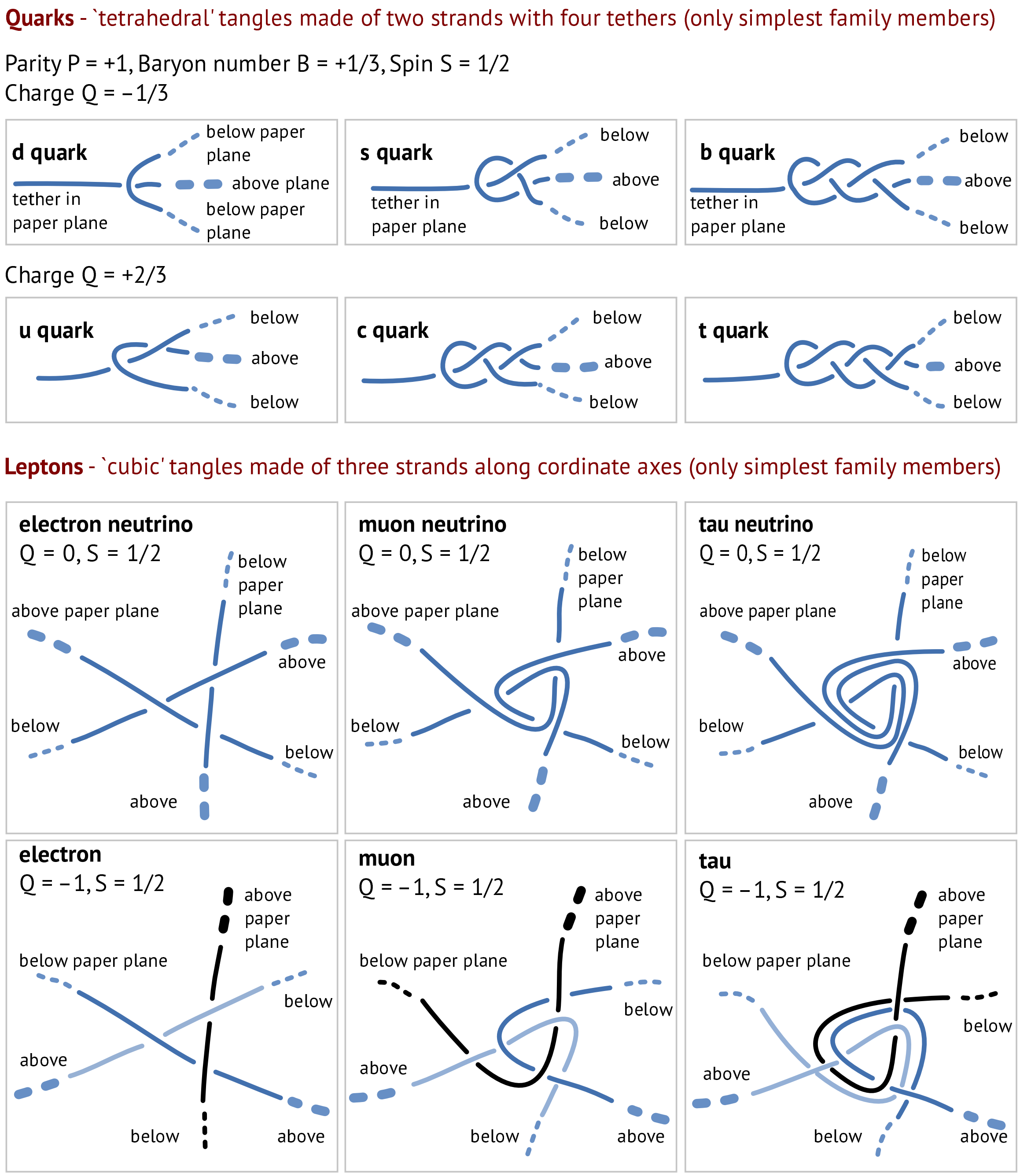

FIG. 25: The figure shows the simplest conjectured tangle for each elementary fermion. Elementary fermions are rational tangles that are formed by braiding tethers. All elementary fermion tangles consist of two or three strands. The tangles yield spin $1/2$ and fermion behaviour. The cores are localized, realize the belt trick, and thus yield positive mass values. The localized cores lead to Higgs coupling, as illustrated in Figure 35. At large distances from the tangle core, the four tethers of the quarks follow the axes of a tetrahedron, and the six tethers of the leptons follow the coordinate axes. The neutrino cores are simpler when observed in three dimensions: they are twisted strand triplets. The tangles of the electron, muon, and tau are topologically chiral and thus electrically charged. Neutrino cores are geometrically chiral, but not topologically chiral; thus, they are electrically neutral. All massive particles have additional, more complex tangles in addition to those shown here, shown in Figure 26 and Figure 27, and form three generations. No additional elementary fermions appear.

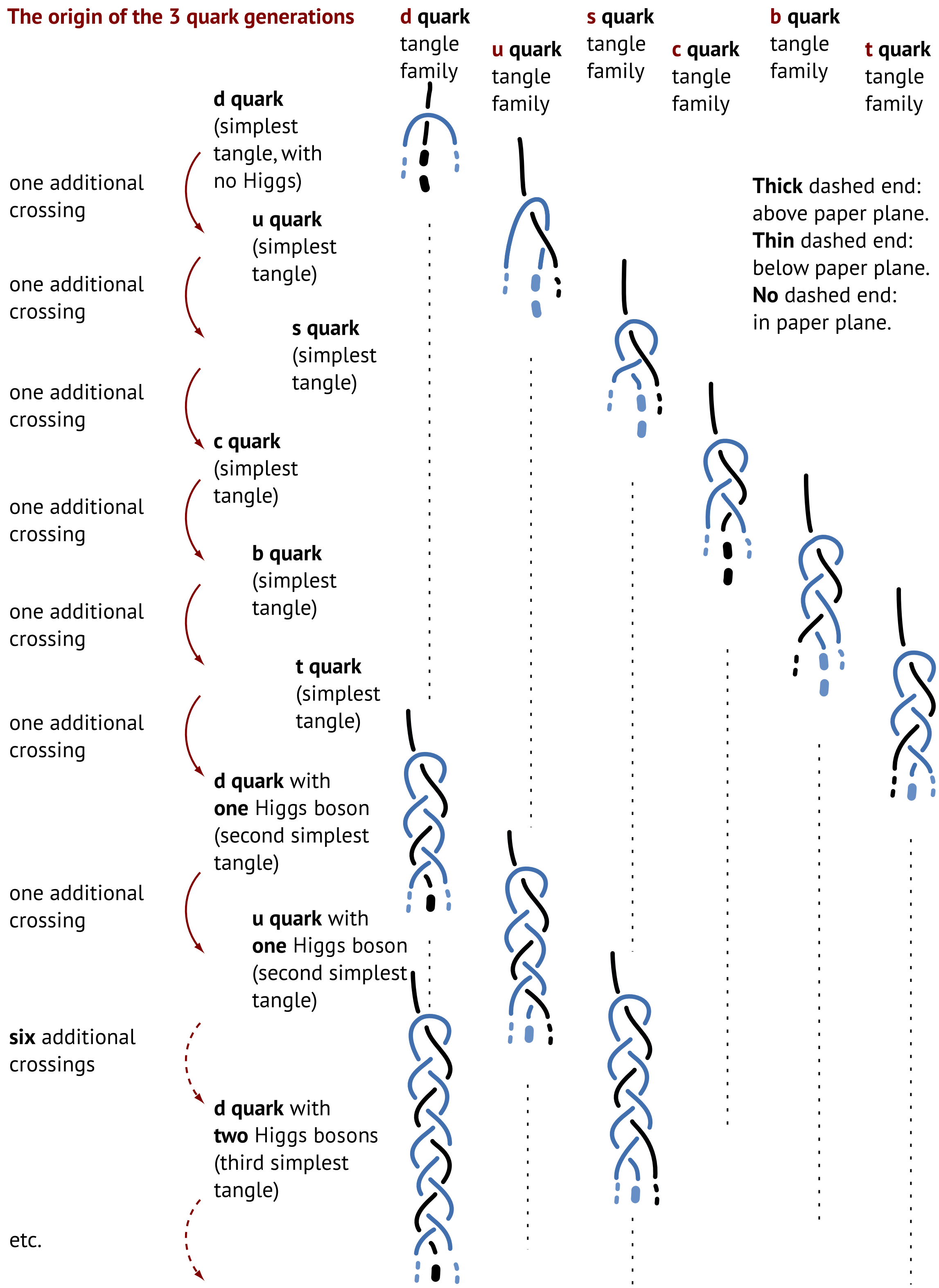

FIG. 26: Each *quark* is a tangle family of two strands, with an infinite number of tangles. The six families define the six quark types and the three generations. The same classification arises for anti-quarks, which are represented by the respective mirror tangles; they are not shown here. In the strand tangle model, the number of generations is thus related to the structure of the Higgs braid, which is itself a result of the three dimensions of space. (Figure improved from reference [86].)

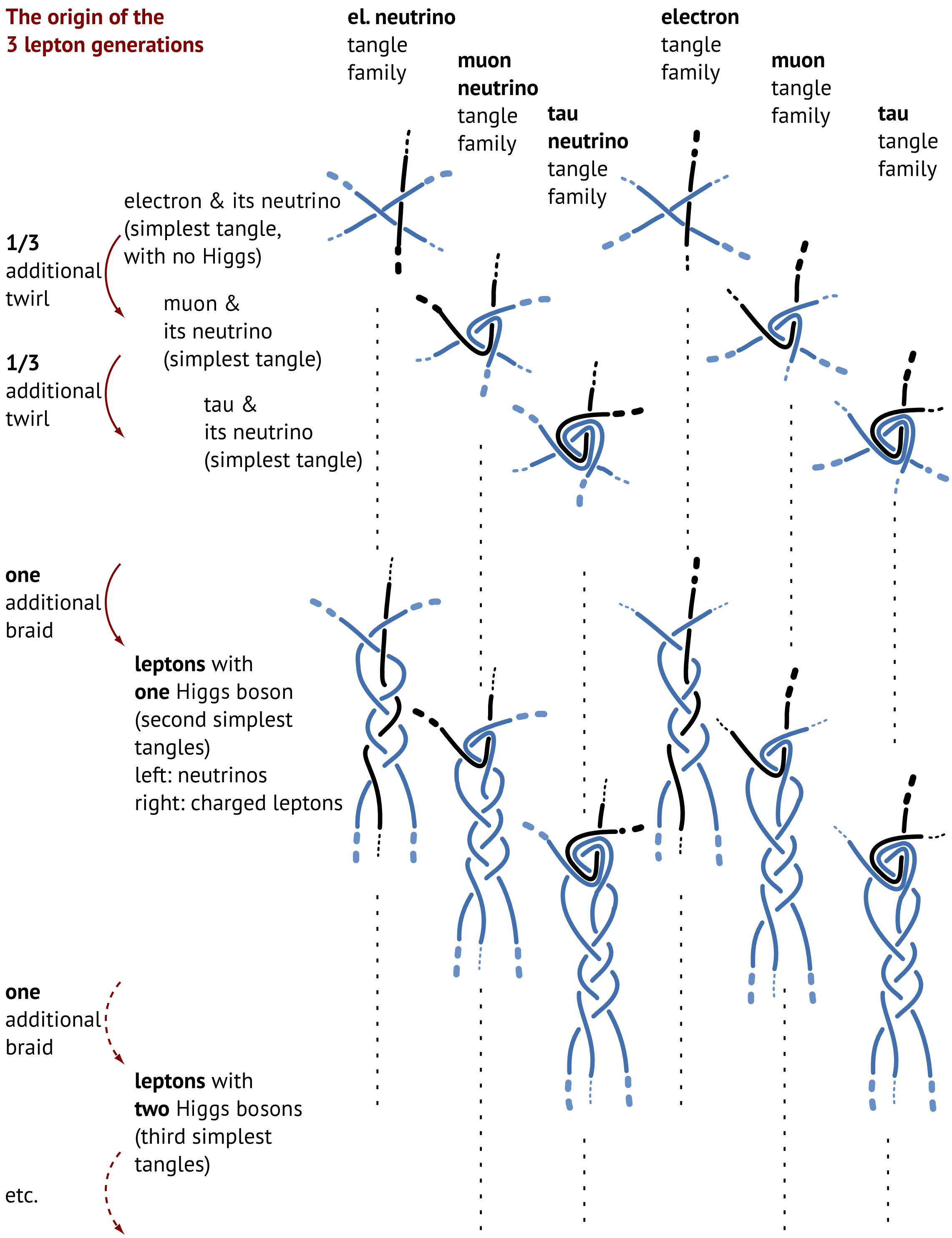

FIG. 27: Each *lepton* is a tangle family of three strands, with an infinite number of tangles. The simplest tangles for each lepton are shown at the top of the figure. The tangles due to one added Higgs boson are shown further down. The six families define the six lepton types and the three generations. Anti-leptons are represented by the corresponding mirror tangles; they are not shown.

- The more complex tangle can be taken apart into simpler tangles. For example, this occurs if a strand is added to a gluon tangle or a W tangle. The resulting four-stranded core consists of two cores of two strands each. The same occurs if a strand is added to a lepton; such a four-strand tangle can also be separated into two simpler localized cores. Likewise, adding a strand to a graviton leads to the possibility of separating the resulting tangle into several elementary bosons.

As a result, all elementary tangles have at most three strands. Specifically, *elementary fermions* consist of either two or three strands. One-stranded rational particle tangles can neither have spin $1/2$ nor have mass, because the belt trick does not apply to them, as argued at the beginning of this article. It turns out that

▷ Two-stranded fermions are *quarks*; three-stranded fermions are *leptons*.

The simplest rational tangles for elementary fermions are shown in Figure 25. Simpler rational tangles made of two or three strands are not possible. One notes that the cores of the heavier charged leptons can be seen as combinations of electron cores and W cores, as is expected from their decay behaviour. Likewise, the cores of the heavier quarks can be seen as combinations of lighter quark cores and W cores. The six tails of the leptons blend into the vacuum strands and allow them to appear as free particles. The four tails of the quarks prevent them from arising as free particles; they need to form composites in order to appear as free particles. This yields the quark model of hadrons, which is presented elsewhere [86].

All massive elementary particles also have *additional* tangles, as illustrated in Figure 26 for quarks and in Figure 27 for leptons. Each massive elementary particle is described by an infinite *family* of tangles that contains the simplest possible core, the simplest core plus one Higgs braid, the simplest core plus two Higgs braids, etc. Thus, the strand tangle model reproduces Yukawa coupling as an important aspect of particle mass.

The structure of the Higgs braid limits both quarks and leptons to three generations. Figure 26 shows how the infinite class of quark-like tangles is split into six infinite families, corresponding to three generations with two quarks each. In other words, the infinitely many possible quark and antiquark tangles consist of $6+6$ separate infinite tangle families. Each family is an infinite series of tangles. In a family, each tangle differs from the next by one Higgs braid. A similar structure of six tangle families arises for the leptons. They are illustrated in Figure 27.

An important check for the tangle–particle assignments are the resulting quantum numbers. In nature, quantum numbers lead to conservation laws that restrict particle reactions.

Baryon and lepton number simply count the corresponding tangles. (Tether braiding could model non-perturbative effects, including baryon number non-conservation and sphalerons, that arise in the early universe.)

Spin was already discussed in Part I. Because elementary fermions are composed of only two or three strands, one obtains:

Test 13: The strand tangle model implies that all fermions are massive.

Test 14: Strands imply that there is no elementary fermion with spin $3/2$ or larger.

The first statement confirms that neutrinos are massive Dirac fermions. It agrees with all experiments so far. Also the second statement, which contradicts supersymmetry, agrees with all experiments so far. Indeed, all observed fermions with higher spin values, such as several nuclei, are *composed*. All *elementary* fermions have spin $1/2$.

Flavour quantum numbers count the number of tangles of a specific flavour (or family). Quark tangles automatically provide the correct values by counting the respective cores. Flavour change is achieved by tether braiding. This process only occurs in the weak interaction, because only the weak interaction moves tethers against each other. In the strand tangle model, only the weak interaction can braid and unbraid tethers. The strand tangle model thus reproduces the observation that only the weak interaction can change quark flavours, and thus lead to quark mixing. Similarly, the strand tangle model reproduces the observation that only the weak interaction leads to neutrino mixing. The similarity between the tangle of the electron neutrino and any section of the vacuum tangle has important consequences for dark energy. This topic will be explored elsewhere.

Charge parity C is the topological chirality of a tangle core. This assignment reproduces all the observed C parities of the elementary particles.

Electric charge is the number of crossings involved in the topological chirality of the tangle, taking the crossing signs into account [85]. Each crossing yields a charge $e/3$ or $-e/3$, depending on its sign. This assignment reproduces all observed electric charge values of the elementary particles. Electric charge is automatically conserved. Magnetic charge cannot arise.

Parity P describes the behaviour of a tangle under spatial reflections. This assignment reproduces all the observed P parities of the elementary particles. (Parity violation by the weak interaction is discussed below.)

Time reversal T changes the spin, i.e., the rotation direction of a tangle. This assignment reproduces the observed behaviour under time reversal of all elementary particles. The CPT theorem is automatically satisfied, as observed.

Weak and strong charge are also explained for each fermion and boson, as explored in references [84] and [86]. The charges explain the interactions to which the corresponding particles are subjected. All observations about weak and strong charges are reproduced.

In the strand model, quantum numbers are *topological invariants*. The topological basis for quantum numbers is the general reason that they are integers. The above strand definitions also explain why certain quantum numbers are additive, whereas others are multiplicative.

Using the tangle model of the strong interaction, the quark–tangle assignments in Figure 25 fully reproduce the quark model of hadrons, as shown in references [84] and [86]. The allowed and forbidden meson quantum numbers are explained. The meson tangle structures also provide natural and correct retrodictions of which mesons violate CP symmetry. The hadron tangles also reproduce all meson and baryon mass sequences.

No other elementary fermions appear topologically possible.

Test 15: Strands predict the lack of contradictions between tangle properties and observed particle properties, such as forbidden values of quantum numbers, new quantum numbers or unexpected non-conservation. For example, millicharged particles, leptoquarks, Majorana particles, or weakly charged particles without mass are predicted not to exist.

Test 16: Strands predict the lack of any unknown energy scale in high-energy physics.

Test 17: Strands predict the lack of any substructure in elementary particles that *differs* from tangles of strands. This includes the preons and ribbons discussed in Appendix D, the superstrings discussed in Appendix E, Möbius bands, prequarks, knots, tori, loops, and any other localised or extended substructure [102, 214–217].

Test 18: Strands predict the lack of any new elementary fermion of any kind. This includes the lack of sterile neutrinos, magnetic monopoles [218], and supersymmetric partners.

Any experimental counter-example would falsify the strand tangle model. The last prediction will be extended to a full prediction about dark matter below.

It should be noted that the strand tangle particle model of fermions is compatible with several particle models deduced from general relativity with and without torsion. In models such as those of Popławski [219] or of Burinskii [220, 221], an elementary fermion is assumed to be a Planck-scale *torus*. In the tangle model, Figure 24 shows that an electron can be seen – once the unobservable tethers are ignored and only crossings are kept – as three spinning crossings. In practice, this yields a structure similar to a fuzzy torus that is always larger than a few Planck lengths and that can be as large as a Compton wavelength. The same properties are deduced in the cited papers on torus models.

In short, in the strand tangle model, the classification of elementary fermion tangles leads to the three generations of leptons and quarks. Every observed quantum number is due to a *topological* property of particle tangles. No additional elementary fermion appears possible. As a consequence, the fermion content due to strands cancels the anomalies of the standard model, as required [222, 223]. In contrast to quantum numbers, the fundamental constants – mass values, mixing angles and coupling constants – are due to *geometric* properties, namely to the (average) *shape* of tangles. All statements deduced from the strand tangle model agree with observations.

32 Classifying tangle deformations leads to interactions and gauge groups

Thus far, only *free* particles have been described with tangles. Free particle motion and free Lagrangians arise when tangle cores rotate and move as *rigid* structures. In free particle motion, only the *tethers* fluctuate by continuously changing their shape. The next step is to explore the *interactions* of particles and to determine their properties. The present section provides a summary of previous papers on the strand tangle explanation for the origin of the gauge groups, the origin of quantum electrodynamics, the origin of quantum chromodynamics with the quark model, and the origin of the standard model [83–86, 111, 112].

In line with the geometric effects of Hermitian operators, gauge interactions arise when the tangle cores do not behave like rigid structures:

▷ Gauge interactions are deformations of tangle cores.

This statement is of interest because all tangle core deformations are local and can be classified. The classification is possible with the help of the moves published by Reidemeister in 1927 [135]. There are only three possible Reidemeister moves: *twists*, *pokes*, and *slides*. These three moves are illustrated on the left side of Figure 28. Every tangle core deformation at a given position is composed of the three Reidemeister moves. Exploring the three Reidemeister moves yields a surprising result: the three moves generate the local gauge groups U(1), (unbroken) SU(2) and SU(3) [111, 112]. Each of the three Lie groups also corresponds to a local gauge symmetry. In addition, each Reidemeister move generates a gauge boson tangle.

Figure 28 shows how any approaching gauge boson core deforms a fermion core. The gauge boson is absorbed and disappears, and the induced local fermion core deformation leads to a local phase change in the fermion core. Exploring the details yields complete agreement with

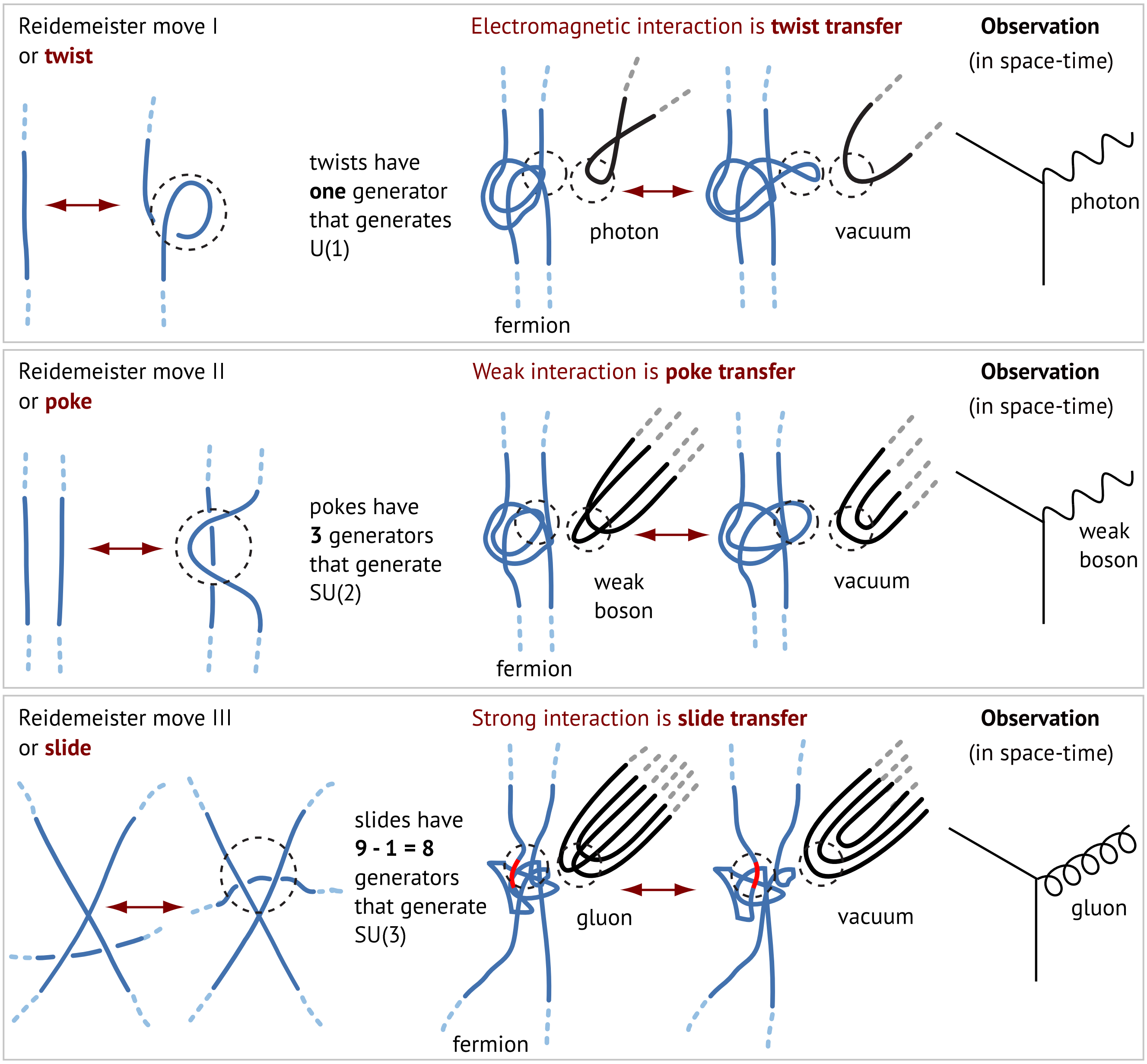

FIG. 28: In an interaction, each gauge boson rotates the bent strand segment enclosed by a dotted circle. The possible observable deformations of tangle cores are classified by the three *Reidemeister moves*. The three Reidemeister moves specify the generators of the three observed gauge interactions and determine the generator algebras, as shown in the subsequent figures. The full gauge group arises when the rotations by the angle π induced by the generators are generalized to arbitrary angles. As a result, the three Reidemeister moves determine the three gauge groups [83]. (Figure improved from reference [84].)

experiments. In particular, modelling interactions as core deformations implies minimal coupling, as the deformation depends on the density of arriving gauge bosons. Minimal coupling and local gauge symmetry lead to the usual Lagrangians of QED, of QCD, of the weak interaction with its symmetry breaking arises, and thus to the full Lagrangian of the standard model [83–86].

Test 19: Strands predict the lack of the tiniest deviation from minimal coupling, for any gauge interaction, at *any* measurable energy or scale.

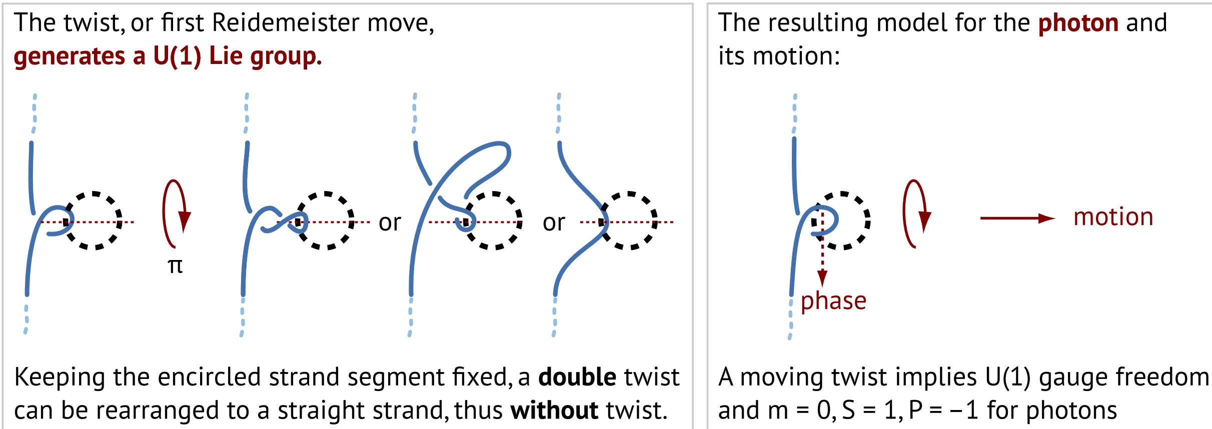

FIG. 29: The *twist,* the first Reidemeister move, generates the local Lie group U(1). When twists – rotations of the region inside the dotted circle by π – are generalized to arbitrary angles, they yield the group elements. The twist also implies the strand tangle model for the photon.

This prediction agrees with all observations [62]. The result can be visualized with strands.

Figure 29 shows that a twist, or first Reidemeister move, can be visualized as the local rotation by π of a strand segment. In the figure, the segment is enclosed by a dotted circle that can be imagined as a transparent plastic disc glued to it. *Generalized* twists can be seen as local rotations by an *arbitrary* angle of the segment or disc. One notes directly that performing a *double* twist is equivalent to *no* rotation. It is straightforward to check that the set of generalized twists also fulfils all other axioms of the U(1) Lie group [111]. The freedom in defining the twist phase yields local U(1) gauge symmetry.

Core twists reproduce quantum electrodynamics in all its details, as shown in [85]. Figure 30 illustrates a selection of processes involving virtual particles. Perturbative QED, including the running of the fine structure constant, appears automatically [85]. In the limit of many photons, classical electrodynamics arises from strands. *Electric fields* are volume densities of virtual photons, i.e., of bound twists. *Magnetic fields* are flow densities of (bound) twists. In other words, photon exchange or twist exchange implies that the *electromagnetic field* is defined by the space-time rotation rate that it induces on an electric charge. This leads to minimal coupling and visualizes the descriptions by Feynman [144], Hestenes [187–189], and Baylis [197, 198]. *Electric charge* results from topological tangle chirality and is conserved.

Test 20: No massless, electrically charged elementary particle will be found.

Test 21: Not the slightest deviation from the local U(1) gauge group or from quantum electrodynamics, at *any* measurable energy, will be observed.

Thus far, all observations have confirmed the strand tangle model of electromagnetism [85], which fully reproduces quantum electrodynamics. Grand unification is ruled out. For example, Furry's theorem is automatically realized by strands. First estimates of the fine structure constant agree with measurements [85].

In the tangle model, the gauge group SU(2) arises from *pokes*, that is, from the second Reidemeister move. Pokes can be seen as local rotations by the angle π. Again, pokes are best visualized by imagining that the dotted circle encloses a transparent plastic disc that contains, in this case, *two* parallel strand segments. The three possible pokes are illustrated in Figure 31. Pokes be-

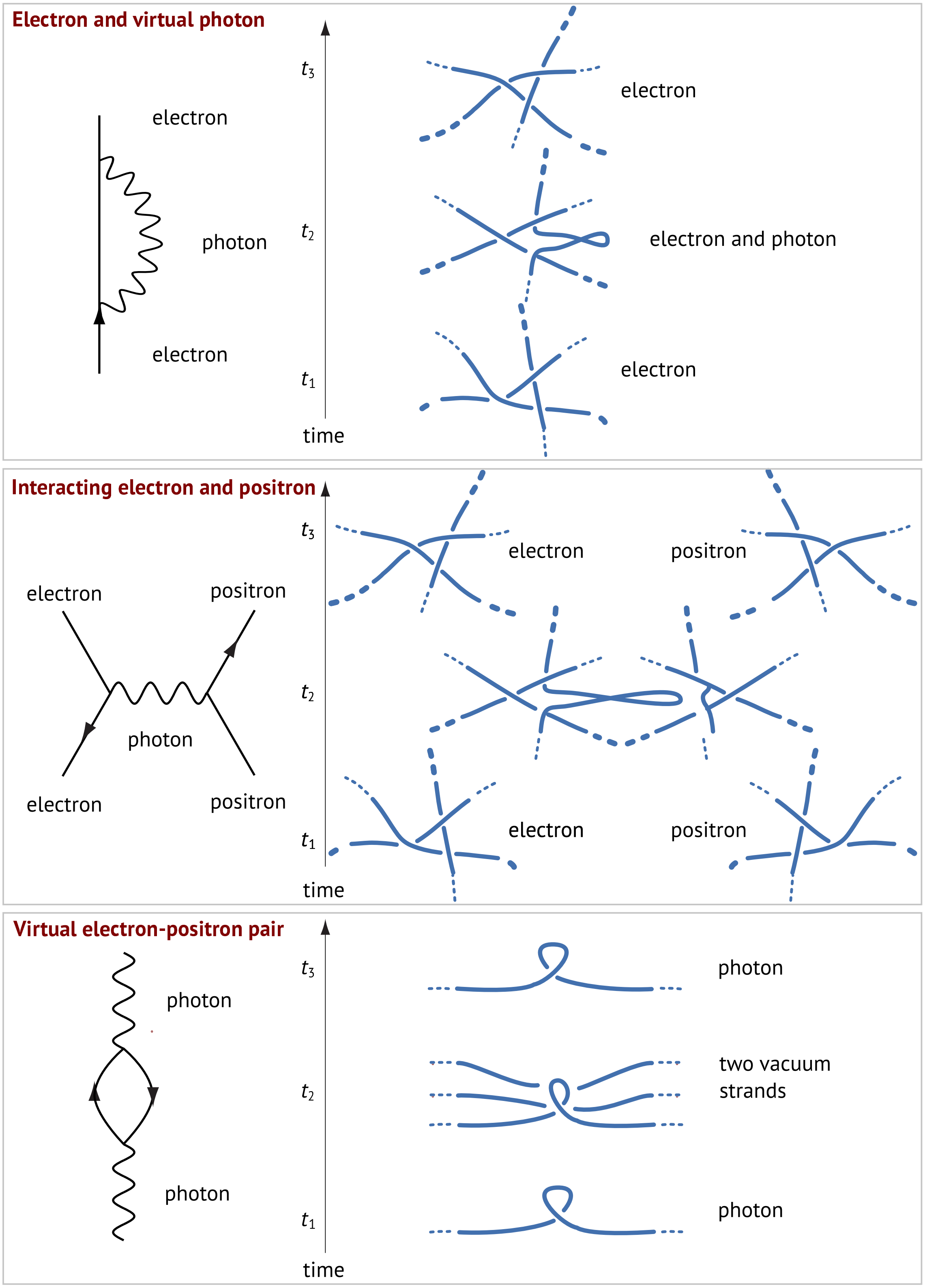

FIG. 30: Three processes from quantum electrodynamics are shown in their strand representation. Top: a free electron emitting and absorbing a virtual photon. Centre: two electrons interacting via a virtual photon. Bottom: a photon generating a virtual electron-positron pair. No difference to quantum electrodynamics occurs at any measurable energy.

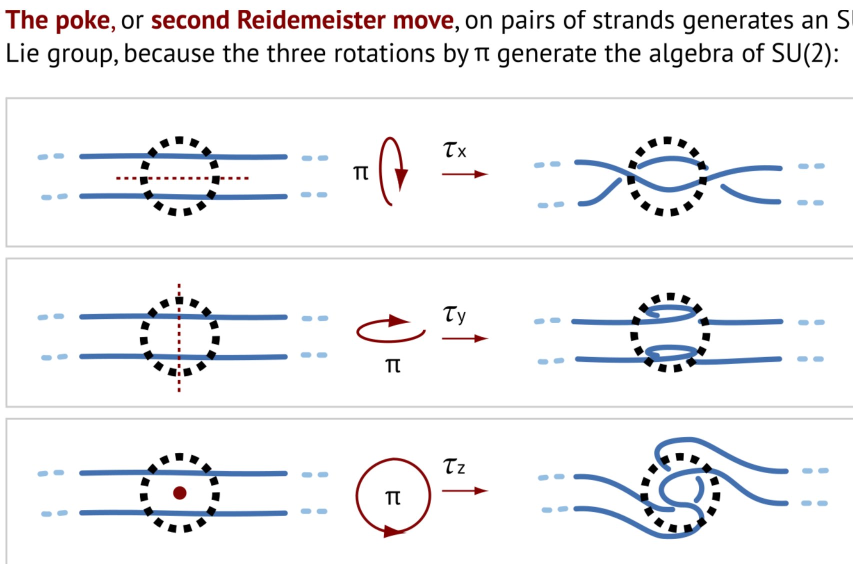

FIG. 31: The three types of *pokes*, second Reidemeister moves, are visualized. Pokes can be modelled by rotating by the angle π a region – inside the dotted circle – containing both strands. (Other visualizations, deforming only one strand, are also possible.) The three pokes generate the Lie algebra of SU(2) and imply a model for (unbroken) weak bosons. The Pauli matrices can be read off directly [111].

have like a belt buckle and form the *algebra* of SU(2), which is generated by the three possible rotations by π of the plastic disc [83, 111, 112]. Indeed, Figure 31 implies that the three pokes behave under concatenation in the same way as i times the Pauli spin matrices under multiplication [111]. *Generalized* pokes are defined as local rotations by an *arbitrary* angle of the two strand segments enclosed by the plastic disc. It is straightforward to verify that generalized pokes realize the SU(2) Lie group axioms [111]. The freedom in defining phases yields the corresponding local gauge symmetry. Together with the minimal coupling illustrated in Figure 28, this yields the full (unbroken) weak interaction Lagrangian.

Strands imply that only massive fermions can exchange weak bosons: only massive particles interact weakly. For example, strands imply and predict massive neutrinos. Due to the tangle structure of particles, *maximal parity violation* arises: parity violation occurs because core rotations due to spin $1/2$ resemble the core deformations due to the group SU(2) of the weak interactions [83, 84]. In addition, SU(2) *breaking* arises: when a vacuum strand is included in the unbroken, massless weak bosons, it leads to the W and Z boson tangles [83, 84]. Also the Higgs mechanism is reproduced. Particle *mixing* is a consequence of the strand tangle model of the weak interaction: it arises from tether braiding, which itself is a specific type of poke deformation.

Test 22: Strands predict the lack of deviations from the weak interaction properties of the standard model with massive mixing Dirac neutrinos, at *any* measurable energy.

So far, all observations confirm the strand tangle model of the weak interaction, which completely reproduces the conventional description of the standard model of particle physics and the corresponding observations [62].

In the tangle model, the local gauge group SU(3) arises from *slides*, the third Reidemeister moves. The set of all slides is illustrated in Figure 32. Slides are best visualized by imagining that the dotted circle encloses a transparent plastic disc that contains, in this case, two crossing

Slides, or **third Reidemeister moves**, acting on strand pairs in three-strand structures, can be generalized to the generators of the Lie group SU(3).

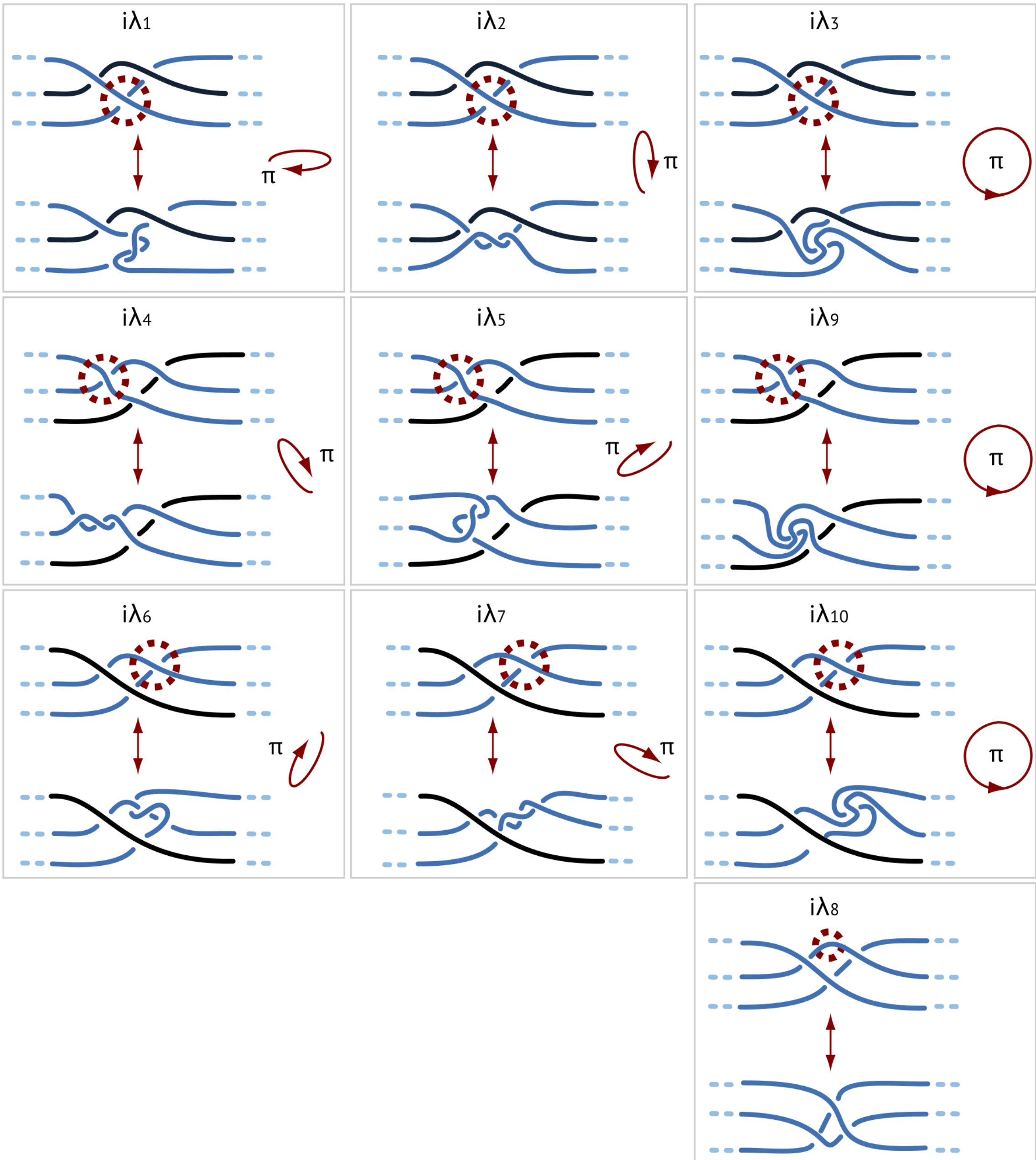

FIG. 32: The ten important deformations deduced from the *slide*, the third Reidemeister move, are visualized. Slides can be modelled by rotating the region inside the dotted circle by the angle π. Each triplet row defines an SU(2) subgroup. (Other visualizations are also possible, e.g., by deforming only one strand.) With the definition of λ_8, the eight slides $i\lambda_1, \ldots, i\lambda_8$ (thus without $i\lambda_9$ and $i\lambda_{10}$) generate the Lie group SU(3) and imply a model for gluons. Each of these eight deformations corresponds to i times a Gell-Mann matrix [111].

strand segments. Exploring the algebra of the local rotations by π numbered from 1 to 8 in the figure, one finds that they behave like i times the Gell-Mann matrices [111]. (Slides 3, 9, and 10 are linearly dependent; it is conventional to use only slides 3 and 8 instead. Slides 9 and 10 help to visualize the three SU(2) subgroups of SU(3).) In other words, the first eight slides generate the Lie algebra of SU(3). *Generalized* slides, or *generalized* third Reidemeister moves, are rotations by an *arbitrary* angle of the two crossing strand segments enclosed by the plastic disc. These generalized slides realize all the required axioms and yield the full, local SU(3) Lie group [111]. This is the main result of previous publications [83, 84, 111, 112]. The implied freedom of phase definition yields local SU(3) gauge symmetry.

As shown in reference [86], the strand tangle model for the strong interaction implies the quark model for hadrons, in all its details. Colour fields are densities of virtual gluons. Modelling the strong interaction as gluon exchange implies minimal coupling. The gluon tangles yield glueballs, with their observed quantum numbers. The strand tangle model of hadrons also yields their correct quantum numbers. The tangle model of hadrons yields the correct sequence of tangle complexity and thus yields the correct mass sequences among hadrons. The strand tangle model also implies that the strong interaction *cannot* produce CP violation. The colour charge of a quark tangle is given by the orientation of its three-ended side in space. Due to their strand structure, gluons carry a double colour charge. As a consequence, colour charge is conserved.

Test 23: Not the smallest deviation from quantum chromodynamics at *any* measurable energy, will ever be observed.

Thus far, the strand tangle model of the strong interaction, including the first estimate of the strong coupling constant, agrees with quantum chromodynamics and with all observations [86].

Thus, strands yield an emergent model for electromagnetic fields, weak nuclear fields, and strong nuclear fields. Gauge field intensities are densities of the corresponding gauge bosons, which are strand configurations specified by the Reidemeister moves, as the next section shows. Because only three Reidemeister moves exist, the strand tangle model implies:

Test 24: No other gauge group – such as SU(10), E8, or any gauge supergroup – will be observed, at any energy.

This prediction agrees with all data [62]. The prediction eliminates many unification attempts and has consequences for the Yang-Mills millennium problem, as told in Appendix F. Strands, due to their rejection of other symmetries, thus respect the Coleman-Mandula theorem [224].

The visualization of the three gauge interactions with strands agrees with and extends the ideas of Boudet [225]. He describes gauge theory using geometric concepts, in parallel to the approach by Hestenes [187–189]. Strands can be seen as a way to simplify those descriptions.

In short, when gauge interactions are modelled as *deformations* of elementary particle tangle cores, classifying these deformations yields only three possible types: each Reidemeister move leads to a gauge interaction. The three possible Reidemeister moves – called twists, pokes and slides – determine the gauge groups U(1), SU(2) and SU(3), as implied by Figure 28 and as explored in references [83–86, 111, 112]. The resulting charges, conserved quantities, and other aspects agree with observations. In the strand tangle model, gauge fields are densities of the corresponding gauge bosons and lead to minimal coupling. No other gauge groups are predicted to exist in nature. The calculations of the coupling constants that were started in references [85, 86] need to be improved further.

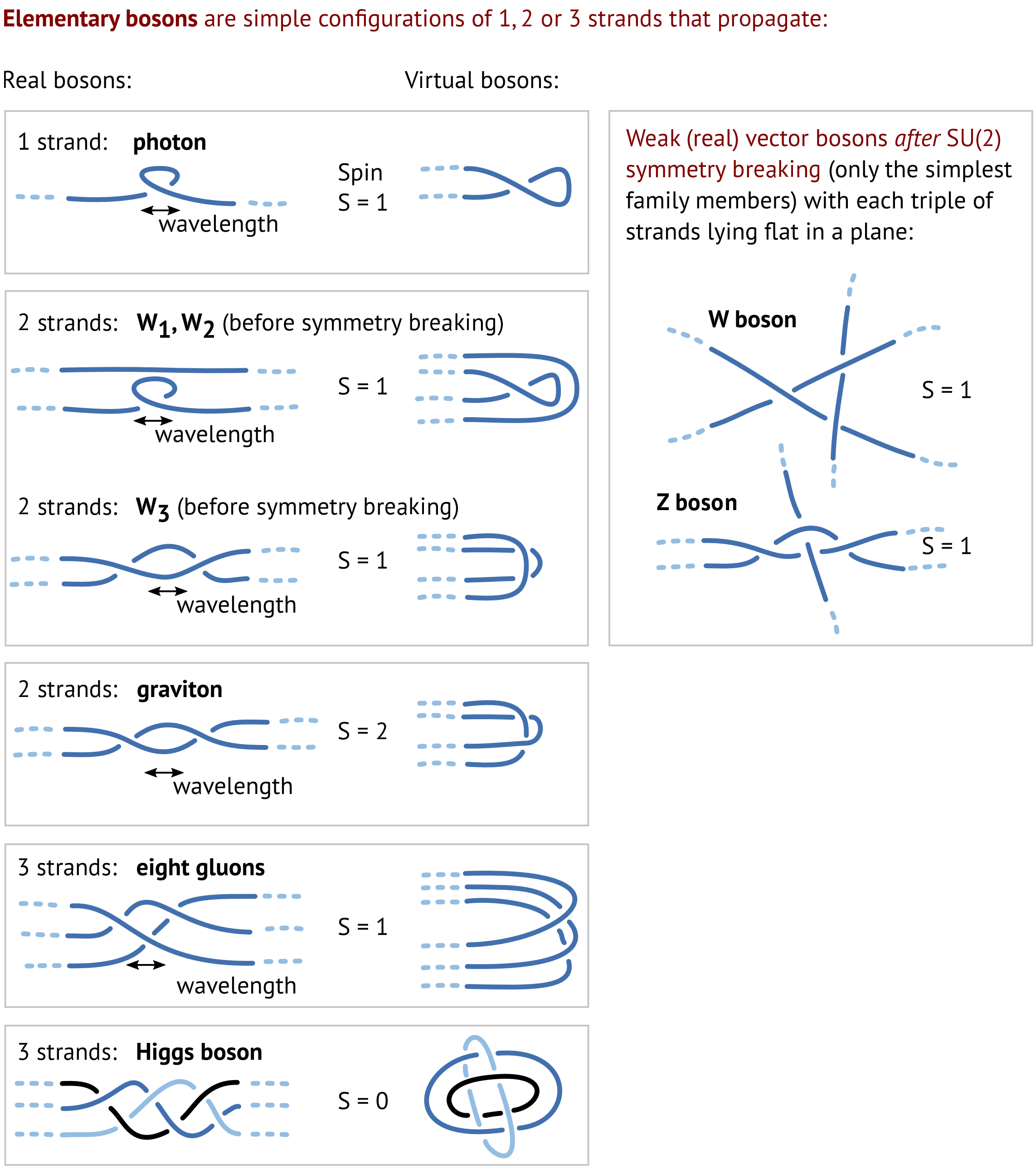

FIG. 33: The proposed rational tangles for each elementary boson are illustrated. The tangles are composed of one, two, or three strands. For each gauge boson, the tangle structure determines the spin value of 1 because any curved strand can rotate by 2π and return to its original shape. The graviton has spin 2 because each of its curved strands can rotate by π and return to the original shape. All boson tangle cores rotate during propagation. Photons and gluons are massless and both are described by a single tangle. The W and Z tangles are asymptotically planar. The W, Z and Higgs are localizable and thus have mass; therefore they have also more complex tangles in addition to the simplest ones shown. No additional elementary bosons are predicted to exist. The W tangle is the only topologically chiral core; thus, it is the only electrically charged elementary boson.

33 Classifying tangles leads to the spectrum of elementary bosons

In the strand tangle model, *elementary bosons* consist of one, two, or three strands. More strands imply composites or groups of *several* bosons. The Reidemeister moves, illustrated in Figure 28, directly suggest that one-stranded bosons correspond to photons, three-stranded bosons to gluons, and two-stranded bosons to the weak unbroken W_1, W_2 or W_3 bosons. This holds before the breaking of SU(2) gauge symmetry; in the strand model, symmetry breaking is the process in which two-stranded weak boson tangles incorporate a vacuum strand, yielding the three-stranded, massive W and Z bosons. A complete overview of all the possible elementary boson tangles is given in Figure 33.

All (unbroken) elementary gauge boson tangles are *trivial*, that is, topologically untangled, as Figure 33 shows. Their tangles can be described as one bent strand among straight strands. The bends in the tangles can *transfer* or *hop* from one strand to the next, yielding boson behaviour. In particular, the one bent strand implies spin 1 for all gauge bosons.

The W and Z bosons, also shown in Figure 33, form a special case. They are composed of three strands that (asymptotically) lie in a plane. This symmetry plane – which distinguishes their tangle from electrons and electron neutrinos – leads to spin 1 and to non-vanishing mass. Their specific tangle structure leads to boson behaviour. No other elementary boson with spin 1 appears topologically possible.

Coupling constants also follow from the strand tangle model. Interactions – absorption or emission of gauge bosons – are due to deformations of tangle cores. The probability of such deformations determines the coupling constant uniquely [83–86]. The challenge of calculating the coupling constants with high precision remains open.

In the strand tangle model, the *Higgs boson* is a braid of three strands. Exploring the Higgs braid shows that it is deduced from the Borromean rings. Random fluctuations in its tangle do not lead to its rotation. For this reason, the Higgs tangle has spin 0. The tangle is not chiral and has positive P parity. Among rational tangles, only a braid has spin 0 and reproduces the observed positive parity values. Only a braid has a vanishing electric charge and a vanishing strong charge.

Test 25: Strands predict the lack of any *additional Higgs boson*.

No other elementary boson with spin 0 appears topologically possible. Observing an additional Higgs boson would falsify the strand tangle model.

For every massive particle, Higgs braids can be added to the simplest tangle core. This does not change any quantum number because the Higgs has the same quantum numbers as the vacuum. Therefore, every massive particle – fermion or boson – is described by an *infinite set* of tangles. The set contains the simplest possible core, the core plus one Higgs braid, the core plus two braids, etc. These options are illustrated in Figure 26 and Figure 27. The precise particle mass value is influenced by this single or multiple Higgs boson addition. Only the Higgs braid reproduces the mass-producing property.

The processes illustrated in Figure 26 and Figure 27 also imply that the Higgs boson couples to itself. The self-coupling, illustrated below in Figure 35, implies that the Higgs boson itself is massive. Both aspects are observed. In contrast, no addition of a Higgs braid to cores of massless elementary particles is possible. Thus, massless elementary particles are described by a *single* tangle.

The *graviton*, the last elementary boson, has an unlocalized core that returns to itself after a rotation by π; thus it has a vanishing mass and a spin of 2. With the help of the graviton, strands describe general relativity with its field equations and its Lagrangian, including the cosmological constant. Also black hole thermodynamics and relativistic quantum gravity arise [81–83]. Cosmology is reproduced, as will be explored elsewhere. No other elementary boson with spin 2 appears topologically possible.

As mentioned above, trivial tangles explain why all elementary particles with integer spin are bosons, and vice versa. Also the case of composed particles was explored above, in Section 9. As a further consequence,

Test 26: Strands predict that there is no elementary boson with spin 3 or larger.

Thus far, this deduction agrees with observations: all known particles with such high spin values are composed.

No additional elementary gauge boson appears possible: neither is a higher number of strands possible in an elementary particle nor is a more complex tangling of strands with boson properties possible. The boson tangles for photons, gluons and gravitons imply that these particles are massless; their tangle cores can rotate unhindered by tethers, in contrast to the W, Z and Higgs bosons, which cannot; they have to perform a kind of belt trick to rotate and therefore have mass.

Test 27: Strands predict the lack of any *additional elementary gauge boson*.

Once tangles are assigned to the gauge bosons, the corresponding charges can be defined from the topological properties of their tangles. *Electric charge* is related to the topological chirality of a tangle [85]. Consequently, electric charge is conserved in interactions. *Colour charge* is related to the threefold spatial symmetry of quark and gluon tethers [86]. Consequently, colour charge is conserved in interactions.

Test 28: Strands predict conservation of electric and colour charge.

Test 29: Strands predict the lack of additional charge types.

Weak charge is due to a combination of the topology and geometry of the tangles. It is not generally conserved.

Strands naturally imply the lack of localized composites made of photons. The possibility of extremely short-lived ZZ-composites or W^+-W^- composites is minimal. In contrast, glueballs, composites of gluons, are allowed by the strand tangle model; so are many other boson composites, from meson molecules to organic molecules. Some boson composites are forbidden:

Test 30: Strands predict the lack of ‘photonballs’.

Test 31: Strands predict the lack of glueballs with incorrect quantum numbers [86].

In contrast, strands reproduce the existence of entangled bosons, such as biphotons. These are regularly observed [226].

The classification of elementary bosons implies

Test 32: Strands predict the lack of *further elementary bosons*, of any spin.

In particular, the strand tangle model predicts the absence of additional gauge bosons, supersymmetric bosons, and additional Higgs bosons. In combination with the previous section on fermions, the prediction becomes even more general:

Test 33: Strands predict the lack of *any new elementary* particle – including dilatons, inflatons, anyons, axions [227], metaparticles [228], and any elementary dark matter particle.

This prediction follows from the strand tangle model of elementary particles and its classification of the possible rational tangles – assuming no mistakes or oversights. Dark matter could still consist of a mixture of black holes and standard model particles.

In short, classifying elementary boson tangles reproduces the observed gauge bosons with the help of trivial tangles of one, two or three strands, reproduces the observed mass-producing Higgs boson with a braid, implies the doubly twisted two-stranded spin-2 graviton, and leaves no room for additional elementary particles beyond the standard model.

34 All measurements are electromagnetic

In nature, every measurement process and every measurement device makes use of electromagnetism. For example, all human senses, even hearing or touch, are electromagnetic. Scales and all devices that measure mass, weight, or force rely on electromagnetism. All seven base units of the International System of Units (SI) are defined and applied using electromagnetic means of observation. Also every comparison with a standard or unit uses electromagnetism. Examples are the reading of a compass needle, the hand of a watch, or a thermometer scale.

In the strand tangle model, the fundamental principle illustrated in Figure 7 defines all observations, all measurements, and all physical observables as consequences of crossing switches. The strand details of the electromagnetic interaction – briefly summarized in Figure 19 as being due to twists – imply that *crossing switches are observable because they couple to electromagnetic fields*. Strands thus confirm at the most fundamental level that without electromagnetism, there are *no* observations and *no* measurements.

> **Test 34:** Performing any non-electromagnetic observation or measurement is impossible. Performing one would falsify the strand tangle model.

Strands thus explain why crossing switches are observable: crossing switches couple to electromagnetism. In contrast, single strand segments or simple strand deformations do not couple to electromagnetism and are not observable. Therefore, strands *explain* the fundamental principle.

The relation between crossing switches and electromagnetism also explains how the minimum time $\sqrt{4\hbar G/c^5}$ arises in the fundamental principle. A crossing switch could, in principle, take an arbitrarily short time. However, such an ultra-rapid, trans-Planckian crossing switch cannot couple to the electromagnetic field; a photon wavelength shorter than a (corrected) Planck length is impossible. Such an ultra-rapid crossing switch would not have any physical effects, and would not be observable.

> **Test 35:** Strands predict the lack of any effect whatsoever that is due to time or length intervals shorter than the corrected Planck limits.

Observing any such effect would falsify the strand tangle model. Again, strands *explain* the fundamental principle.

In short, the fundamental principle implies that all measurements are electromagnetic. Strands also imply that only crossing switches that take longer than the corrected Planck time are observable. No trans-Planckian effects are predicted to occur, to be measurable, or to play a role in nature. These deductions agree with all observations.

35 Strands make predictions about elementary particle mass values

In the strand tangle model, the mass value of every elementary particle is due to the frequency of the belt trick that appears because of spontaneous strand fluctuations when a tangle moves [83]. The mass value is thus *emergent*. The mass is also influenced by the Higgs mechanism. These connections imply several predictions that can be tested even before calculating any mass value. As mentioned in Section 30, only localized particle tangles have mass. This statement can be made more precise.

Test 36: Only localized tangles have *inertial* mass. Strands predict that only *localized* tangles interact with the Higgs field, and thus have Yukawa couplings. In particular, in the strand tangle model, only fermions, W, Z and the Higgs boson have positive mass.

Test 37: Because particle masses are due to the belt trick frequency, massive particles are surrounded by a cloud of virtual gravitons. The gravitons are the two-stranded twists arising in their tethers, illustrated in Figure 2. Virtual gravitons have spin 2 and vanishing mass. Therefore they induce a $1/r^2$ dependency of gravity in flat space. Only localized tangles have *gravitational* mass.

Test 38: The mass values of all elementary particles are *positive*, *fixed*, *unique* and *constant* in time and space, across the universe.

Test 39: Mass values for particles and antiparticles, i.e., for tangles and mirror tangles, are predicted to be *equal*.

All of these deductions from the tangle model agree with all observations [62].

The strand tangle model allows comparison of the *inertial* and *gravitational* mass values. On the one hand, the belt trick generates a displacement and thus relates rotation and displacement. This relation, illustrated in Figure 2, defines the inertial mass. On the other hand, the tether twists generated by the belt trick correspond to virtual gravitons. Thus, the belt trick also defines and generates gravitational mass. Because both mass values are due to the same process,

Test 40: Strands predict that the inertial and gravitational mass of all particles are intrinsically *equal*, at all times, places and energies.

Thus far, the deduction agrees with all the observations. This prediction is of interest because some versions of modified Newtonian dynamics (MOND) suggest a difference between the two mass types, though only for large masses and curved space [229, 230].

Strands imply that particle mass values depend on the tangle structure of the particles. For all hadrons, the prediction that more complex tangles have larger mass values (for the same number of tethers) is valid in all cases, as shown in [83, 84, 86]. For elementary particles, the most important prediction is about leptons.

Test 41: The neutrino tangle assignments explain their extremely small mass values, especially for the electron neutrino. For this particle, the simplest tangle is similar to a vacuum configuration. Strands predict that the electron neutrino mass arises only through the terms due to the Higgs mechanism, and thus is the smallest of all elementary particles.

Test 42: Strands imply and predict

$$m_e < m_\mu < m_\tau \quad \text{and} \quad m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \quad . \tag{29}$$

Test 43: Strands predict that neutrino masses and mixings can be calculated before they are measured with precision.

The prediction of the normal ordering of neutrino masses should be testable in experiments in the coming years.

In the strand tangle model, all particle tangle cores get *flatter* at higher energy – or higher four-momentum. This change will influence the frequency of the belt trick. As a result,

Test 44: Strands imply the *running* of elementary particle masses with energy.

The deductions qualitatively agree with the observations and with standard model calculations, such as those by Xing et al. [231]. Performing a quantitative test is a research topic.

Thus far, only rough estimates of particle masses from belt trick probabilities have been achieved [85, 86]. No precise calculation method has been developed yet. It is expected that exploring the behaviour of asymmetric bodies and their rotation in a viscous (Stokes) flow might lead to better approximations [232]. This field of research is still in its infancy.

In short, calculating particle masses – and their running with energy – with the help of computer simulations that determine the relationship between the structure and shape of a tangle core, the belt trick, and the motion of a tangle will allow independent *tests* of the strand tangle model. Even before such calculations, additional tests of the strand tangle model are possible.

36 The principle of least action follows from strands

In nature, a higher value of action corresponds to a higher amount of change in a system. According to the fundamental principle of the strand tangle model, the action of a physical process is the number of occurring crossing switches. Each crossing switch yields an action increment of $\hbar$. In the strand model, a higher value of action corresponds to a higher number of crossing switches. Therefore,

- ▷ Least action is the smallest number of crossing switches.
- ▷ Every motion and every process in nature minimizes the number of crossing switches.

This equivalence is consistent with the fundamental principle because a crossing switch is less probable than other strand deformations. A crossing switch is a form of change; a crossing switch requires action, and thus it requires a 'cost', whereas a strand shape deformation without a crossing switch does not. One can even speculate about the role of the vacuum strands in this difference. Thus, strands confirm that that physical *action is the measure of change*.

In other words, strands suggest an underlying reason and an explanation for the principle of least action: the fundamental principle. This may be the only explanation in the literature. Thus,

Test 45: Strands predict the lack of the slightest deviation from the least action principle and from the description of motion with Lagrangians. Discovering such a deviation would falsify the strand tangle model.

In particular, the result applies to motion in the quantum domain. Indeed, the Dirac Lagrangian for a free particle arises from the minimization of the number of crossing switches. The results of the last sections confirm that least action also applies to gauge interactions.

In short, the strand tangle model implies the principle of least action for all processes. This result allows evaluation of the standard model as a whole.

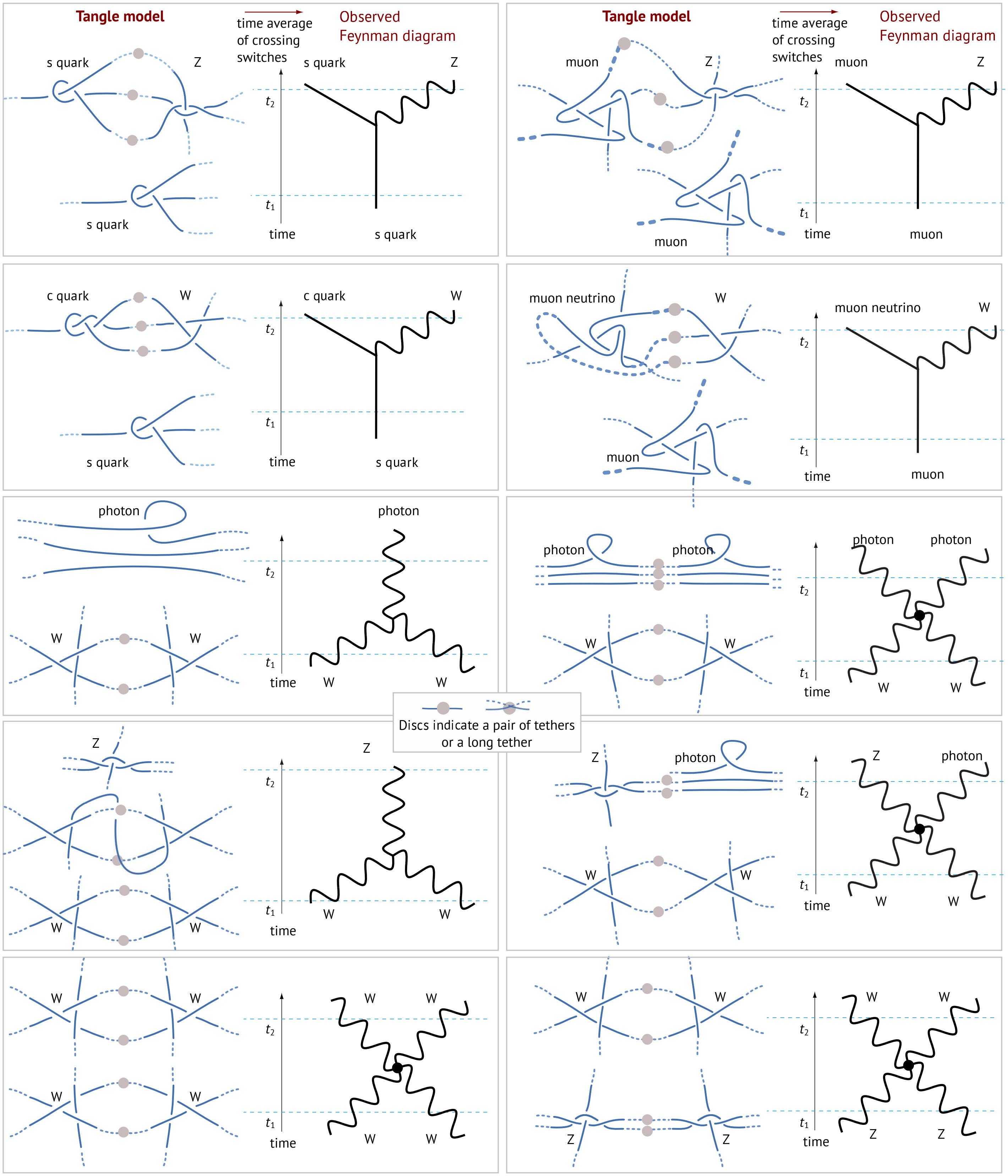

FIG. 34: The first interaction vertices allowed by fermion and boson tangle topologies imply the full Lagrangian of the standard model and its renormalizability. (Figure improved from reference [84].)

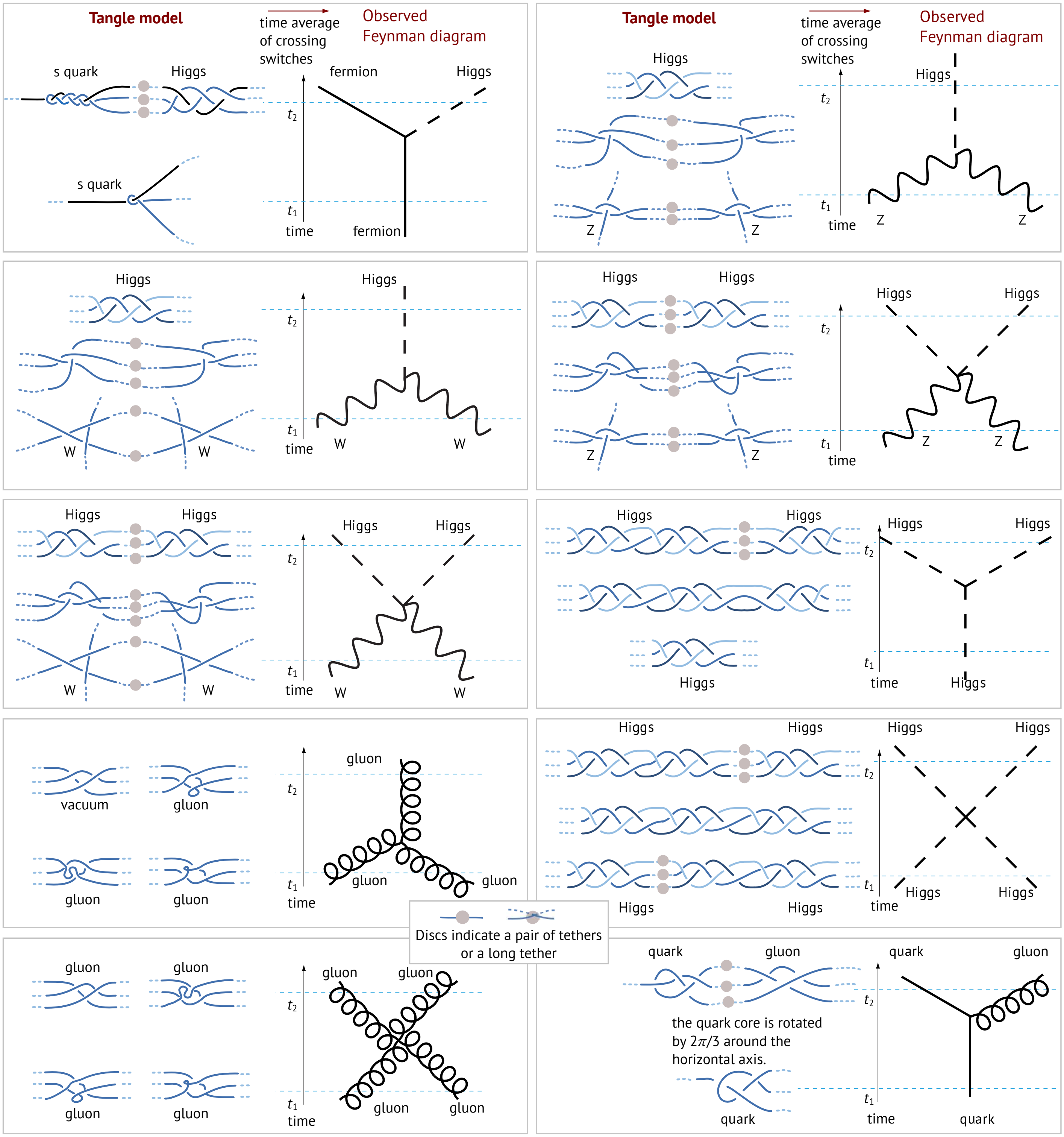

FIG. 35: The remaining interaction vertices allowed by fermion and boson tangle topologies imply the full Lagrangian of the standard model and its renormalizability. (Figure improved from reference [84].)

37 Strands limit the possible interaction vertices of the standard model

This section provides a summary of previously published results on particle physics [83–86]. In the strand tangle model, gauge interactions are due to tangle core deformations that occur by transfer of twists, pokes or slides, as illustrated in Figure 28. The figure also shows that as a consequence, particle reactions and decays are described by Feynman diagrams.

Using the particle tangles of the elementary fermions and bosons given in Sections 31 and 33, only a limited number of interaction vertices are topologically possible. All the possible interaction vertices are listed in Figure 34 and Figure 35. These vertices cover all observed Feynman diagrams of the standard model with massive Dirac neutrinos [233, 234]. The vertices reproduce the conservation of charge, parity, energy, momentum, and spin. No additional vertex arises and no observed vertex is missing [84]. The lack of additional interaction vertices is due to the structure of the rational tangles of the elementary particles. This result agrees with observations [62]. In other words, strands restrict the possible particle reactions and decays to the observed ones.

In the full Lagrangian of the standard model, strand tangles imply, due to the Dirac equation and the few allowed elementary rational tangle structures, mass terms for the charged leptons, the neutrinos and the quarks. Strand tangles imply, due to the Reidemeister moves, the terms for the bosons of the three gauge interactions. Strands imply the existence of antiparticles. Strand tangles imply, due to the interaction vertices and the resulting minimal coupling, the dynamical or interaction terms for all leptons. Strands imply, due to the Higgs tangle and its vertices, the Higgs mass and dynamical terms. Strand tangles imply, due to the chiral fermion structures, maximal parity violation. Strand tangles imply, due to tether movements, the Cabibbo-Kobayashi-Maskawa mixing matrix and the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix. Strand tangles imply, due to their average shape, all fundamental constants. As a consequence, the full standard model, extended with massive mixing Dirac neutrinos, arises from rational tangles of strands [235]. Strands imply that no alternative or modification to the standard model is possible. Thus far, this conclusion agrees with all the experimental data.

The figures with the interaction vertices show that only *triple* and *quadruple* interaction vertices arise. This limitation is due to the limited number of strands in the tangles of elementary particles. This limitation implies that QED, QCD, and the theory of the weak interaction are *renormalizable* when they are extrapolated to point-like particles. Because of the minimum length, strands also eliminate the Landau pole. These results are expected, as the strand tangle model yields a finite description of quantum field theory.

In short, the strand tangle model, using the fundamental principle, limits the rational tangles for elementary particles to the observed one. These limited elementary tangles limit the interaction vertices of the standard model to the observed ones. Strands also imply the finiteness of the standard model in the conventional approximation with point particles. In other words, *the fundamental principle implies quantum physics and particle physics.* Therefore, the standard model of particle physics is simple, finite, unique and elegant.

38 Strands imply experimental tests of the tangle model

Rational tangles imply the standard model of particle physics with massive mixing Dirac neutrinos. The connection leaves no freedom of choice; it is unique.

Test 46: The tangle model predicts that the standard model with massive mixing Dirac neutrinos is valid at all measurable energies, without any modification. Strands predict the *lack* of measurable effects *beyond* the standard model.

So far, this conclusion agrees with all observations [62]. This conclusion is equivalent to stating that strands explain the origin of the interactions and all particles. The strand tangle model adds a statement that completes the answer to all open questions of particle physics.

Test 47: The strand tangle model predicts that the calculated values for the fundamental constants – elementary particle masses, coupling constants, mixing angles and phases – *agree with experiment*. The values can be calculated from the average tangle shapes of fluctuating tangles. Values that differ from the observed ones are *impossible*.

Thus far, the few estimates deduced from the tangle model so far agree with observations [83–86]. The possible tests include the comparison with observations of calculations for neutrino masses and mixing angles, lepton masses, quark masses and mixing angles, the CP violating phases, the Higgs, W, and Z boson masses, and the three coupling constants.

The strand tangle model also implies the same perturbation expansions as conventional quantum field theory [85, 86]. The perturbation expansions are due to more complex tangles arising from fluctuations at smaller scales.

Test 48: The strand tangle model predicts that the calculated runnings with four-momentum of the fundamental constants *agree with experiment*.

Test 49: Due to the lack of grand unification, supersymmetry, and new energy scales, the running coupling constants do not need to converge.

The calculation is a topic for future research.

Alternatives to the strand tangle model are rare. In the research literature, only one other unified model attempts to deduce the fundamental constants: the octonion model due to Singh [236]. So far, the octonion model appears to predict additional particles and possibly additional interactions [237]. These have not yet been observed.

Stated simply, the strand tangle model predicts the *lack of breakthroughs* in fundamental experimental physics. Equivalently, strands predict that the standard model and general relativity limit what humans can achieve. Of course, this prediction needs to be challenged continuously.

The strand tangle model implies that there is a natural *cut-off* at the Planck scale and a well-defined *continuum limit* that arises when the strand radius is assumed to vanish.

Test 50: Strands predict that physical space has *three dimensions* at all measurable scales.

Test 51: Strands predict that there is *only one vacuum state* in nature – in flat space. There are no domain walls between 'different' vacuum states.

So far, the predictions agree with observations.

Additional possibilities for tests of the strand tangle model appear to be rare. The predicted lack of trans-Planckian effects includes a lower limit on the effective elementary particle diameter:

Test 52: No elementary particle has an intrinsic diameter smaller than twice the Planck length.

The predicted size of elementary particles also implies a limit on the locality of interactions. However, such tiny deviations from locality will not produce any measurable effect that differs from the already known particle properties. Can a Planck-scale deviation from locality be observed in measurements? One can dream that the electric dipole moments of elementary particles correspond to the tangle assignments.

Measurements of the electric dipole moment for the electron are closest to achieving Planck scale sensitivity [67, 68]. However, to first order, the strand tangle model predicts no dipole moment for the electron. For other particles, the dipole moment values should be at most a few times the smallest length times the unit charge. Small measurement errors imply long measurement times. Due to their short lifetime, measuring the electric dipole moment of the W and Z bosons or the Higgs seems impossible. Measuring the dipole moment of a single quark to high precision also appears out of reach. It is unlikely that the measurements of the dipole moment of the neutron will ever reach the sensitivity of the measurements of the electron.

Deviations from point-like behaviour can also be searched for in other settings. However, electron-electron scattering experiments will not reach the required short distances, neither for the electromagnetic nor for the weak interaction. Also decay processes do not provide the required sensitivity. In the nuclear interactions, the tangle structure might lead to experimental effects, possibly in the non-perturbative domain. However, the possibility appears to be remote [86].

An important deviation from point-like behaviour is entanglement. The relationship between entanglement, decoherence, and gravity has been and is being explored by many scholars [238–240]. In the strand tangle model, the three processes exhibit differences and similarities. On the one hand, gravity is due to the exchange of virtual spin 2 particles made of two strands, whereas entanglement and decoherence are not due to particle exchange. On the other hand, in the strand tangle model, all physical processes are built from crossing switches, so that all processes share fundamental similarities. So far, entanglement does not contradict the tangle model. Further exploration of the possibility of detecting additional relativistic quantum gravity effects is useful. Again however, the possibility seems remote.

In short, strands predict the lack of new physics, the lack of trans-Planckian effects, and the encoding of all observable deviations from point-like behaviour in the usual particle properties. Apart from the calculation of the fundamental constants – particle masses, coupling constants, and mixing angles – and their running with four-momentum, the feasibility of any additional experimental test for particle tangles appears unlikely.

39 Strands resemble Galileo's book of the universe

In 1623, Galileo published his successful book *Il saggiatore*. Chapter 6 contains the well-known statements:

> La filosofia naturale è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi agli occhi, io dico l'universo, ma non si può intendere se prima non s'impara a intender la lingua e conoscer i caratteri nei quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi ed altre figure geometriche, senza i quali mezzi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro labirinto.

The translation into English (by the present author) is: "Natural philosophy is written in this huge book that is continually open in front of our eyes, I mean the universe, but one cannot understand it if one does not first learn to understand the language and know the characters in which it is written. It is written in mathematical language, and the characters are triangles, circles and other geometric figures, means without which it is impossible for humans to understand a word; without these it is a wandering in vain through a dark labyrinth."

The tangle model confirms that the book of nature is not written with equations, but is indeed written, as Galileo stated, in mathematical language, with geometric figures, namely the shapes of strands. As shown above, the fluctuating shapes appearing in the fundamental principle yield quantum mechanics and the standard model of particle physics. In addition, the fluctuating shapes of strands yield space, general relativity and black hole entropy [81, 82].

40 Particle tangles have open issues

The above presentation of the strand tangle model may contain inconsistencies or errors. First, it could be that the tangle assignments for the quarks, leptons, or Higgs boson require corrections. For example, a smooth transition between quarks and antiquarks is difficult to visualize.

▷ Even if specific tangle corrections will be necessary, the strand tangle model is predicted to remain valid.

A second open point concerns the completeness of the classification of the rational tangles given above. For example, when a braid is formed from three sets of two strands each, which set of elementary particles corresponds to the resulting configuration? Similar questions regarding more complex tangles are yet to be answered comprehensively.

Another point was not explored in this article. The propagation of a quantum particle does not require that its strands are always the same ones. A propagating particle can take in a vacuum strand and, in exchange, leave behind a strand from its tangle. This process is easiest to visualize for photons; a twist can be imagined to jump from one strand to the next. It is unlikely that this process leads to any observable effect; however, this point requires further study.

In short, it is possible that future investigations lead to minor corrections to the tangle model.

41 Conclusion

> Ciò che per l'universo si squaderna:
> ...
> La forma universal di questo nodo
> ...
> Dante, *Paradiso*, Canto XXXIII, 87, 91.

Starting from the tethers of Dirac, the ideas of Battey-Pratt and Racey and the Planck limits, fermions are modelled as tangles of unobservable fluctuating strands with Planck radius. Every crossing switch yields a quantum of action $\hbar$. This is the fundamental principle.

As a consequence of the fundamental principle, the wave function is a crossing density and propagating particles are modelled as advancing and rotating rational tangles that fluctuate. As a further consequence, fluctuating rational tangles reproduce the probabilistic behaviour of quantum theory in all details, including superpositions, entanglement and decoherence.

Only strands derive, using the belt trick, spin $1/2$ and fermion behaviour. Only strands yield emergent wave functions, spinors, and the Dirac equation. Only strands derive, by classifying strand deformations with the Reidemeister moves, the gauge interactions and the observed gauge groups. Only strands derive, by classifying the possible tangles, the three generations of elementary fermions and the elementary bosons, with their observed quantum numbers. Only strands derive massive mixing Dirac neutrinos. Only strands derive, from the topology of the tangles, the Higgs mechanism, the breaking of SU(2), lepton mixing, and maximal parity violation of the weak interaction. Only strands derive, from their Planck radius and from the average shapes of tangles, unique elementary particle mass values, coupling constants, and mixing angles.

Only strands derive, from the topology of the tangles, the observed Feynman vertices, particle reactions and decay modes. Only strands derive the quark model, glueballs, and the lack of CP violation in the strong interaction. Only strands derive quantum field theory, including quantum electrodynamics, quantum chromodynamics, and the theory of the weak interaction. Only strands derive the Lagrangian of the standard model without any measurable modification.

From their tangling, strands derive the three dimensions of space. From their crossing switches, strands derive maximum force, space curvature, Bekenstein's entropy bound, the thermodynamics of black holes, and Einstein's field equations. Only strands derive both the Hilbert Lagrangian and the standard model Lagrangian, both without any measurable deviation.

In short, the strand tangle model derives all known observations and numerous experimental predictions of high precision from a simple fundamental principle. So far, all derived consequences agree with observations. Strands predict the lack of new physics and the possibility of calculating the observed fundamental constants, including their running with four-momentum, to high precision. The strand tangle model implies that the standard model of particle physics – with massive mixing Dirac neutrinos – is simple, unique, and elegant.

42 Outlook

In the experimental domain, the strand tangle model derived, from the fundamental principle, more than 50 precise predictions and tests. Strands predict the lack of measurable physics beyond the standard model with massive mixing Dirac neutrinos, predict the lack of measurable corrections to general relativity, predict the lack of measurable trans-Planckian effects, and predict the normal ordering of neutrino masses. If only one of the listed tests does not agree with observations, the strand tangle model is falsified. In the last decade, despite intense experimental efforts, no disagreement with observations appeared.

The main challenges for future research are the mathematical basis and the numerical simulation of the strand processes presented above. Exploring the Heisenberg picture of quantum theory will deepen understanding. Deducing Schwinger's quantum action principle from strands is appealing. Deducing the Dyson–Schwinger equations and the Ward–Takahashi identity from strands of zero radius is attractive. Deducing axiomatic quantum field theory from tangles of zero radius is worthwhile. The relation between crossings, qubits, entanglement and quantum gravity should be explored. The consequences of strands for entanglement entropy should be deepened. The relation of rotating tangles to Hestenes' geometric algebra should be studied. Animations of particle propagators and vertices in Feynman diagrams will be useful. Strands should help to clarify topics about perturbation calculations. An example might be the Kinoshita–Lee–Nauenberg theorem [241]. Ultraviolet issues and questions about anomalies might become more accessible using strands. Non-perturbative effects might be clarified.

In cosmology, an estimate of the cosmological constant from strands must be derived, including its time dependency or lack thereof. Strands should be used to clarify the issue of galaxy rotation curves by deducing the existence and nonexistence of MOND from the tangle model for gravity.

In the domain of numerical simulation, efficient and precise calculation methods for the fundamental constants and their running with four-momentum need to be developed. The study of the quantification of the chirality of tangle cores should be deepened. The study of the rotation of asymmetric and tethered bodies in viscous flows should allow better approximations for the mass values of elementary particles. Geometric ab initio approximations for the fundamental constants should be developed, including for quark and neutrino mixing, for the Higgs mechanism, and for the W and Z masses. All calculations are predicted to agree with the measurements. If there is no agreement, the strand tangle model is refuted. With a finite effort, neutrino masses and mixing angles can be calculated before they are measured.

Acknowledgments and declarations

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Appendix

A General relativity and all of nature deduced from strands

In the strand tangle model, everything observed in the universe – space, horizons, and particles – is composed of strands. The corresponding strand structures are illustrated in Figure 36. Thus

▷ The whole universe is expected to be one long closed strand – i.e., a strand with the topology of a ring – that crisscrosses all of nature and forms space, particles and horizons. This strand is becoming increasingly tangled over time. The strand tangle model of cosmology is illustrated in Figure 37.

Strands realize Heraclitus' idea that everything in nature is connected to everything else. In the strand tangle model, nature has no exact parts.

In the strand tangle model, flat empty space is an aggregate composed of *untangled strands*, packed (almost) as densely as possible. Curved space is an *irregular aggregate* of strands, in which some strand pairs are twisted, representing real or virtual gravitons. Black hole horizons are *weaves* of strands. Because of the Planck limits that are part of the fundamental principle of the strand tangle model, strands imply the maximum force $c^4/4G$. Strands imply the usual expressions for black hole mass, the moment of inertia of black holes, black hole entropy, and black hole temperature [81–83]. The usual logarithmic correction to black hole entropy also arises – but it is unmeasurably small. Despite their countless tethers, black holes can still rotate, making use of the belt trick. In the strand model of black holes, for an observer at spatial infinity, the mass is distributed over the horizon. This mass distribution explains the non-vanishing moment

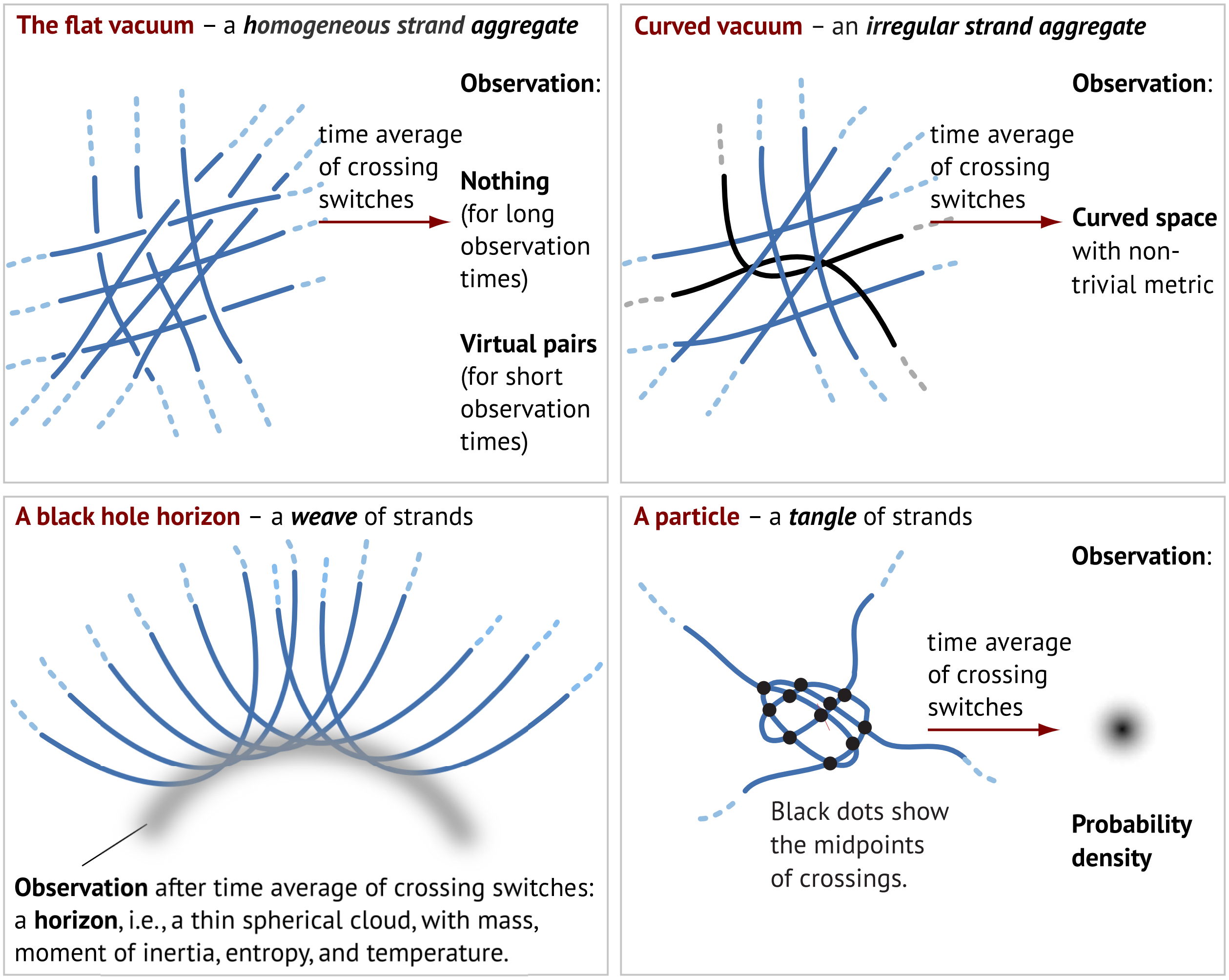

FIG. 36: Strands can form aggregates (space), weaves (horizons), and tangles (particles). The present text concentrates on particles. For the exploration of space, curvature and black hole horizons, see references [81] and [82].

of inertia of black holes deduced by Ha [242]. Together, these properties imply Einstein's field equations and the Hilbert Lagrangian, without any measurable deviation, as shown in references [7, 9, 16, 81, 82]. This derivation is an example of how an inequality, in this case $F \leqslant c^4/4G$, leads to precise equations of motion. All deduced properties of gravity agree with all experiments, as shown by the summary by Will [46]. Also all LIGO observations about gravitational waves confirm general relativity. In particular, no black hole merger exceeds the power limit $P \leqslant c^5/4G$ [44, 45], and thus none of the other black hole limits.

All particles are *rational tangles* of strands. Fields are particle densities. The present article focuses on elementary particles and fields. To simplify the explanations, only particle strands are drawn in the illustrations in the present article. Vacuum strands are not drawn. When the vacuum strands are taken into account, every elementary particle, modelled as a tangle of strands, is effectively a *defect* in space. Strands thus realize this old dream, explored by many researchers over the years, including Hehl, Kröner, and Kleinert [243, 244]. Exploring all possible defect structures of the strand vacuum yields, apart from horizons and particles, no other possibility.

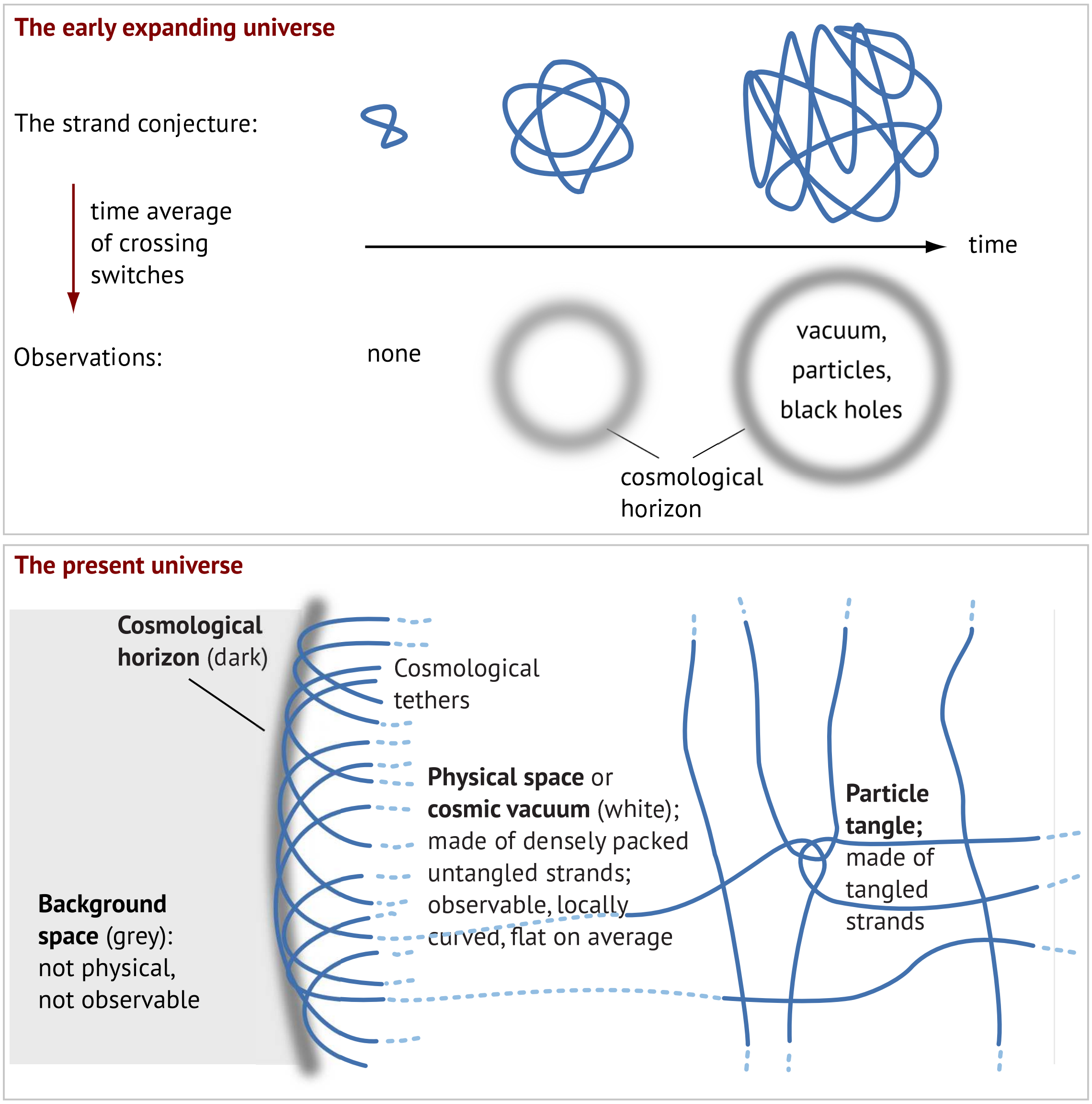

FIG. 37: The whole universe is expected to consist of a single strand, without ends. The figure illustrates its early history (top) and a part of its present state (bottom).

Nature has no domain walls, line defects, singularities, white holes, or wormholes [81, 82].

The strand tangle model of cosmology is illustrated in Figure 37. The strand model defines a preferred reference frame, namely the frame in which the cosmic background radiation is isotropic. This is also the reference frame in which the maximum possible elementary particle energy, $\sqrt{\hbar c^5/4G}$, would have to be measured, were the value achievable. This description with Planck limits resembles the approach by Amelino-Camelia [245]. In extremely boosted reference frames, the background formed by the strands from the horizon would not be isotropic and the derivation of the Dirac equation given in Part III would not apply. However, at all realistic boost values, deviations from the Dirac equation cannot be measured.

In short, in the strand tangle model, *all* systems found in nature – particles of radiation and matter, flat and curved space, cosmological and black hole horizons – consist of strands. Strands imply that black hole horizons have the usual entropy. Space curvature and gravitation are described by Einstein's field equations, without any measurable deviation, in agreement with all experiments. Finally, strands yield a model of cosmology in agreement with observations.

B Strand crossings visualize qubits and Urs

Qubits, a contraction of 'quantum bits', play an important role in two research fields: quantum computing and relativistic quantum gravity. In modern quantum computing research, physical qubits are commonly realized as trapped ions, quantum dots, nitrogen-vacancy diamonds, or superconducting loops. The state of a qubit can be visualized with the Bloch sphere, which visualizes the SU(2) structure of the qubit state space [246, 247]. In quantum computing, quantum gates are represented by unitary operators. The different types of quantum gates can be represented as different operations on the Bloch sphere.

The strand tangle model for fermions – as illustrated in Figure 20, Figure 21 and Figure 22 – is useful for visualizing qubits. *In the strand tangle model, the Bloch sphere describes the orientation of the tangle core and the twist state of its tethers.* In the strand tangle model, unitary operators are represented by strand deformations that keep the average shape of the tangle core untouched, but change its orientation in space. In this way, a strand tangle visualizes a single-qubit state; many tangles visualize multi-qubit states. In this context, Figure 7, illustrating the fundamental principle, has an additional aspect. As long as the strand segments are kept close to each other, the fundamental principle itself can be seen as a belt buckle realizing SU(2).

The other use of qubits is in relativistic quantum gravity. Several research questions are being explored. How can one imagine a region of space-time that includes vacuum, matter, and particles in terms of qubits? In particular, do qubits form lattices, random structures, or other types of structures? Is the number of qubits finite, countably infinite or uncountably infinite? These questions ask about the microscopic, Planck-scale constituents of nature as a whole [248]. In 2001, Zizzi formulated the aim of this quest with the expression 'it from qubit' [249]. In the strand tangle model, as told in Section 2, this aim becomes: 'equations from no equation'.

Given the possibility of describing quantum theory, gravity, and quantum field theory with strand tangles [81, 83–85], relations between tangles and qubits arise almost naturally. Given that space, curvature, and particles are due to strands and their tangling, one can interpret the fundamental principle of Figure 7 as a maximally simplified qubit. The fluctuations of the strands change the structure and interdependencies of the qubits over time. Strands clarify that the number of their crossings – the number of qubits – is countable in principle, but is not fixed. With this interpretation of the fundamental principle, strands confirm that *nature is built of qubits.*

The quest to construct nature from qubits is not new. A 'qubit' is the modern expression for the 'Ur', the fundamental and continuous alternative, described by SU(2), that Weizsäcker introduced in the 1970s [250, 251]. Weizsäcker dreamt of deducing all laws of nature by building them from Urs [252]. (The correct German plural is 'Ure'.) The research approach was explored by Kober [253] and by Lyre [254, 255]. The fundamental principle of Figure 7, which generates SU(2), can be seen as the simplest possible realization of an Ur. Thus, the strand tangle model can be said to realize Weizsäcker's wish: *nature is built of Urs.*

An approach to qubits similar to that of strands arose recently. In 2016, Freedman and Headrick introduced *bit threads* to describe entanglement, holography and the Ryu-Takayanagi entropy

[256]. Central to the idea of bit threads is the hope that space can be seen as a consequence of the entanglement of filiform entities with Planck-sized diameter [257]. Various authors explored the topic [258–264]. It is expected that research about bit threads will follow the path prepared by strands.

In short, strand tangles are useful tools for teaching and visualizing qubits in quantum computing. In the field of relativistic quantum gravity, the fundamental principle can be seen as a specific version of building 'it from qubit' and 'nature and her laws from Urs'.

C Strands define quantum fields

Traditionally, quantum fields are introduced as part of the axioms of quantum field theory. In particular, quantum field theory defines fields assuming *point-like* particles and using *local* commutator relations and operator-valued *local* densities. In this approach, particle masses, couplings, and mixing angles must be added by hand and cannot be explained. Strands propose a different view of quantum fields.

▷ In the strand tangle model, *quantum fields* emerge from fluctuating *rational tangles* made of strands with Planck radius.

The minimum length forbids using local operators or distributions as fundamental concepts. The traditional, axiomatic quantum field concept is an *approximation*. All concepts of axiomatic field theory, from C*-algebras to fibre bundles, are approximations. At the Planck scale, the local quantum fields of axiomatic quantum field theory do not exist.

In the strand tangle model, there are no approximations. In exchange, nothing has to be added by hand and all constants are explained. In the strand tangle model, particles are *rational tangles* of strands. Strand shape fluctuations lead to fluctuations in the strand crossings. Averaged over time, strand crossings in massive particles yield *complex wave functions.* Crossing switches define probability densities and all observables. Gauge fields also arise from strand crossing switches, namely those of the related gauge bosons. Thus, all fields emerge from fluctuating strands. Like traditional quantum field theory, the tangle model treats all fields on an equal footing. Axiomatic quantum fields and local operators arise from strands in the limit that the strand radius goes to zero – keeping in mind that strands themselves are unobservable.

In the strand tangle model, identical particles are *countable* and *indistinguishable*; their *spin* and their *statistics* are closely related. Particles are (approximately) localized *excitations* of the (untangled) *vacuum*. Excitations are realized by the tangling of untangled strands of the vacuum. *Rational* tangles allow for virtual particles, antiparticles, particle transformations, and particle reactions. Thus, the (rational) tangle model reproduces *particle creation* by tangling, particle *annihilation* by untangling, particle *absorption* by tangle combination and particle *emission* by tangle separation. Vacuum excitation results in both particles and antiparticles. The strand tangle topology determines the particle type and its quantum numbers. The (rational) tangle model also implies that (certain) *particles can interact*: different tangles can be combined, or a single tangle can be separated into two or three tangles. The (rational) tangle model also implies that (certain) *particles can transform* into each other: tether braiding changes particle type. The strand tangle model reproduces the *three generations* and the *particle spectrum* as a result of tangle classification. The strand tangle model reproduces particle quantum numbers, their mass values, and their mixing angles as a consequence of topological and geometric tangle properties. The strand tangle model, with its built-in length limit, resolves the issues of renormalization.

The strand tangle model thus reproduces all effects traditionally attributed to quantum field theory. As a result, an expression such as 'an electron is an excitation of the electron field' is either misleading or wrong in the strand tangle model. Likewise, the concept of 'second quantization' is superfluous in the strand tangle model. Concepts such as 'electron field', 'neutrino field' or 'wave function of the universe' make no sense at the fundamental scale.

Strands simplify quantum field theory in other ways. Whenever space is assumed to be described by a lattice, the Nielsen–Ninomiya theorem implies the necessity of doubling the number of chiral fermions [265–267]. This theorem has generated numerous studies on its extensions and limitations. The theorem is also the reason for various difficulties in numerical simulations of elementary particles. In the strand tangle model, because of the minimum length, space is *neither* discrete *nor* continuous. In particular, space is *not* a lattice. Thus, there is no Brillouin zone in the strand tangle model. Therefore, the strand tangle model does not realize the conditions for the doubling of fermions and the no-go theorem published by Nielsen and Ninomiya.

In 1955, Haag proved that an interacting quantum field theory is mathematically inconsistent. The proof is based on continuous space, on the properties of vacuum polarization, and on its behaviour under renormalization [268, 269]. However, given that nature is not characterized by continuous space, continuous observables, or local operators, the conditions for the theorem are not satisfied. Again, strands simplify quantum field theory.

Other issues of quantum field theory, such as UV completeness, are dissolved by strands. Renormalization and anomalies were (briefly) addressed earlier.

In short, fluctuating tangles and their crossing densities reproduce, visualize and simplify both wave functions and quantum fields. Both concepts emerge from the fundamental principle of the strand tangle model. Strands predict that there is *only one* possible quantum field theory that describes particle properties and gravity: the standard model with massive Dirac neutrinos and the observed fundamental masses, couplings and mixing angles. A few aspects of this uniqueness are presented in the following appendices.

D On preons

In 1979, Harari [270] and Shupe [271] proposed models of composite quarks and leptons. In both models, there are only two fundamental particles, which Harari called *rishons* and Shupe *quips*. Nowadays, such hypothetical particles are generally called *preons*. Both approaches assumed that the preons had a charge $e/3$, anti-preons $-e/3$ and that leptons were composed of three preons. Shupe explored states with fewer and more preons as models for gauge bosons and gravitons, deduced the vanishing mass of preons, and tried to model Feynman diagrams.

Many explorations followed and continue to this day, too numerous to mention all [272–278]. Approaches that try to add interactions or dynamics had no success. In retrospect, this lack of success resulted from the insistence to find a Lagrangian, often with new symmetries.

If one calls a strand crossing a 'preon' and its mirror inverse an 'anti-preon', one obtains several similarities: fractional electric charge $\pm e/3$, masslessness, and a description of both fermions and bosons. However, preons do not explain the gauge groups, whereas strands do. Likewise, preons do not explain particle masses, gauge coupling constants, or general relativity, whereas strands do.

Similar limitations also apply to the fascinating preon approach by Bilson-Thompson based on braided ribbons [103–107], which has generated numerous subsequent studies. Despite their similarity to strands, ribbons do not seem to explain wave functions, their dynamics, the gauge groups, the fundamental constants, or general relativity. However, replacing ribbons with their edges yields an approach similar to the strand model and could yield new insights.

In short, point-like preons seem to be more similar to fundamental quantum numbers and to strand crossings than to fundamental particles. Ribbon-like preons resemble strands but do not appear to provide an inequivalent unified description. It appears that strand tangles solve the problems that preons left open: strands explain the gauge groups, the fundamental constants, general relativity, and wave functions.

E On strands and superstrings

At first sight, strands differ from superstrings in several respects. Superstrings are characterized by Planck tension, have a Lagrangian, live in 10 dimensions, realize dualities, carry supersymmetric superfields, and possibly imply grand unification [279, 280]. Superstrings have been generalized to various types of supermembranes, such as D-branes. In contrast, strands have no tension, have no Lagrangian, live in usual $3 + 1$ dimensions, carry no fields or observables, and exclude supersymmetry and grand unification. There is only one type of strand, with Planck radius. At the same time, the entire universe is supposed to be a single closed strand.

At second sight, the differences between strands and superstrings might be much smaller. If one adds two 'strand surface coordinates' for each strand, a higher number of dimensions can be imagined to arise. Elementary particles with three strands, with two surface dimensions each, would give a ten-dimensional space-time. Even Calabi-Yau manifolds might arise in this way. This might explain the different number of dimensions in the two approaches.

Strands include non-commutativity. An aggregate of crossing strands resembles a fermionic space. Supersymmetry might be seen as due to ubiquitous crossing switches.

In the strand tangle model, crossings yield observable Lagrangians. Instead, the original superstring Lagrangian might just be a calculation guide, not a real observable Lagrangian. Therefore the fundamental supersymmetry that is assumed in superstring theory might not be observable, and neither might its super-Lie group. On the other hand, the idea of grand unification that is often assumed to exist in superstring theory [279, 280] must be abandoned.

Certain supermembranes respect the Planck limits, like strands do. Like strands, certain types of supermembranes arise in finite numbers in finite volumes of three-dimensional space. If one imagines that the superstring tension is responsible for forming strands, the resulting strands themselves might still be wobbly, as assumed in the strand tangle model. Strands might thus be seen as tubular supermembranes. As a result of the formation of strands, the original superstring/supermembrane Lagrangian itself might not be observable.

Finally, strands can be thought of as realizing specific forms of gauge/gravity duality, of AdS/CFT duality, and of holography, provided that these superstring concepts are modified by taking Planck limits into account. Strands can also be thought of as a way of realizing the freedom from anomalies that started superstring theory.

Testing any hypothesis connecting supermembranes and strands will require that research in superstring theory focuses on a concrete model for elementary particles. Such a model would have to go beyond the statement that 'particles are vibrating superstrings' and might well be achievable in the coming years. The particle tangles proposed above could be an inspiration.

In short, there might exist a mapping between certain types of superstrings and strands.

F On the Yang-Mills millennium problem

In the year 2000, the Clay Mathematics Institute asked, in its list of millennium problems, to prove the existence of a non-trivial quantum field theory for every (non-Abelian) compact simple gauge group in continuous flat space-time. The problem, formulated by Witten and Jaffe, also asks to prove that each such quantum field theory leads to a finite mass gap. In the millennium problem, a quantum field theory is defined as a structure realizing the axioms by Streater and Wightman as well as those by Osterwalder and Schrader. All these axioms are based on perfect locality and continuous quantum fields. This millennium problem has two aspects: a physical aspect about nature, and a mathematical aspect about axiomatic quantum field theory.

The *physical* aspect of the Yang-Mills millennium problem is answered negatively both by nature and by the strand tangle model. In nature, continuous space-time does not exist, due to the smallest length. In nature, additional gauge groups do not exist: only the two nuclear interactions are observed to be non-Abelian gauge theories. Also strands answer the question in a way that leaves *no* imaginable alternative to the standard model (with massive Dirac neutrinos) at all: due to the existence of just three Reidemeister moves and of the smallest length, additional gauge groups and additional gauge particles are impossible, as told in the main section of this article. Strands clarify the relation between the three dimensions of space, the three possible gauge groups, and the three fermion generations. In particular, strands imply that the emergence of three-dimensional space and the lack of higher gauge groups are two sides of the same coin. Strands thus answer the challenge that motivated the millennium problem *negatively* [281].

Three reasons imply that the physical Yang-Mills millennium problem has no relation to nature. First, the Yang-Mills millennium problem explicitly assumes that continuous space, continuous fields, perfect locality, perfectly local operators and perfect point particles exist. Thus, the millennium problem explicitly disregards the smallest length and all the other Planck limits of relativistic quantum gravity. Secondly, the millennium problem disregards the way that particles, quantum fields, and gauge groups arise in nature. Thirdly, the millennium problem disregards the way that spatial three-dimensionality arises in nature.

All three reasons for the lack of any relation between the Yang-Mills millennium problem and nature are *independent* of the strand tangle model. Whatever the unified description of nature may be, it does not comply with the assumptions of spatial continuity and of locality in the Yang-Mills millennium problem. Whatever the unified description may be, it will, by definition, explain why only the observed gauge interactions exist. Whatever the unified description may be, it will, by definition, explain why three-dimensionality arises. Thus, the statement of the Yang-Mills millennium problem contradicts nature and observations.

In summary, if the Yang-Mills millennium problem is restricted to the *physical* aspect, the existence of additional quantum field theories *in nature*, the answer is that such theories do not exist. This has been known experimentally for roughly a century. Strands confirm and *derive* this observational result, possibly for the first time. The best one can do is to restrict the Yang-Mills millennium problem to proving the existence of a finite mass gap in the special case of the gauge group SU(3) of the strong nuclear interaction (in vacuum). Only in this case is the answer positive, due to the existence of glueballs – both in observations [62] and in the strand tangle model [86].

The *mathematical* aspect of the millennium problem has various issues that arise independently of strands. *Continuous* space is in contrast with the minimum length in nature. Continuous space has no units of length and time. Units of length and time are needed to discover and determine the quantum of action $\hbar$. Continuous space is *in contrast* with quantum field theory.

In mathematics, one can choose to ignore nature and physics and assume both continuity and quantum field theory at the same time. This leads to a further issue. The statement of the millennium problem defines quantum field theory as a structure realizing specific axioms. This implies the assumption that the quantum of action $\hbar$ and the speed of light c are finite. But exact continuity implies that the Planck length is zero. This implies that G must vanish, in contrast to observations. A vanishing G implies that mass units cannot be defined and mass values *cannot* be measured.

In mathematics, one can choose to ignore nature and physics and assume continuity, quantum field theory, and non-vanishing G. However, if one insists that G is finite after all, then one has a non-vanishing minimum length. The minimum length implies that there are no points, no sets and no axioms. The lack of axioms, sets and points is deduced in detail in Appendix G. Quantum field theory and finite G, taken together, imply that there is *no* axiomatic quantum field theory.

In mathematics, one can choose to ignore nature and physics further and assume both continuity and axiomatic quantum field theory at the same time. The two assumptions taken together, imply the description of particles as exact point particles. The assumptions are purely mathematical and contradict general relativity with its impossibility of objects that are smaller than the Schwarzschild radius determined by their mass. Thus the concepts of 'particle' and 'quantum field' assumed in the millennium problem contradict general relativity and have no relation to actual particles in nature or actual quantum fields. The two mathematical assumptions have an unfortunate effect. Different elementary particles differ at least at the Planck scale. Planck scale differences explain the existence of different quarks and the existence of electrons, muons and tau leptons. Eliminating the Planck scale by assuming point particles prevents deriving the existence of any specific elementary particle. Eliminating the Planck scale by assuming point particles prevents deriving the three particle generations. Eliminating the Planck scale implies that the existence of elementary particles cannot be derived but must be *assumed*. Therefore, it is impossible to derive whether additional gauge bosons exist. This implies that it is impossible to derive whether additional gauge interactions with additional gauge groups exist.

In mathematics, one can choose to ignore nature and physics completely and assume continuity, axiomatic quantum field theory, and point-like gauge bosons for gauge groups that do not exist in nature. All these unphysical assumptions are suggested by the formulation of the millennium problem. But all these assumptions also prevent deducing the existence of any additional particle asked for in the millennium problem, such as generalizations of glueballs. In particular, the unphysical assumptions imply that particle mass is a mathematical parameter and not the result of an underlying mechanism of nature. Because of all these unphysical and contradictory assumptions, the existence of any mass gap asked for in the millennium problem *cannot* be deduced.

In short, the three limits of nature – the speed limit c, the quantum of action $\hbar$, and the maximum force $c^4/4G$ – imply a minimum length, given by twice the Planck length. In nature, the minimum length implies the lack of continuity of space, and thus the lack of perfect locality. In nature, only this lack of continuity of space allows the appearance of wave functions, quantum field theory and different particles, the definition of measurement units, the definition of physical observables, including mass, and the determination of the dimensionality of space. Instead, the Yang-Mills millennium problem assumes counterfactual continuity of space, counterfactual perfect locality, counterfactual point-like particles, counterfactual axiomatic quantum field theory, counterfactual properties of gravitation, and a counterfactual origin of three-dimensionality. The Yang-Mills millennium problem continues by asking about counterfactual gauge interactions, counterfactual gauge bosons, counterfactual generalized glue balls, and counterfactual mass mechanisms. On such a basis, the proof of additional axiomatic quantum field theories and their mass gaps is either impossible or worthless. In any quantum field theory that is mathematically consistent and that has a relation to nature, no Yang-Mills theories beyond SU(3) exist. A finite mass gap arises only for the gauge group SU(3) of the strong interaction [86].

G On physics' axioms and Hilbert's sixth problem

The following arguments do not rely on strands. However, strands allow visualizing them.

Combining maximum speed c, the quantum of action $\hbar$, and maximum force $c^4/4G$ yields the existence of a minimum length in nature. The minimum length is both the smallest possible length measurement result and the smallest possible length measurement error. The same applies to time. The smallest length error implies that space is not made of points, time is not made of instants, and coordinates are approximations. The same limitation holds for any other observable quantity: measurement results without errors are impossible.

The intrinsic measurement errors and the lack of exact measurement results for every physical observable imply the *lack of sharp boundaries.* There is no precise way to distinguish two values or even two objects. There are no exact boundaries in nature. In nature, there is no way to separate space into separate regions. At the Planck scale, nature is inherently fuzzy. Nature has no parts that can be distinguished *exactly*. Nature has no parts.

The lack of exact measurements and distinctions implies that it is impossible to define sharp boundaries. The lack of sharp boundaries implies the *lack of sets* in nature. Furthermore, the impossibility of exact distinction implies the impossibility of distinguishing elements from each other. This implies the impossibility of defining *elements* of sets in nature.

In mathematics, all axioms are based on distinctions, elements, boundaries, and sets. But all these concepts do not exist in nature, because in nature, there is a smallest length measurement error. As a result, the minimum length implies the *lack of axioms in nature.* Physics as a whole *cannot* be based on axioms. Hilbert's dream [282], most clearly stated in his sixth problem [283, 284], in which he asked for an axiomatic system for all of physics, cannot be realized as soon as gravity and quantum theory are combined.

In contrast, *branches* of physics *can* be based on axioms. Axioms are possible in quantum theory without relativistic gravity, or in relativistic gravity without quantum theory – because in these domains the minimum length does *not* arise. In branches of physics, length and time measurements can be – or at least can imagined to be – without errors, thus infinitely precise. Only the combination of quantum theory, relativity, and gravity introduces minimum errors and thus eliminates the possibility of defining axioms.

Despite the lack of axioms in physics, a complete and unified description of physics and nature remains possible. Such a description of nature has to be *logically circular.* Logical circularity makes describing nature possible despite the lack of axioms. Strands provide such a logical circular description: (background) space and time are part of the fundamental principle while, at the same time, (physical) space and time arise from averaging strands.

Even though space and time are approximations, they *must* be used to talk about nature. There is no way to do physics without space and time. One reason: the fundamental constants c, $\hbar$, G and k – and thus all measured quantities, directly or indirectly – have the metre and the second in their units and use space and time in their fundamental implementations. Another reason is that space and time are required to distinguish and to think; there is no way to do otherwise. Even the act of counting requires space and time. Above all, space, time and boundaries are needed to communicate. A description of nature without a space-time background is impossible.

Space, time and axioms do not exist in relativistic quantum gravity. More directly: in relativistic quantum gravity, equations are not available, as argued in Section 2. At observable scales, nature's intrinsic measurement errors require a *statistical* description of nature.

The arguments just given against the existence of axioms in a unified theory of physics can be summarized: *at the Planck scale, there are no parts.* At the fundamental level, nature is a unity without parts. The lack of parts goes hand in hand with the lack of axioms. A well-known proponent of this approach was Dante. In 1320, in the last canto of his *Commedia*, he described the universe as a knot spreading all over nature.

The first mathematical proposal that eliminates exact parts has been given by Kauffman [285, 286]. In his proposal, all sets and elements are approximate concepts that result from the folding of a unique fundamental entity. Kauffman's proposal is realized by the fundamental principle given in Figure 7. In the strand tangle model, nature consists of a single closed strand, and all *parts* are approximate. The strand tangle model confirms the unity of nature and the lack of exact parts in nature. All parts of nature – points, instants, particles – are low-energy approximations.

In short, every unified description of motion and in particular every description of relativistic quantum gravity implies the smallest length and time measurement errors. Because of the smallest measurement errors, precise distinctions are impossible at the Planck scale. Therefore, an *axiomatic* theory of relativistic quantum gravity, an *axiomatic* unified theory, is impossible. However, the conclusion does not prevent finding a unified description of nature. Any unified description must be *logically circular, statistical,* and contain *no parts.* Although relativistic quantum gravity implies that points and instants do not exist, emergent space and time must be used as *communication tools* to describe nature. Fluctuating strands of Planck radius whose crossing switches define the quantum of action realize these requirements.

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