The Maximum Force $\frac{c^4}{4G}$ In Nature

Origin, arguments, paradoxes, experiments, consequences –
20 years later

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Statement

There is a maximum physical speed in nature:

$$v \leq c = 299\,792\,459\,\mathrm{m/s} \approx 3 \cdot 10^{8}\,\mathrm{m/s}.$$

It implies *special* relativity.

There is a maximum physical force in nature:

$$F \leq \frac{c^4}{4G} \approx 3 \cdot 10^{43}\,\mathrm{N}.$$

It implies *general* relativity (see below).

There is also a maximum physical power in nature:

$$P \leq \frac{c^5}{4G} \approx 9.1 \cdot 10^{51}\,\mathrm{W}$$

and more (see later).

A Challenge For You

Think about all counter-arguments to maximum force or maximum power you can find.

Then check whether they are answered in the following.

Note that

- $c^4/4G$ and $c^5/4G$, unlike c, do *not arise* in daily life.
- $c^4/4G$ and $c^5/4G$ are hard to reach: they require gravitational *horizons*.
- $c^4/4G$ and $c^5/4G$ are not related to quantum theory. The limits are *classical*.

Reasons

1. Force is energy per length: any force acting along a path *deposes* energy along its length. The highest energy per length ratio is achieved when a Schwarzschild black hole of energy Mc^2 is deposed over a length given by its diameter $4GM/c^2$. This yields a maximum force

$$\frac{c^4}{4G} \quad .$$

2. Maximum force is also a consequence of the definition $F = ma$. In special relativity, the acceleration of (the front of) a body of length l *is limited* by $a \le c^2/l$. As a result, the force on a body of mass m and length l is limited by $F \le c^2(m/l)$. The largest ratio m/l arises for a black hole, with a value $c^2/4G$. This yields a maximum force value

$$\frac{c^4}{4G}$$

that is independent of the mass and the length of the body.

A Further Reason, From General Relativity

3. Consider the force produced by a Schwarzschild black hole when a test mass m is lowered, using a rope, towards the horizon. The force of gravity $F(r)$ is

$$F = \frac{GMm}{r^2\sqrt{1 - \frac{2GM}{rc^2}}} \quad .$$

Despite appearances, there is no divergence at the horizon! Every test mass is *extended* in space and needs to be located *outside* of the horizon. The test mass itself has a minimum size given by its own Schwarzschild radius $2Gm/c^2$. Neglecting spacetime effects by assuming $m \ll M$, the minimum distance between the mass centres is $r = 2G(m + M) = c^2$. This yields

$$F = \frac{c^4}{4G}\frac{M\sqrt{m}}{(M+m)^{3/2}} \leq \frac{c^4}{4G} \quad .$$

History

No mention by Einstein. That is strange, because

$$T_{\mu\nu} = \frac{c^4}{8\pi G} G_{\mu\nu} \quad .$$

Maximum force c^4/G was (probably) first mentioned in writing in 1968 by Elizabeth Rauscher. Maximum force was mentioned, rediscovered and explored by Treder, by Heaston, by de Sabbata and Sivaram, and various other scholars. The factor $1/4$ was deduced by Gibbons (2002) and independently by Schiller (2003).

Independently, maximum power c^5/G was (probably) first mentioned in 1973 by Dennis Sciama.

Experiments

- Force: no measurement comes even close.

- Power or luminosity: $P_{max} = c^5/4G \approx 9.1 \cdot 10^{51}$W is 50700 solar masses per second.

No visible or electromagnetic source comes close, because of the Schwinger limit due to electron-positron pair production.

Black hole mergers arrive at 230 ± 80 solar masses per second, about 0.5% of P_{max}.

Even the universe does not exceed P_{max}, as measurements show. And it never did.

Curvature

The force near a black hole implies that space is *curved* around a mass.

In short, maximum force implies that vacuum is *elastic*.

The elasticity of a material can be described with the *shear modulus*. The shear modulus also determines the *shear strength*, i.e., the maximum shear that a material can support (before breaking). The two quantities are related by a factor of order O(1).

Likewise, the elastic constant of the vacuum, $c^4/8\pi G$, determines, within a factor O(1), the maximum force $c^4/4G$ that the vacuum can support.

From Maximum Force To The Field Equations

The energy density ϵ in vacuum is a force per area. A maximum force $c^4/4G$ describing the elasticity of vacuum implies

$$\epsilon = \frac{c^4/4G}{A} \quad .$$

This is the maximum energy density for a spherical surface. For a spherical surface of radius r and curvature $R = 1/r^2$, the area is related to curvature by $A = 4\pi/R$. The relation above then becomes

$$\epsilon = \frac{c^4}{16\pi G} R \quad .$$

This is also the maximum possible curvature for a sphere. For a general observer, the energy density ϵ is replaced by the energy–momentum tensor $T_{\mu\nu}$ and the curvature $R/2$ is replaced by the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} R/2$. This yields

$$T_{\mu\nu} = \frac{c^4}{8\pi G} G_{\mu\nu} \quad .$$

Once More From Maximum Force To The Field Equations (Condensed)

A maximum force $c^4/4G$ implies, on a horizon with surface gravity a and area A, a limit on the energy of a falling system:

$$E = \frac{c^2}{8\pi G}\, a\, A$$

and thus

$$\delta E = \frac{c^2}{8\pi G}\, a\, \delta A \quad .$$

This is the *first law of horizon mechanics.* It yields (Jacobson 1995)

$$\int T_{ab} k^a d\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab} k^a d\Sigma^b$$

which implies

$$T_{ab} = \frac{c^4}{8\pi G} \left(R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right) \quad .$$

These are the full field equations of general relativity.

The Principle of Maximum Force

The slides so far show two results.

- General relativity contains maximum force. (Rauscher, 1968)
- Maximum force contains general relativity. (Schiller, 2003)

One can speak of the *principle* of maximum force.

Note: one *also* needs maximum speed c to deduce general relativity.

Inverse Square Gravity

A system moved in a circle deposes energy along the circumference. The energy per enclosed surface area is limited by

$$\frac{E}{A} C_{min} = F_{max} \quad .$$

Insert the area $A = 4\pi r^2$, the maximum force $F_{max} = c^4/4G$ and, from special relativity, energy $E = c^2 M$ and minimum circumference $C_{min} = \pi L_{min} = \pi c^2/a$. Together, this yields

$$a = \frac{GM}{r^2}$$

as a direct consequence of maximum force in flat space.

Counter-Arguments

• Lorentz boosts *reduce* force.

• Adding forces? Only if at the same point. Limit applies to the sum. But *no* limit applies to sums of forces at different points (like for speed).

• Gravity is weak – how can it provide an upper limit? The limit comes from curvature and is valid for *all* interactions.

• Charged black holes? No problem.

• Some counter-examples to $c^4/4G$ in the literature claim different factors, or even no limit at all. All have errors.

• Renormalization of G? There is no proof for its existence.

• Force between two black holes? Research topic!

• Exceeding maximum power through addition of sources? Does not succeed.

• Alternative theories of gravitation: usually do not agree with maximum power.

• Modified Newtonian dynamics (MOND): not in contrast.

More Limits

$\frac{c^3}{4G} = 1.0 \cdot 10^{35}$ kg/s is the maximum mass flow rate.

$\frac{c^2}{4G} = 3.4 \cdot 10^{26}$ kg/m is the maximum mass per length.

$\frac{c}{4G}$ is the maximum product of cosmic matter density, (Hubble) radius and (Hubble) time.

$\frac{1}{4G}$ is the maximum product of cosmic matter density and (Hubble) time squared.

The maximum electric field $E_{max} = 3c^4/(4Ge) \approx 5.7 \cdot 10^{62}$ V/m and the maximum magnetic field $B_{max} = 3c^3/(4Ge) \approx 1.9 \cdot 10^{54}$ T do not occur in nature, because of the Schwinger limit.

A selection of consequences for quantum gravity

Combining the limits on speed v, force F and action W using the general relation $Fvt = W/t$ leads to a limit on time measurement given by

$$t \geq \sqrt{\frac{4G\hbar}{c^5}} \approx 1.1 \cdot 10^{-43}\,\mathrm{s} \quad ,$$

i.e., twice the Planck time. Shorter times cannot be measured or observed. Similarly, the relation $Wa = Fv^3/a$ leads to the acceleration limit

$$a \leq \sqrt{\frac{c^7}{4G\hbar}} \approx 2.8 \cdot 10^{51}\,\mathrm{m/s^2} \quad ,$$

or half the Planck acceleration. The mixing of space and time yields a limit for length

$$l \geq \sqrt{\frac{4G\hbar}{c^3}} \approx 3.2 \cdot 10^{-35}\,\mathrm{m} \quad ,$$

twice the Planck length. Thus there are no points in space.

- Area, volume, curvature, jerk (see the dynamic Casimir effect) etc. are limited.
- Leads to a limit on mass density given by $\rho_{max} = c^5/(16G^2\hbar) \approx 3.3 \cdot 10^{95}\,\mathrm{kg/m^3}$.

There are no singularities.

- The factor 1/4 in black hole entropy is due to the 1/4 in maximum force.

What happens maximum force is approached or exceeded?

Exceeding the force limit would mean the ability to act beyond a horizon.

Exceeding the speed limit would mean the ability to circumvent causality.

What happens if one approaches the force limit?

Just before a material loses its elastic properties, *defects* arise.

Similarly, just before the vacuum loses its elastic properties, defects arise; and vacuum defects are *particles*.

Indeed, whenever one approaches maximum force by approaching a horizon, particles arise, e.g., in the form of Hawking radiation.

What if maximum force did not exist?

- What if maximum speed c would not exist?

Special relativity would not be valid, light would not be the fastest moving system, and, without a natural invariant standard, speeds *could not be measured.* And thus not observed.

- What if maximum force $c^4/4G$ would not exist?

The field equations would not be valid. Curvature and energy-momentum tensors would not be connected.

Force, power, luminosity, mass rate, or mass to length ratio *could not be measured* – and thus not observed – because no natural, invariant standards for them would exist.

Outlook

Special relativity: $v \leq c$.

Quantum theory: $W \geq \hbar$.

General relativity: $F \leq c^4/4G$.

Physics can be summarized in 9 lines. See
`https://www.motionmountain.net/9lines.html`.

Predictions:

There is *no deviation* from general relativity at strong fields.

There is *no* trans-Planckian physics.

The three limits also hold in a *unified* theory.

Maximum force $F \leq c^4/4G$ is the *last law of physics*.

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References on the last slide.

Short bibliography

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