An emergent model for wave functions explaining gauge interactions and elementary particles

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A geometric model for wave functions is tested against observations. The model is based on Dirac's proposal to describe spin 1/2 particles as tethered objects. A quantum particle is modelled as a fluctuating tangle made of unobservable tethers of Planck radius but with observable crossing switches. Time-averaged spatial distributions of crossings have all the properties of conventional wave functions. Crossing distributions reproduce entanglement, collapse and decoherence, and yield the path integral formulation. For spin-less particles, the non-relativistic evolution of the crossing distribution follows the Schrödinger equation. Fermion behaviour, antiparticles, spinors and the spin-statistics theorem arise naturally. Relativistic spinors derived from crossing distributions of fluctuating tangles are confirmed to be described by the Dirac equation, as Battey-Pratt and Racey showed already in 1980.

Several consequences of the strand tangle model go beyond quantum theory. Classifying the possible tangle structures determines the possible elementary particles, their quantum numbers and their mass values. Classifying tangle deformations with the three Reidemeister moves determines the possible gauge interactions, their symmetry groups and their coupling constants. More than 50 precision tests are derived. All the tests predict that there is no physics beyond the standard model with massive Dirac neutrinos. The Yang-Mills millennium problem is clarified. All the tests agree with the observations.

Keywords: origin of wave functions; origin of quantum theory; strand tangle model; origin of elementary particles; origin of elementary particle masses; origin of gauge groups.

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Contents

Part I: A geometric model for emergent wave functions

Quantum theory uses wave functions to describe the state of quantum systems, such as the state of a quantum particle. The wave function is described by one or several complex fields that follow the Schrödinger equation or the Dirac equation. The present article argues step by step, that wave functions and their evolution equations emerge from a more fundamental description, namely from a *fluctuating tangle of strands.* All known results about wave functions are recovered from the strand tangle model. In addition, several differences are found. Only these differences make the model worth exploring. The differences explain many open problems in the standard model of particle physics: they explain the origin of elementary particles, of the three generations, of the particle masses, of the gauge interactions, of their gauge groups, and of their couplings. All differences turn out to agree with observations.

The presentation is self-contained. Each section ends with a summary paragraph that begins with *In short*. These summaries contain all important results and allow for quick reading. More than 50 high-precision tests of the strand tangle model are derived. They are numbered with **Test *n***.

The first part of the article explains how wave functions arise from the fundamental principle for strands. The second part explains superpositions and measurements, including decoherence and entanglement. The third part derives the emergence of evolution equations and the first experimental predictions. The fourth and most important part deduces the standard model of particle physics with massive Dirac neutrinos and many experimental predictions, covering neutrino masses, supersymmetry, dark matter and more. The appendices cover special topics such as qubits, gravitation, cosmology, renormalization, and some aspects of axiomatic quantum field theory.

In short, the article presents the strand tangle model for wave functions and for quantum particles. Experimental tests and predictions are derived. They agree with all data. Moreover, the strand tangle model explains properties of nature that have no other explanation so far: in the model, wave functions, the observed spectrum of elementary particles and the observed gauge interactions arise naturally. In particular, every evolution equation and every continuous observable used in physics *emerges* from fluctuating strands.

1 Searching for emergent wave functions

The search for an *emergent* description of quantum theory has been a research topic for almost a century. The efforts of de Broglie, Schrödinger, Bohm, Bell, Kochen, Specker and many other researchers in the twentieth century had a lasting impact that fuelled hope on the one hand and limited possibilities on the other. In this century, the books by Adler [1] and by Peña, Cetto and Valdés [2] have examined this subject for consistency. They conclude dthat an emergent description of quantum theory using a statistical basis is possible, but did not propose any specific model. Many other scholars have studied the topic over the past two decades [3–16]. Emergent quantum theory is also a topic of regular conferences. The collection presented in reference [17] is a recent example.

Despite all the efforts, no successful model for emergent wave functions has been proposed thus far. The main reason for this seems to be that the options left by the aforementioned investigations are both difficult to identify and quite different from the habits of thought. Indeed, the *strand* *tangle model* for wave functions is rather unusual. Nevertheless, it agrees with the experimental results and with the requirements for any emergent description of quantum theory. The model is based on the counterintuitive notion of *tethers*, that is, on *unobservable* connections between any quantum particle and the cosmological horizon. Tethers were introduced around 1929 by Dirac to explain spin $1/2$ and were further explored by Battey-Pratt and Racey in 1980. The strand tangle model extends the idea of tethers and suggests that elementary particles, even though they are effectively point-like, are themselves made of *fluctuating filiform constituents:* elementary particles are modelled as tangles of strands.

In the following, the term *strand* is used for a filiform entity in three-dimensional space, without ends, of Planck radius, that fluctuates in shape. The mathematical definition will be given shortly. The concept of a *tangle* is taken from topology and is defined intuitively in Figure 1 as a collection of strands tied together [18]. The region of space in which this occurs is called the tangle *core*. The term *tether* is used for the strand segments that connect the core to the cosmological horizon. Thus, a tangle consists of a core and tethers. As will be shown below, tethers are responsible for spin $1/2$, for fermion behaviour, and for all other quantum effects.

In physics, extended filiform particle constituents avoid arguments against constituents inside elementary particles. Experimentally, all the known elementary particles lack particle-like ('point-like') constituents [20]. Also theory argues against particle-like constituents because such constituents would have to be confined inside an extremely small volume, resulting in extremely large kinetic energy. This would lead to mass values for the known elementary particles that are much higher than those observed. In contrast, *extended filiform constituents* imply that the region where they are tangled is the region where the wave function of the quantum particle is localized. As shown below, tangles of extended filiform constituents can be counted, exhibit fermion behaviour, and have spin $1/2$, parity, chirality and many other properties of quantum particles. However, tangles are also challenging: a mechanism must be found that allows the mass to emerge from tangling.

In short, already a long time ago, the counter-intuitive approach that quantum particles are tethered explained spin 1/2 and the Dirac equation. The idea that particles consist of extended filiform constituents suggests that an emergent description of quantum theory may be possible. Four tasks remain to prove this suggestion: specifying a strand tangle model for wave functions, demonstrating that it leads to quantum theory, explaining the origin of quantum particle mass values, and comparing the differences between tangles and conventional quantum theory with experiments.

2 A short history: from the Planck limits to the strand tangle model

In 1905, Einstein derived special relativity from the maximum energy speed c observed in nature [21]. The *principle* of maximum speed, together with all its consequences, was confirmed in all experiments ever carried out.

In the years around 1910, Bohr used to present quantum theory [22] as the sum of all consequences of the smallest measurable action value $\hbar$, the quantum of action, that had been discovered by Planck. The limit given by the quantum of action was thoroughly checked through thought experiments and real experiments. The *principle* of the quantum of action led Schrödinger and many others to develop the concept of the wave function, to describe its evolution and to present its use

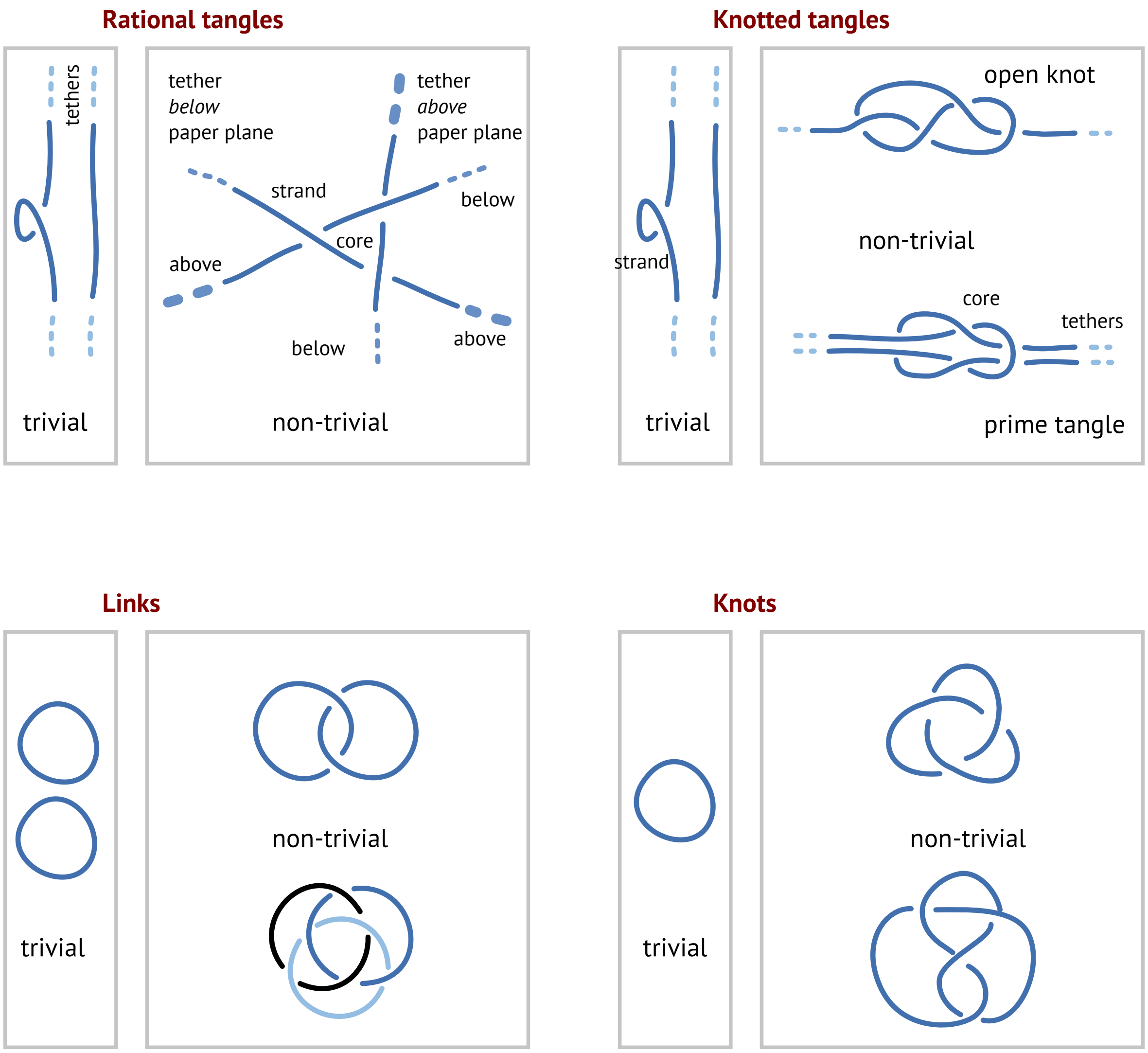

FIG. 1: The differences and overlaps among rational tangles, knotted tangles, open knots, prime tangles, links and knots – as defined in mathematics – are illustrated. *Rational* tangles are formed by braiding their tethers (shown with dashed lines); *knotted* tangles cannot be formed in this way. Equivalently, rational tangles, unlike knotted tangles, can be disentangled by moving their tethers in space. Rational tangles owe their name to their close relation to rational numbers, in the case of two strands [19].

for physical measurements. Later, Dirac incorporated spin $1/2$ and the energy speed limit c into quantum theory. Dirac's equation and Planck's description of light led to quantum electrodynamics, which agrees with all experiments ever performed.

Around the year 1973, Elizabeth Rauscher discovered that general relativity implies a maximum force [23]. In the years 2000 to 2002, it became clear that Einstein's field equations can be deduced from the *principle* of maximum force $c^4/4G$ or from the *principle* of maximum power $c^5/4G$ [24–28]. The force and power limits, as well as general relativity, agree with all thought experiments and with all real experiments ever performed [24–62]. If preferred, one can also start from the equivalent *principle* of maximum mass to length ratio $c^2/4G$.

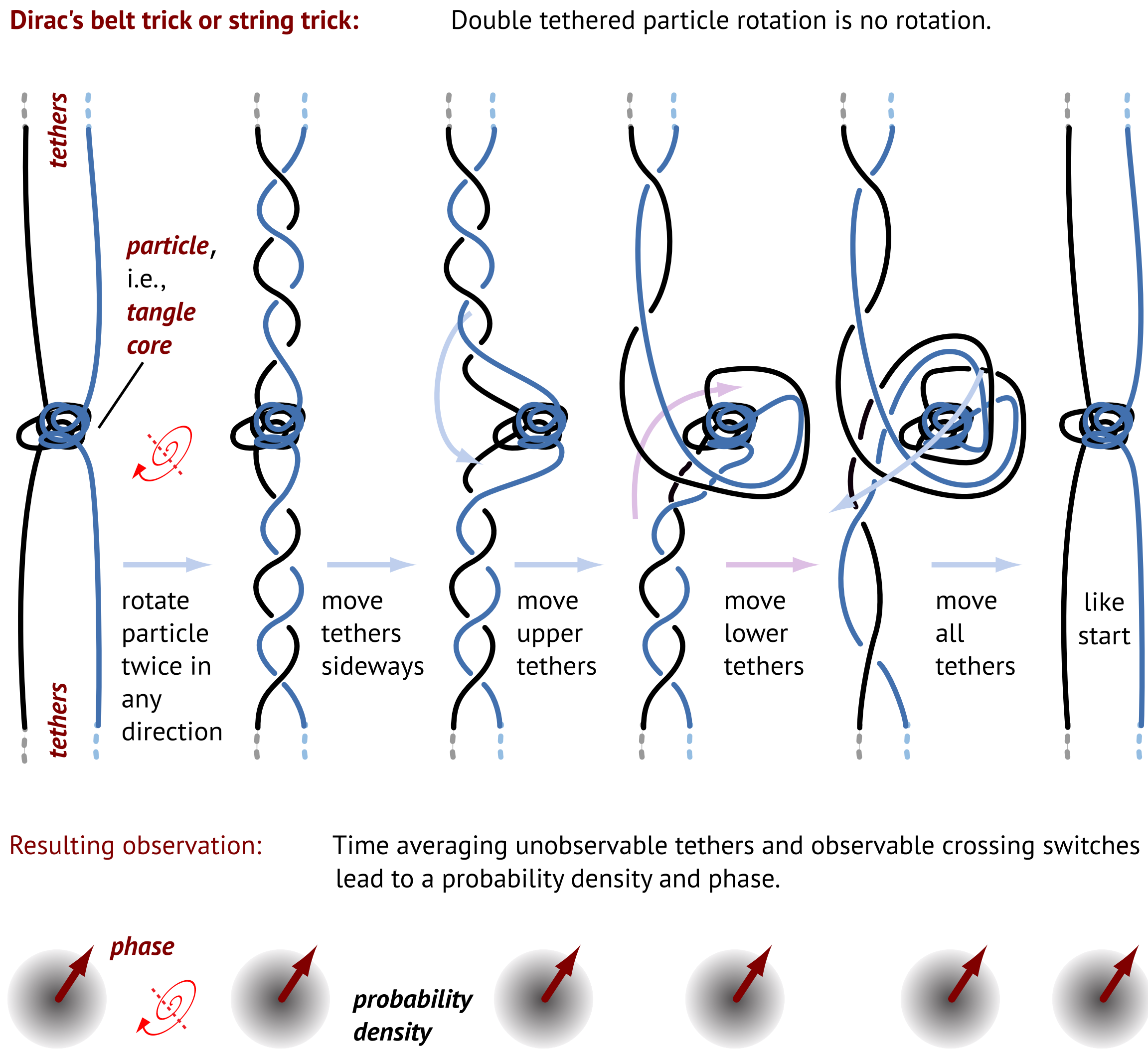

FIG. 2: Upper part: Dirac's *string trick* or *belt trick* shows that a *double* rotation by 4π of a tethered, indivisible particle – here an oriented tangle core – is equivalent to no rotation at all. Lower part: the particle tangle leads to a probability density and a phase once the ideas from Figure 10 and those explained in the text are used. The belt trick works as long as the tethers are allowed to fluctuate, untangle, and are assumed to be unobservable, whereas their crossings are assumed to be observable. A *single* rotation by only 2π does *not* allow this to occur. Indivisible localized cores with several tethers thus exhibit the properties that characterize spin $1/2$ particles. As a result, a tethered particle can *rotate continuously*. Careful observation also shows that Dirac's trick couples rotation and *displacement* of the tethered core. The coupling of rotation frequency and displacement determines the inertial mass, as discussed in Section 30. The twists in the tethers are virtual gravitons that determine the gravitational mass, as outlined in Section 34. Dirac's string trick or belt trick can be easily reproduced with a real belt, with any long strip of paper, or with two or more ropes. Links to animations illustrating the belt trick are given later on. (The figure is modified from reference [63].)

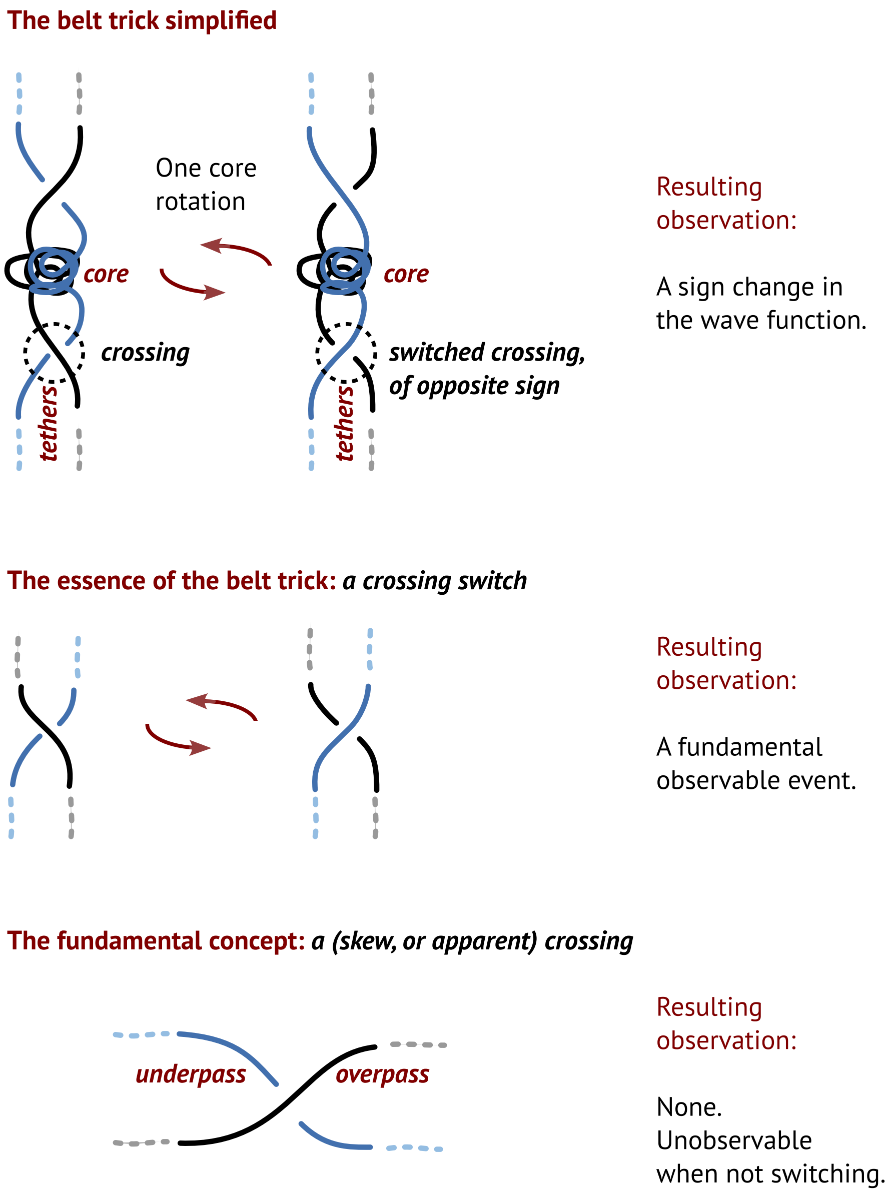

FIG. 3: Dirac's belt trick is based on the idea that a *crossing switch* of tethers leads to an *observable* change in the sign of the wave function. The figure shows that a crossing switch is related to a core rotation by 2π. Simplifying further, one sees that a crossing switch is the sign change of a crossing, a common process in topology (and defined in Section 8). This sign change is *observable* even though the tethers themselves are *unobservable*. The bottom diagram highlights that all crossings are apparent. Because strands cannot intersect, they have a *skew* configuration: each crossing consists of a strand passing over another one.

Around the year 2003, the mentioned results allowed summarizing special relativity as the consequence of maximum speed c, quantum theory as the consequence of the smallest action $\hbar$, and general relativity as the consequence of maximum force $c^4/4G$ If desired, the maximum mass to length ratio $c^2/4G$ can also be used to characterize general relativity. Both limits are realized by black holes. Taken together, the limit triplet implies that no *trans-Planckian* effect of any kind

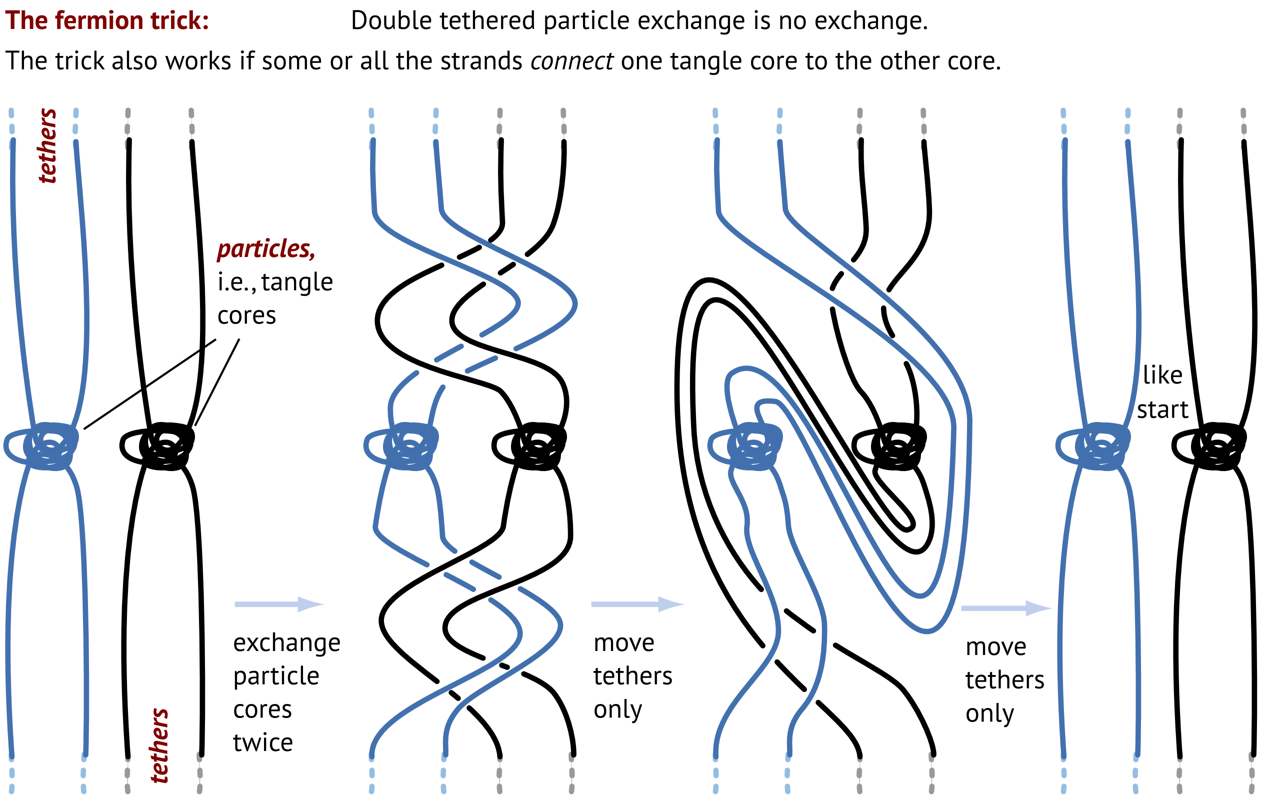

FIG. 4: The *fermion trick* shows that a *double* particle exchange of tethered particles is equivalent to no exchange at all. This works as long as the tethers are allowed to fluctuate and untangle. Tethers are assumed to be unobservable, whereas crossing switches are assumed to be observable. In contrast, a *single* particle exchange does *not* allow untangling. Indivisible tangle cores made of several tethers thus show all the properties that characterize fermions. (The figure is modified from reference [64].) Animations illustrating the fermion trick are presented later on. The fermion trick is easily reproduced with two long strips of paper, or with two ropes.

exists in nature: no observation has found or will find a result beyond the corrected Planck limits, and no argument depended or will depend on the existence of values beyond these limits.

The Planck limits are called *corrected* because, traditionally, the quantity c^4/G is called the Planck force. However, in nature, the force limit $c^4/4G$ has an additional factor 4 in the expression [28]. In general, the corrected Planck limits of nature arise when G is replaced by $4G$ in the usual Planck limits. (Several Planck limits – the Planck energy, the Planck momentum and the Planck mass – are valid only for a *single elementary particle*. This condition is necessary to derive these particular limit values.)

Thus, modern physics predicts that all searches for observations or effects beyond the corrected Planck limits will fail [65]. The corrected Planck limits include the minimum length $\sqrt{4G\hbar/c^3}$ and the minimum time $\sqrt{4G\hbar/c^5}$. Indeed, no known observation contradicts the corrected Planck limits, despite intensive search efforts in special relativity [66], in quantum field theory and particle physics [20], in general relativity [28] and in quantum gravity [67].

The foundation of the main theories of physics using simple invariant limits brings up a question. Can the force spectrum, the particle spectrum and the fundamental constants arise in an equally simple manner? Dirac provided the first important clue to the answer. From around 1929 onwards, he used his *scissor trick* – also called the *string trick* or the *belt trick* – in his lectures. Dirac's trick, illustrated in Figure 2, captures the basic properties of spin $1/2$: the original situation reappears after a rotation by 4π – but *not* after a rotation by 2π. Indeed, after a rotation by 2π only, the sign of the wave function of a spin $1/2$ particle is observed to change, and it returns to the original sign only after a rotation by 4π. The only difference about a tethered system before and after a rotation by 2π is the sign of the tether crossings. This is illustrated in Figure 3. If a system is not tethered, no difference is detectable between the situation before and after a rotation by 2π. Tethers are the only visualization of spin $1/2$. No other visualization of spin $1/2$ exists.

In Dirac's belt trick and in the rest of this article, strands never actually cross; they cannot intersect. All crossings are *apparent* or *skew* and are recognizable as 'crossings' only in a two-dimensional projection. In three dimensions, crossings are recognizable by the shortest distance between the two involved strands.

In other words, the belt trick implies that *crossing switches*, are somehow observable – even if the tethers themselves and their crossings are not. A crossing switch is the change of *sign of a crossing*, i.e., the change of which strand passes under the other. (The sign of a crossing is defined mathematically in Section 8.) Dirac's trick implies that crossing switches lead to a sign change in the wave functions.

As Dirac explained [68], his trick also demonstrates that a spin value smaller than $\hbar/2$ is not possible. This result agrees with Bohr's statement: a smallest angular momentum value $\hbar/2$ implies a smallest observable action given by a spin flip, and the action for a spin flip has the smallest possible observable value $\hbar$. In other terms, crossing changes are related to $\hbar$. This connection was already stated by Kauffman in the 1980s [69, 70].

A variant of Dirac's belt trick is the *fermion trick*, illustrated in Figure 4. It describes the basic property of fermions: after a double particle exchange, two fermions return to the original situation – but not after a single exchange. Similar to Dirac's trick, the fermion trick is also due to *tethers*. Again, crossing changes are observable through their effects on the sign of wave functions, even if the tethers themselves are unobservable. The effects of tethers have been visualized in numerous internet videos. References [71–75] are examples.

No tethers are observed in everyday life. The idea of tethers attached to every quantum particle seems absurd. However, tethers clearly describe the main properties of quantum matter particles, namely their spin and their exchange behaviour. In fact, *no other model* for matter particles describes spin and exchange behaviour. Dirac's trick was the first hint that matter should be described with extended, filiform, *unobservable* constituents, for which *only crossing switches* lead to observable effects. More than fifty years later, in 1980, Battey-Pratt and Racey showed that tethers imply the full free Dirac equation [76]. Their argument is given below. In simple terms, Battey-Pratt and Racey proved that *Dirac's trick implies Dirac's equation.* They wrote to Dirac about their discovery, but he never answered.

Taken together, the properties of tethers suggest that

- ▷ *Every quantum effect* is due to observable crossing switches of unobservable tethers.
- ▷ The quantum of action $\hbar$ is due to a crossing switch.

These statements are valid for every quantum system: for particles, black holes, and space itself.

Tethers are *filiform* entities. Various researchers have explored filiform entities. In the 1990s, it was common lore that black hole entropy can be explained by filiform constituents [77]. Later, it became clear that filiform constituents are compatible with general relativity [78, 79], allow the description of black holes, and imply the field equations of general relativity [80]. In addition, photons have been visualized as helically deformed filaments [81]. In other words, suspicion arose that everything in nature could be made of strands.

However, if everything is composed of strands, what exactly are particles? The straightforward conjecture that particles might be modelled as *open knots*, i.e., as knots in infinitely long tethers, as illustrated in Figure 1, is unsuccessful. Despite intense attempts in this and similar directions [82–94], open knots, closed knots, bands, branched structures, and graphs do not explain the particle spectrum or the appearance of wave functions. Above all, none of these structures explains gauge interactions and particle reactions.

Between 2008 and 2014 it became clear that modelling elementary particles as *rational tangles* – i.e., as unknotted tangles – *does* explain wave functions, the particle spectrum, particle reactions, and particle interactions. The concept of a rational tangle is illustrated in Figure 1. It was found that the classification of rational tangles yields the elementary particle spectrum. It also turned out that the three known gauge symmetry groups arise from classifying strand deformations using the three Reidemeister moves [95]. These results are presented in detail below. Gravity and general relativity also arise. Unique and specific values for the particle masses, coupling constants, mixing angles, charges, and other quantum numbers arise naturally.

Together, the aforementioned results allowed us to derive both gravity and particle physics from a single fundamental principle inspired by the crossing switch of figurerefi-tether-obs. This fundamental principle, illustrated in Figure 6 and presented shortly, models particles as rational tangles, space as untangled strands, and gravitational horizons as weaves of strands. The various possible strand structures are presented in Appendix D. In particular, strands yield the Lagrangian of general relativity and the Lagrangian of the standard model with massive Dirac neutrinos. The presentation of the strand tangle model in reference [96] was followed by extensive tests of the consequences for particle physics, including quantum electrodynamics and quantum chromodynamics [63, 64, 97], and by tests of the consequences for general relativity, black holes and quantum gravity [80, 98]. All the consequences agree with the observations. The present article is a *prequel* that describes the basic *quantum mechanics* as a consequence of the strand tangle model.

In short, the results by Dirac, by Battey-Pratt and Racey, by Rauscher, and by Gibbons prove that three ingredients are required to describe all observations in nature: unobservable tethers, observable crossing switches, and invariant (corrected) Planck limits – including $\hbar$, c, $c^4/4G$, $\sqrt{4G\hbar/c^3}$ and $\sqrt{4G\hbar/c^5}$. These three ingredients are combined in the fundamental principle of the strand tangle model by using fluctuating strands with Planck radius, by defining $\hbar$ with crossing switches, and by limiting time intervals to $\sqrt{4G\hbar/c^5}$. In this way, the strand tangle model realizes all the (corrected) Planck limits and fulfils all the requirements for a unified description of nature. In particular, the fundamental principle explains the standard model and general relativity [63, 64, 80, 96–98]. The rest of this article explains why and how crossing switches are observable in quantum theory, and how tangles of strands lead to wave functions, particles, and interactions.

3 Quantum gravity forbids equations of motion and requires strands

The search for a theory of emergent quantum mechanics is closely tied to the search for a theory of relativistic quantum gravity. The first search is about an emergent description of wave functions, whereas the second search is about an emergent description of space. It only takes a few lines to deduce that both searches require finding the constituents of particles, wave functions, space and black holes, that all these entities are made of common constituents, and that these constituents must be strands.

In nature, action W can be defined as

$$W = F\,l\,t = \frac{F\,l^2}{v} \quad , \tag{1}$$

where F is force, l is distance or length, and v is speed. Equivalently, action can be defined as

$$W = E\,t = m\,c^2\,t = \frac{m}{l}\,c\,l^2 \quad , \tag{2}$$

where E is energy, m is mass. The expressions can be used to insert the maximum speed c measured in nature, the minimum action $\hbar$ measured in nature and for general relativity, either the maximum force $c^4/4G$ or, if preferred, the maximum mass per length ratio $c^2/4G$ measured in nature. The last two limits are only achieved by black holes. The result in both cases is

$$l \geq \sqrt{\frac{4G\hbar}{c^3}} \quad . \tag{3}$$

In other words, the domain of nature where maximum speed, maximum curvature, and the quantum of action play a role at the same time – the domain of quantum gravity – is characterized by a *minimum length*. The minimum length is given by twice the Planck length. Smaller lengths *cannot* be measured or observed. The minimum length also describes the smallest possible length measurement error [99–101]. The existence of a minimum length of nature has several important consequences.

Minimum length implies that continuity, derivatives, differentials, discrete points and discrete instants of time do not exist in nature [65]. As a result, all these concepts are only *approximate:* they are due to averaging. Therefore, all equations of motion and all Lagrangians are due to averaging. Because of this averaging process, at the Planck scale and at the foundations of nature, equations of motion and Lagrangians cannot exist. (For additional reasons leading to the lack of equations, see Appendix A on the impossibility to observe single strands and Appendix B on the impossibility to deduce Lagrangians for single strands.)

The minimum length appears in the form of the minimum area in the expressions for the entropy S and the temperature T of black holes.

$$S = \frac{kc^3}{4G\hbar}A \quad \text{and} \quad T = \frac{\hbar c^3}{8\pi G M k} \quad , \tag{4}$$

where k is Boltzmann's constant and A and M are the surface and mass of the black hole, respectively. Because black holes have *finite* entropy, they are composed of a finite number of discrete constituents that fluctuate. Because black holes can be seen as limit cases for curved space *and* for densely packed particles, the constituents of black holes are also the constituents of space and particles. Because the black hole entropy contains the minimum area, the common constituents of space and particles have an effective cross section given by the minimum area. Because space and wave functions are *extended*, also the common constituents are extended. Consequently, particles and space are made of common constituents of the Planck radius that fluctuate and reach the cosmological horizon [65]. These common constituents are called *strands*.

The minimum length and minimum length uncertainty eliminate many alternative types of constituent [65]. The minimum length is incompatible with loops, knots, strings, bands, branched structures, networks, and ribbons. The black hole entropy eliminates strands of finite length and knotted strands. The minimum length further eliminates many alternative options for space and space-time. In particular, minimum length is in contrast with additional dimensions, non-commutative space, singularities, conformal symmetry, holography, higher dimensions, space-time foam, spatial lattices, supersymmetry, supergravity, conformal gravity, conformal field theory, anti-de Sitter space, de Sitter space, twistors, doubly special relativity, spin networks, and with combinations of continuous space with additional discrete or continuous mathematical structures. Only a small number of approaches to quantum gravity are not eliminated by the existence of a minimum length.

In the literature, fluctuating filiform constituents for space were explored by Carlip [79] and independently by Botta-Cantcheff [78]. A strand crossing, defined below, also resembles a tetrahedron; such structures are used in some approaches to quantum gravity [102]. An interesting approach is the use of causal sets [103]; strands can be seen as a specific realization of causal sets. The behaviour of the spatial complement of strands, the space between them, was investigated in detail by Asselmeyer-Maluga [104, 105]. Instead of a universe made of only a single strand, one then has a universe made of space only, albeit an intricately shaped one. However, to date, none of these approaches have deduced a model for wave functions, particles, or gauge interactions.

Research in theoretical physics usually proceeds in steps. First, a fundamental principle is proposed and specified. Subsequently, a general description or theory is logically deduced. From this theory, all consequences are logically deduced. All the consequences are tested against observations. If a test fails, the principle is falsified. If a test is positive, the next test is performed. This sequence of steps defines physical science and is followed below.

The tools of science are (1) precise concepts and principles, (2) logically correct deductions, and (3) precise comparisons with observations. These are the only criteria for judging scientific proposals. All other claimed criteria are habits of thought. A particularly persistent habit is the longing for equations. This longing cannot be fulfilled at the Planck scale.

In short, relativistic quantum gravity implies a minimum length in nature given by twice the Planck length. The minimum length implies that *no* equation of motion and *no* Lagrangian exist at the Planck scale.

▷ No new mathematics, no new equations, and no new Lagrangian are possible in fundamental physics or in the unified theory of relativistic quantum gravity.

The minimum length further implies:

▷ Horizons, space, and particles – thus all things in nature – are made of fluctuating strands of Planck radius.

The only way to test fundamental physics at the Planck scale is to test the *statistical* behaviour of fluctuating strands against experiments. Quantum effects, from probabilities to wave functions, are examples of this connection. So are the Unruh effect and black hole thermodynamics. Fluctuating strands appear to be the only constituents that yield space, general relativity and black hole entropy [80, 98] and that also yield quantum mechanics and the standard model [63, 64, 96, 97]. In the following, emphasis is placed on deducing quantum mechanics. Because strands are not observable directly, it is necessary to explore their crossings and their crossing switches.

4 Crossings of strands resemble wave functions

In this study, the term *crossing* is used in the sense of mathematical knot theory [106–108]. In other words, a 'crossing' appears only when the configuration of two strands is projected or drawn in two dimensions. In three dimensions, as is usual in knot theory, a crossing always implies a *skew* geometry of two strand segments. In three dimensions, the strand segments are *always* at a distance, as illustrated in Figure 5.

▷ A *strand crossing* is the region of the smallest distance between two skew strands.

The definition assumes that in the region of the crossing, both strands have a radius of curvature that is larger than the Planck length. Due to the Planck-sized radius of strands and their impenetrability, the shortest distance between two strand segments is always larger than the smallest length.

Crossings can occur between any two strand segments. In particular, crossings can occur in two segments of the same strand or of two different strands. Crossings also arise between two extremely distant strands – although they have no physical importance because no crossing switches – which define all physical observables – will arise between such distant crossings. For a specific strand configuration, the *observation* of a crossing depends on the position of the observer. The *existence* of a crossing, which is due to the smallest distance between two strands, is independent of the observer.

Crossings have mathematical properties that resemble wave functions. The resemblance will be used in the following to define wave functions with the help of crossings. The simplest wave function, as it appears in the Schrödinger equation or the Klein-Gordon equation, is defined by a real amplitude R and one phase φ:

$$\psi(\mathbf{x},t) = R(\mathbf{x},t)\mathrm{e}^{\varphi(\mathbf{x},t)} \quad . \tag{5}$$

In contrast, Pauli spinors have three phases and Dirac spinors have seven phases, as explained below. As shown in Figure 5, also strand crossings are described by amplitude and phase(s). The *amplitude* is large when the shortest distance between the two strands is small. (Intuitively, the probability density is high when the strand density is high, i.e., when the strand distance is small.) The (main) *phase* describes the orientation of the crossing around its axis. Both quantities are precisely defined in Section 8. Similar to strands and crossings, also wave functions are not observable. Similar to strands and crossings, also wave functions can be used to define physical observables. The other angles illustrated in Figure 5 play a role in spinor wave functions, as shown below.

Dirac's belt trick implies that crossing switches are observable. As expected, crossing switches are local processes defined with the help of crossings:

▷ A crossing *switch* exchanges which strand passes over the other.

The fundamental principle of the strand tangle model, illustrated below in Figure 6, defines a crossing switch visually and relates it to the (corrected) Planck units. Strands cannot intersect and cannot be cut. Due to the impenetrability of strands, crossing switches always arise via strand deformations or fluctuations, and in no other way. The observation of a crossing switch, like the observation of a crossing, depends on the observer.

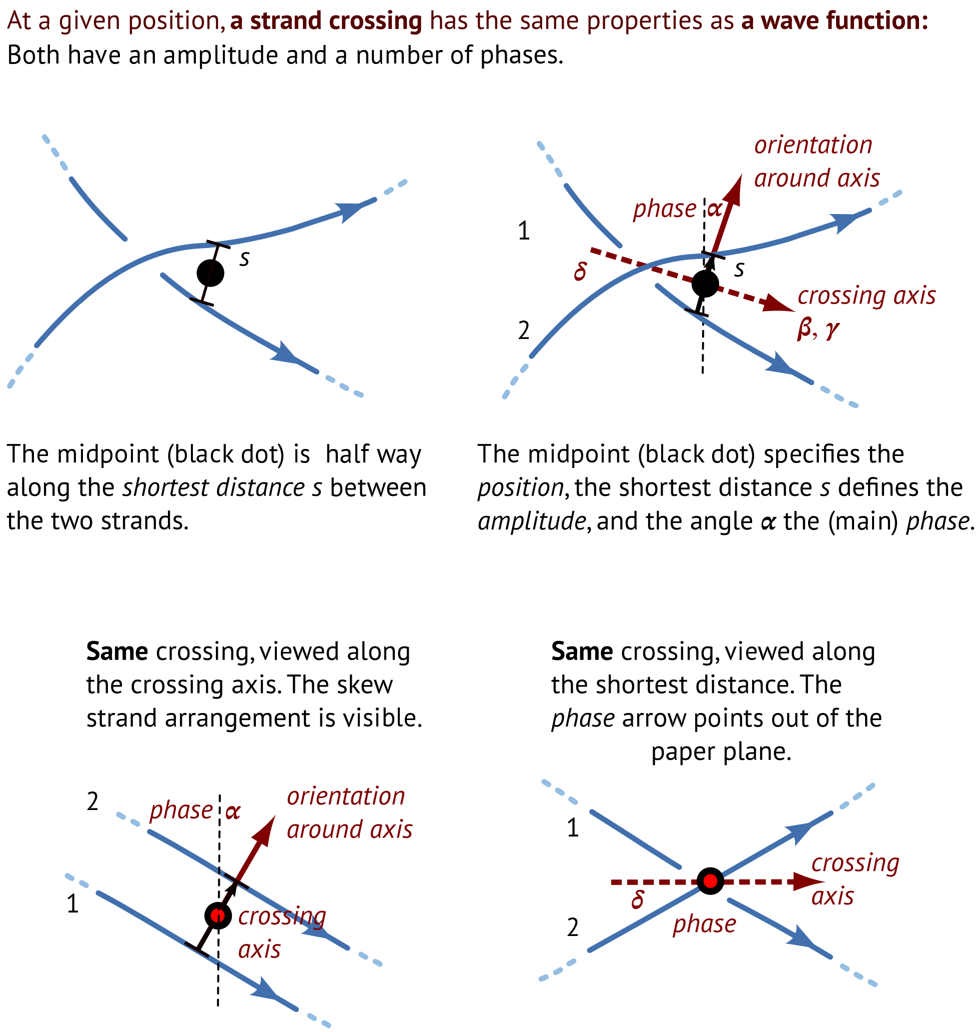

FIG. 5: A strand crossing consists of two skew strand segments separated by the shortest distance s. The shortest distance s is never shorter than the minimum length, due to strand impenetrability. A strand crossing allows defining amplitude and phase(s) around an axis. Therefore, a strand crossing at a point has the *same properties* as a wave function at a point. The *crossing axis* is defined, using the drawing in the centre, as the sum of the two tangent vectors at the two endpoints of s. (This requires defining a positive direction on each strand.) The (main) *phase* α describes the direction in which the shortest distance vector (pointing from the strand with the smaller number to the one with the larger number) points around the axis. The dotted direction defining *vanishing* phase around the axis is a matter of choice, as expected. Regardless of the choice of zero phase, phase differences are always defined uniquely. The *angles* or *phases* β, γ and δ play a role in spinors.

▷ The essence of Dirac's trick are *observable crossing switches* due to *unobservable strands*.

The final reason why crossing switches are observable – and that nothing else is – is given below, in Section 36, where it is shown that crossing switches couple to the electromagnetic field. Appendix A explains in detail why, in contrast to crossing switches, single strands or strand segments are always unobservable.

The fundamental principle states that a crossing switch defines a *physical event*. Events are local. In the strand tangle model, each event is a *process*. In particular, events are *discrete* and *countable*. The crossing switch is both the most fundamental *event* and the most fundamental *process* in nature. The fundamental principle states that every observed process in nature – motion, interaction, decay, particle transformation, or measurement – is *composed* of crossing switches. Not only is every *observation* and every *physical observable* due to and composed of crossing switches; but also every type of *change* is due to and is composed of crossing switches. Crossing switches are irreducible 'building blocks' of change.

▷ Crossing switches are *quanta of change.*

▷ Each crossing switch yields a *quantum of action* $\hbar$*.*

This statement is a generalization of Dirac's trick. In physics, change is measured with action, and each crossing switch realizes a quantum of action. Thus, the statements about crossing switches agree with the quantization of action. Above all, crossing switches visualize the quantization of action.

In short, the geometric properties of strand crossings – amplitude and phase(s) – closely resemble the mathematical properties of wave functions. In the strand tangle model, unobservable strands and unobservable crossings lead to observable crossing switches. In the strand tangle model, crossing switches are discrete and indivisible processes of nature, because crossing switches define the quantum of action $\hbar$: each crossing switch is an observable quantum of change. Therefore, the strand tangle model suggests that every motion, every transformation, every interaction, every measurement and every observation is a process built of crossing switches. The remainder of this study proves this suggestion in detail.

5 The fundamental principle of the strand tangle model

The strand tangle model states that matter, radiation, space, and horizons – thus all systems observed in nature – consist of strands of Planck radius that fluctuate at all scales larger than the Planck scale [63, 96]. Here,

▷ A *strand* is defined as a smooth curved line surrounded by a cylinder with a tiny, but finite radius. More precisely, the line is a one-dimensional, open, continuous, everywhere infinitely differentiable subset of $\mathbb{R}^3$ (or of the spatial part of a curved Riemannian space) without self-intersection, unknotted, and without endpoints. In addition, the line is surrounded by a volume defined by a perpendicular disk of Planck radius $\sqrt{\hbar G/c^3}$ at each point of the line. The entire strand volume is not allowed to self-intersect, meaning that the curvature radius of the strand is never smaller than the Planck length $\sqrt{\hbar G/c^3}$.

From a mathematical viewpoint, the definition of strands is the usual definition used in knot theory, for example, in the ropelength calculations of tight knots [109, 110]. From a physics viewpoint, strands differ in various ways from flexible ropes, cables, or cooked spaghetti. First of all,

▷ Strands randomly fluctuate in shape.

The fundamental, Planck-scale principle of the strand tangle model

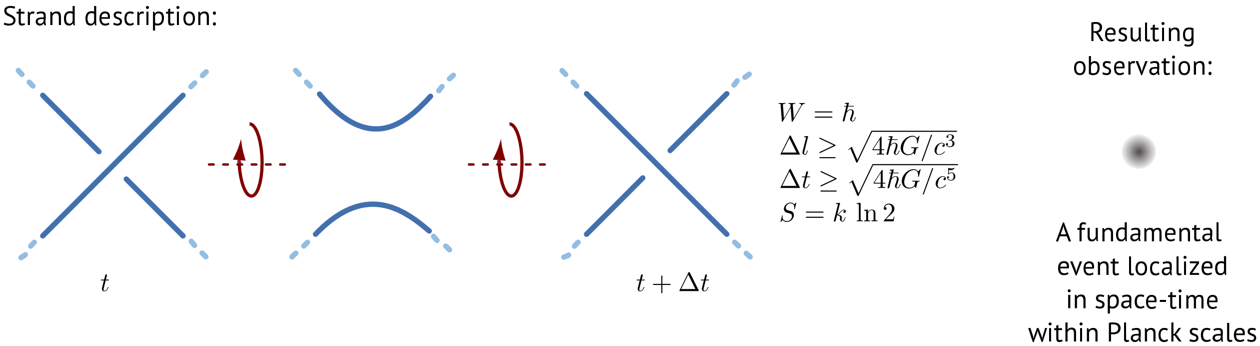

FIG. 6: The fundamental principle of the strand tangle model, deduced from Dirac's trick, describes the simplest observation possible in nature: a fundamental event. In the strand tangle model, a fundamental event arises at the Planck scale and is *almost* point-like. A fundamental event results from a *strand crossing switch* – the change of which strand passes above the other – at a location in three-dimensional space. The *location* of the crossing is defined by the centre of the shortest distance between the two strand segments. The *crossing switch* is a deformation in space, illustrated by the circling arrows – that results, for example, from rotating together the two tethers on the right-hand side against the two tethers on the left-hand side, around the west-east axis in the paper plane. The strands themselves, best imagined as ropes or cables with a Planck-size radius, are not observable and are impenetrable. The crossing switch is the simplest observable process in nature; it defines $\hbar$ as the unit of the physical action W. The Planck length and Planck time arise from the most localized and the most rapid crossing switch possible, respectively. Section 36 discusses the fastest possible crossing switch. Appendix C explores the relation between crossing switches and qubits. All processes and all observations consist of crossing switches.

The origin of the shape fluctuations is the continuous pushing of strands against strands. This is a consequence of their impossibility to intersect. Furthermore, the Planck-size radius of strands is so small that it is *negligible* in almost all situations – except in the domain of quantum gravity. In addition, strands have the following specific property:

▷ Strands cannot be cut.

In other words, contrary to habits of thought, strands are *not* made of parts. Strands are *not* made of anything else. In further contrast to everyday life, strands have *no ends.* But the main difference to everyday life is

▷ Strands themselves have *no* observable properties – even though their tangling *changes* are observable.

The final reason for this aspect of strands will be clarified later in Section 36. In particular, strands do not have mass, colour, energy, tension, momentum, charge, or any quantum number. Furthermore, strands have *no* fixed length, and *cannot* exert forces: strands cannot be pulled, they cannot pull anything, and they do not offer any resistance when they are deformed. Strands just define connections, have a tiny radius, and that's it.

The strand tangle model states:

▷ **Crossing switches of strands with Planck radius – changes of crossing sign – determine the quantum of action $\hbar$, as illustrated in Figure 6, and all physical units and observables.**

This statement is the *fundamental principle* of the strand tangle model. The sign of a crossing is defined mathematically in Section 8, with the usual definition of knot theory. Several statements follow from the fundamental principle.

▷ Although strands are themselves unobservable, *crossing switches are observable*, because of their relation to $\hbar$, c, k and G.

▷ Physical space is a *network* of untangled strands. Horizons – including black hole horizons, Rindler horizons and the cosmological horizon – are *weaves* of strands. Particles are *rational tangles* of strands, i.e., unknotted tangles of strands. Appendix D provides a brief overview of these configurations.

▷ Wave functions are due to tangle *crossings*. Probabilities and fields are caused by crossing *switches* that occur in specific tangles. This is the topic of the present study.

▷ Physical motion *minimizes* the number of observable crossing switches of unobservable fluctuating strands. This is the principle of least action.

Because of the definition of the constants $\hbar$, c, k and G with the help of crossing switches, all physical units and observables can be defined with crossing switches. Every observed type of change is due to crossing switches of strands, from nuclear reactions to black hole mergers. The tiny, but finite radius of strands visualizes the minimum measurable length $\sqrt{4\hbar G/c^3}$ as the shortest distance between two strand segments. (Appendix E explores the topic of the strand radius in more detail.) Nevertheless, strands themselves remain unobservable. (See Appendices A and B for the underlying reason.)

In short, the fundamental principle of the strand tangle model states: crossing switches of fluctuating strands of Planck radius are at the basis of particle physics and of general relativity. The strand tangle model thus claims that the fundamental principle of Figure 6 not only defines the fundamental event, every physical unit and every physical observable but that it contains *all of physics.* As mentioned, this claim has been tested successfully for general relativity [80, 98] and for the standard model with massive Dirac neutrinos [63, 64, 96, 97]. As shown below, both discrete quantum numbers and continuous field observables emerge from crossing switches. In particular, particles and wave functions arise from strands. In this article, the strand tangle model is tested against observations in the domain of *quantum mechanics.* As is usual in introductory quantum mechanics, physical space is assumed to be flat throughout. The strands that lead to empty space are not discussed or explored in the following (with one exception).

As a note, event-centric approaches to quantum theory are being also explored by Powers and Stojkovic [111].

6 Emergence of rotating and orbiting quantum particles

Tethers explain spin $1/2$ behaviour and fermion behaviour. These consequences of Dirac's trick are illustrated in Figure 2 and Figure 4. Both results apply independently of the number of tethers, as long as the number is three or more. But the two figures show more.

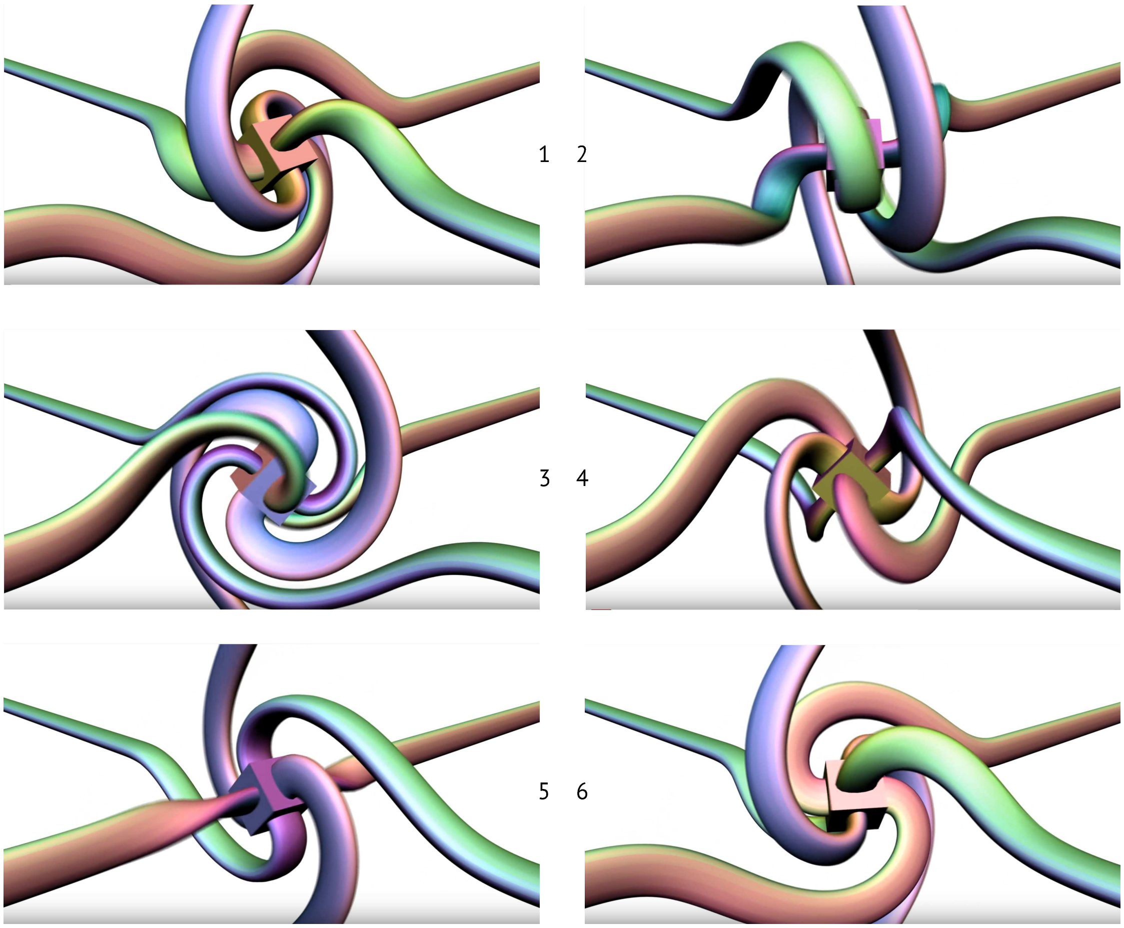

FIG. 7: The particle rotation for a (free and moving) lepton can be visualized with the animation produced by Jason Hise [71]. The rotating (spinning) central cube symbolizes the tangle core, i.e., the (moving) region where the particle is localized with the highest probability. The continuous rotation of a tethered particle is possible. (The figure is taken from reference [63].) The cube rotates twice before returning to the starting configuration. The factor of 2 appears in many expressions involving the rotation angle and the quantum phase of fermions.

Dirac's belt trick implies that a tethered particle can *rotate continuously*. This is best observed in animations. A video showing the continuous spinning of a particle with six tethers was produced by Hise [71]; still images from the video are shown in Figure 7. A video of a spinning particle with four tethers (or two attached belts) was produced by Martos [72]; still images from that video are shown in Figure 8. Videos of rotating particles with dozens of tethers are also found on the internet [74, 75]. (In section 31, it will become clear that *elementary* fermions have four or six tethers, whereas *composed* particles have more than six.) In other words, when unobservable tethers are allowed to fluctuate, particle rotation can continue forever, without any obstacles, despite the tethering. Therefore, spin can be visualized as the rotation of tangle cores.

A related statement can be made for systems consisting of two particles. The fermion trick implies that tethered particles can *orbit each other continuously*. Martos published a further video showing the fermion behaviour of two tethered particles [73]; still images from the video are shown in Figure 9. The video shows that tethered particles can *orbit each other forever* – if tethers

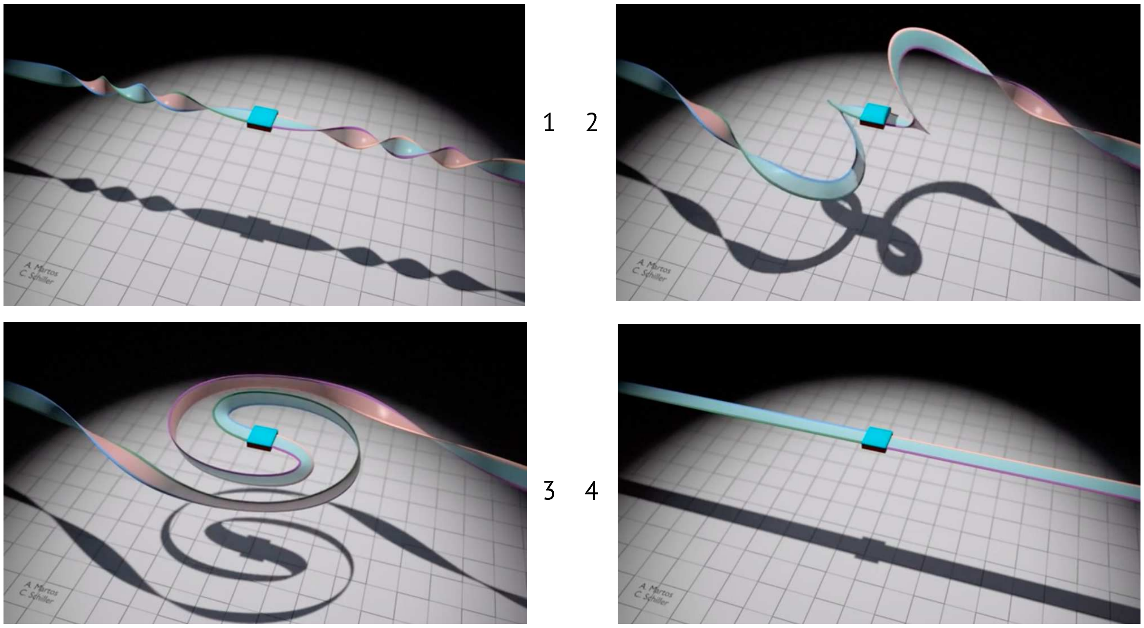

FIG. 8: The return to the original configuration after a double rotation can also be visualized with the animation produced by Antonio Martos [72]. The rotating central belt symbolizes the tangle core. The continuous rotation of a tethered fermion is possible.

are allowed to fluctuate. Again, the number of tethers is not limited. Orbiting particles can be visualized as orbiting tangle cores. For example, Figure 9 visualizes the electron orbiting a proton in a hydrogen atom.

The mentioned properties imply that the tethers of two typical quantum particles do not disturb each other, as long as strands are unobservable, infinitely flexible and have no fixed length, no tension and no mass. The same is true for more than two particles. As a result, the many tethers filling empty space *usually* lead to *no* interaction between particles. (Exceptions are discussed below. It is also explained below that, as a general rule, interactions arise only between particle cores, not between tethers.)

In the strand tangle model, every quantum particle is a *tangle tethered to the cosmological horizon* in which the tethers fluctuate continuously.

- ▷ Tangles with their tethers reproduce spin $1/2$ as tangle core rotation.
- ▷ Tangles with their tethers reproduce fermion behaviour as tangle core exchange.
- ▷ Rotation of tethered cores reproduces particle spin, including its orientation in space.
- ▷ Orbiting tethered cores reproduce orbiting particles.

These results are valid for all measurable energies. In particular, *spin is tangle core rotation.*

In short, tethers are essential for describing and understanding spin $1/2$, rotation, orbiting particles, and particle exchange. This understanding is *impossible* without tethers. The result suggests that *every other* motion in the quantum domain – including translation, interference, scattering and interactions – can also be described with tethered tangle cores. This is indeed the case, as argued in the remainder of this article. However, before doing so, it is worth confirming the idea of tethers in the case of composed particles.

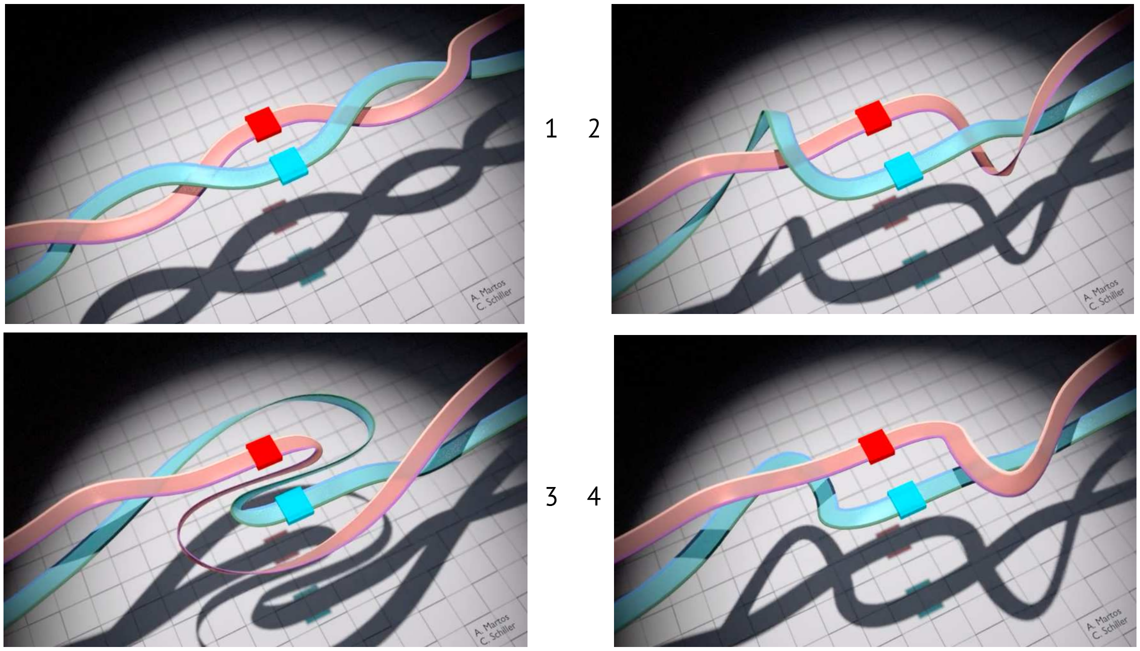

FIG. 9: Particle exchange can be visualized with the animation produced by Antonio Martos [73]. The rotating central belts symbolize the two tangle cores, i.e., the regions where the two particles are localized with the highest probability. (In the strand tangle model, tethers and thus belts are unobservable.) The first image shows two particles whose positions were exchanged twice. The other images show that the shape changes of the belts (each consisting of two tethers) bring back the original, unexchanged situation. Therefore, the continuous exchange of the two tethered particles is possible. The animation thus illustrates, among others, the motion of an electron (red) continuously orbiting a proton (blue) in a hydrogen atom. In particular, the sub-figure 1 on the top left can be taken as the defining configuration for a composed system.

7 The spin of particles composed of spin 1/2 particles

In nature, when two spin $1/2$ fermions form a composite, the composite is observed to have either spin 0 or spin 1, and never any other spin value. In other words, the composite of two fermions is always observed to be a boson. This behaviour can be reproduced with tethers.

In the strand tangle model, a system is *composed* when it forms a unique, non-separable tangle core connected to the cosmological horizon by tethers. Examples showing composite systems most clearly are the second graph from the left in Figure 4 or sub-figure 1 in Figure 9. Composed systems usually have a different spin value than the particles they are made of. The two figures show that a system composed of two tangle cores (two belt buckles) returns to itself exactly after a rotation by 2π (the double exchange). The composite system does not return to itself after rotation by π, that is, after a simple exchange. These are the defining properties of *integer* spin. Thus, a composite system of two spin $1/2$ tangles behaves like a particle with integer spin.

The analogy can be made more precise by recalling that in the strand tangle model, each of the two particles in Figure 9 continuously spins along their axes. The simplest situation is that each particle continuously spins around an axis *along* the straight belts. If the rotation axes and rotation sense of the two fermions and that of the double exchange agree, the situation is described by $S = 1$ and z-component $S_z = +1$. If the rotation axes of the two fermions are the same but that of the double exchange is opposite, one has $S = 1$ and $S_z = -1$. If the rotation axes of the two fermions are the same, but that of the double exchange is perpendicular to them (e.g., in the direction of the line connecting the two cores), one has the situation $S = 1$ and $S_z = 0$.

The defining property of spin 0 is that a rotation by any angle keeps the system unchanged. If the rotation directions of the two fermions are *opposite* to each other, then the total system has $S = 0$ and $S_z = 0$, independently of the rotation axis of the double exchange. A composite for which two cores rotate in opposite directions has a total spin 0. Any tangle core that does not rotate when strands are randomly deformed has spin 0.

The exploration shows that a composite of two tethered spin $1/2$ particles cannot have any other spin value: no other behaviour under system rotation apart from the four mentioned cases is possible. In other words, composing two spin $1/2$ particles can only yield a composite with spin 0 or spin 1. For tangle cores with integer spin, core rotation by 2π is equivalent to no rotation, in contrast to the case of a half-integer spin core. (For the elementary bosons explored later on, only the curved strands making up the core need to be rotated.) One notes that in these arguments, the two particles do not have to be identical or even elementary. Even if their tangles cores differ or if they are themselves composed, the results about spin composition still hold.

The above arguments can be extended to composites of *more* than two spin $1/2$ particles. The result is known: particles composed of odd numbers of spin $1/2$ particles are fermions; likewise, every composed fermion contains an odd number of spin $1/2$ particles. *All composite fermions* have a spin value that is an odd multiple of $1/2$. Tethers reproduce the behaviour of fermions under composition.

In nature, particles with integer spin behave differently from fermions: they are *bosons.* This is reproduced by the strand tangle model. When the positions of two identical tangle cores with integer spin are exchanged, no partial orbit of one core around the other core is required – in contrast to the case of half-inter spin cores. No double exchange is needed to restore the sign of the wave function. Indeed, in the strand tangle model, bosons, whether elementary or composed, can exchange positions without hindrance. For composites, exchange is achieved by the cores passing through each other. In other words, the tethers of such tangles explain why all spin 1 particles are bosons. In total, tethers yield the result that *all composite bosons* have a spin value that is an integer, and all particles with integer spin are bosons.

In short, tethers explain why all particles are either *fermions*, with a spin given by an odd multiple of $1/2$, or *bosons*, with an integer spin. Tethers thus imply the spin–statistics theorem. In other words, *every* quantum particle can be seen as tethered, whether elementary or composed, and whether fermion or boson. This result cannot be deduced without tethers. This is valid for all measurable energies. Given that tethers appear to explain the quantum behaviour of particles, the next step is to use tethers to define wave functions.

8 Emergence of wave functions

Tethers define the quantum behaviour of particles. The strand tangle model goes one step further than the tethered particle models used by Dirac and by Battey-Pratt and Racey. In the strand tangle model, quantum particles are made of tethers or strands. Also all other aspects of nature are *exclusively* due to strands. In particular, quantum particles are tangles. The basic result of the

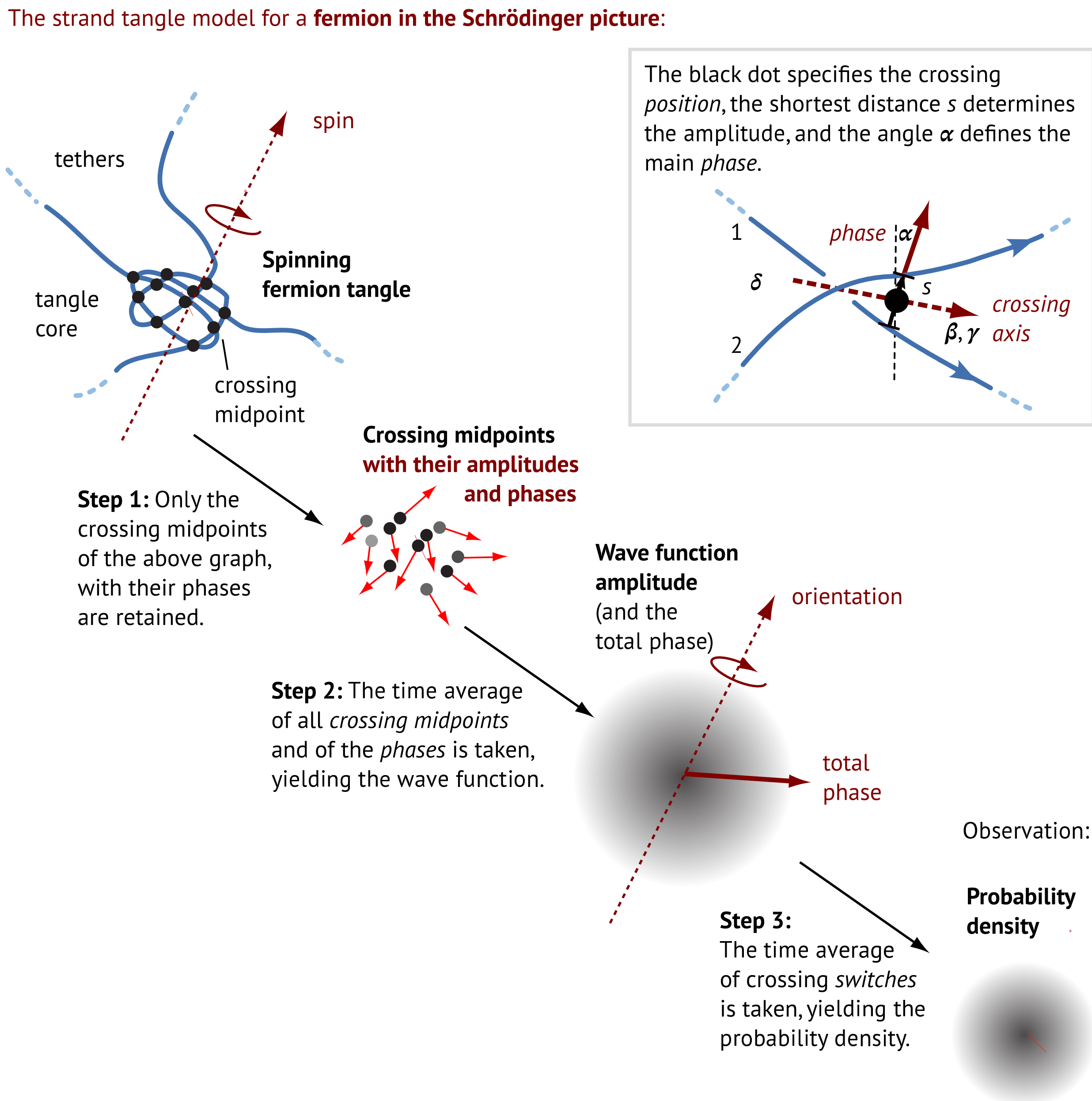

FIG. 10: In the strand tangle model, the *wave function* in the Schrödinger picture is the time-averaged *crossing density*, and the *probability density* is the time-averaged *crossing switch density*. The figure illustrates how a tangle defines crossings and local phases, how these fluctuating crossings lead to a wave function and a total phase, and how the observable *crossing switches* lead to a probability density. There is a simplification in the figure: in reality, the phase of the wave function of a localized particle *depends* on the position.

strand tangle model for quantum theory is:

▷ Wave functions are time-averaged crossing distributions.

The general correspondence is illustrated in Figure 10 and Figure 11. The steps leading from a tangle to a wave function and a probability density are explained in this section. The starting point is the basic similarity of crossings and wave functions.

The inset at the top right of Figure 10 recalls that, at a given point in space, a crossing can be described by *one positive real number* that specifies the smallest strand distance, and by *(up to) four angles or phases.* Also a wave function at a point is described by one positive real number and several angles or phases. Three types of wave functions are important in this context.

1. Every *spin-less wave function*, as used in the Schrödinger equation or the Klein-Gordon equation, can be written as

$$\psi = R\,\mathrm{e}^{i\varphi} = \sqrt{\rho}\,\mathrm{e}^{i\alpha/2} \quad . \tag{6}$$

Here, R is the (positive) modulus, $\rho = R^2$ is the (positive) probability density, φ is the phase, and as will soon become clear, $\alpha = 2\varphi$ is the tangle core rotation angle. The factor of $1/2$ is due to the belt trick. In other words, a spin-less wave function at a point, usually described by a complex number, can be described by one positive real number and one phase. The inset at the top right of Figure 10 visualizes the phase.

2. Every *non-relativistic spinor* with two complex components, as used in the Pauli equation, can be written [112, 113] as

$$\Psi = \sqrt{\rho}\,\mathrm{e}^{i\alpha/2} \begin{pmatrix} \cos(\beta/2)\mathrm{e}^{i\gamma/2} \\ i\sin(\beta/2)\mathrm{e}^{-i\gamma/2} \end{pmatrix} \quad , \tag{7}$$

Again, ρ denotes the probability density. The three angles α, β and γ are the Euler angles that describe the orientation of a rigid body in three dimensions. (In this context the tangle core is the rigid body.) The factors $1/2$ are due to the belt trick. In other words, a Pauli spinor, usually considered as a matrix of two complex numbers, can be described by one positive real number and three phases. Pauli spinors are explored below.

3. *Relativistic spinors* with four complex components, as used in the Dirac equation, can be described by one positive real number, four phases and three real parameters. They are also explored later on.

In the following, the spin-less non-relativistic Schrödinger wave function ψ is explored first. The mentioned similarities suggest the following definition:

▷ The *amplitude* related to a crossing varies inversely with the shortest strand distance s. The *probability density* increases when the strand density increases. More precisely, the *amplitude* or *modulus* $R(\mathbf{x}, t)$ of the wave function at a point $\mathbf{x}$ is defined as

$$R(\mathbf{x}, t) = \left\langle \frac{1}{s^{3/2}} \right\rangle \frac{1}{\sqrt{n}} \quad . \tag{8}$$

The average – denoted as $\langle\rangle$ – is taken over all possible strand shape fluctuations during a few Planck times.

The crossing distance s is defined in Figure 10 as the shortest distance between two strand segments. The average of the crossing distance is normalized with the help of the number n, the so-called *(minimal) crossing number* of the tangle. The crossing number n is the smallest number of crossings that arises if a tangle is laid down on paper. The crossing number n is a topological invariant; for example, $n = 3$

for the electron tangle presented in Figure 22. In particular, the crossing number n is observer-invariant; it is a constant factor introduced in the modulus to normalize the wave function.

As a result of the definition, strand fluctuations imply that the amplitude $R(\mathbf{x}, t)$ is a *positive* continuous and differentiable real function of space and time, as expected.

- ▷ The *(quantum) phase* is due to the average orientation of the shortest distance s at a point in space. (The shortest distance is assumed to point from the strand with the smaller number to that with the larger number.) More precisely, the quantum phase $\varphi(\mathbf{x}, t)$ of a spin-less wave function at point $\mathbf{x}$ is *half* the time-averaged local strand *crossing phase* α of the particle tangle – the rotation around the crossing axis – at that point:

$$\varphi(\mathbf{x}, t) = \langle \alpha(\mathbf{x}, t)/2 \rangle \quad . \tag{9}$$

  The factor of $1/2$ is due to the belt trick. Again, the average is denoted by $\langle\rangle$ and is taken over a few Planck times and over all possible strand shape fluctuations.
- ▷ The crossing *axis* is defined with the two unit tangent vectors of the strands at the endpoints of the shortest distance s, as illustrated in the inset of Figure 10. In particular, the *axis* of a crossing is given by the *sum* of the two tangent vectors. The axis is always perpendicular to the shortest distance vector.
- ▷ The *sign* of a crossing is the direction in which the right hand turns when the thumb and index follow the two strands in the direction of their axes: clockwise hand rotation corresponds to a positive sign.

Thus, the *phase* of the crossing specifies the orientation of the shortest distance around the crossing axis. In the spin-less case used in the Schrödinger equation, the phase angle is measured against a predefined direction, as indicated by the vertical dotted line in Figure 10. The other angles describing the crossing are ignored. As usual, there is freedom in the definition of the direction that corresponds to the vanishing phase of a particle tangle. In Figure 10, the freedom is the ability to choose the direction of the black dotted line. In the case of tangles for physical fields, such as tangles for the electromagnetic field, the freedom to choose the orientation of the zero phase is related to the freedom of gauge choice.

The second type of wave function, the two-complex-component Pauli spinor, takes the angles β and γ of Figure 10 into account. They describe the orientation of the crossing in space and are thus related to the spin orientation. Pauli spinors will be explored below.

The third type of wave function, the Dirac spinor, takes also the angle δ of Figure 10 into account. The angle describes the relative importance of particles and antiparticles. Dirac spinors will be explored further later on.

All wave function definitions with strands use a general relation.

- ▷ In the strand tangle model, *continuous quantities* arise through *time averages* of strand shape fluctuations. The averaging time is a few Planck times. Quantum theory emerges through fluctuation averaging.

Thus, the averaging time is much shorter than any other time interval that plays a role in observations or measurements. (It is recalled that strands and strand shapes are not observable, and thus there is no speed limit for their shape fluctuations.) The emergence of the Schrödinger wave function occurs in three steps, as illustrated in Figure 10.

Step 1 consists of reducing the tangle to its crossing midpoints, crossing amplitudes, and respective crossing phases. This step results from the unobservability of strands.

Step 2 consists of averaging all crossing amplitudes and phases over all strand shape fluctuations occurring during a few Planck times. In the simplest case, this step leads to the Schrödinger wave function. In particular, strands and their crossings imply that the quantum phase usually varies from point to point (in contrast to the simplified illustration of the result of step 2 in Figure 10).

Step 3 consists of averaging the crossing switches observed at each position in space. The averaging leads to the probability density.

Thus, one can summarize the working hypothesis in three claims:

- ▷ The wave function for a tangle – amplitude and phase – is the short-time averaged *(oriented) crossing distribution*. Wave functions are blurred tangle crossings.
- ▷ The probability density for a tangle is the short-time averaged *crossing switch density*. Thus, the probability density is given by the *blurred switch density* due to a fluctuating tangle.
- ▷ For visualization, it is sometimes useful to see tangles as fluctuating *skeletons* of wave functions.

These claims must be checked in detail. The check is performed in the remainder of this article.

In particular, there are *two ways* to perform the time average – i.e., the blurring – for a given tangle. The first, straightforward way is, as just mentioned, to average crossings over all possible strand shape fluctuations realized during the averaging time. *Each segment of strand* fluctuates continuously and changes shape continuously. Therefore, the crossings of a particle tangle continuously move around in space; some also appear or disappear. Averaging the amplitude and phase for the crossings at a point in space yields a local value for the amplitude and phase of the wave function. This averaging method thus yields the common *Schrödinger picture* of the wave function and quantum mechanics. For example, the spinning tangle for a free particle, which is illustrated in Figure 7, leads, after averaging the shape fluctuations (not shown in that figure), to a rotating cloud with a rotating phase. The resulting cloud is illustrated in Figure 2. A productive way to visualize spin-less wave functions in the Schrödinger picture is to use colour for the phase and to use colour saturation for the amplitude. This is done in the fascinating books of Thaller [114, 115] and in the beautiful animations that come with them.

The second way to visualize spin-less wave functions is to imagine a small arrow in a plane at every point in space, and to average over all paths taken by the moving arrow. This path-integral approach is presented in the beautiful book on QED by Feynman [116]. The path-integral approach is the second averaging method. It can also be reproduced by the tangle model, as illustrated in Figure 11. The first step is to imagine that the tangle core is *tightened* to a tiny, Planck-sized region. This yields a point-like tangle core with a phase. (The Planck diameter of the strands is neglected. The tethers, being unobservable, are discarded.) For example, the spinning tangle for a free particle illustrated in Figure 7 can be reduced in scale down to a point with a rotating

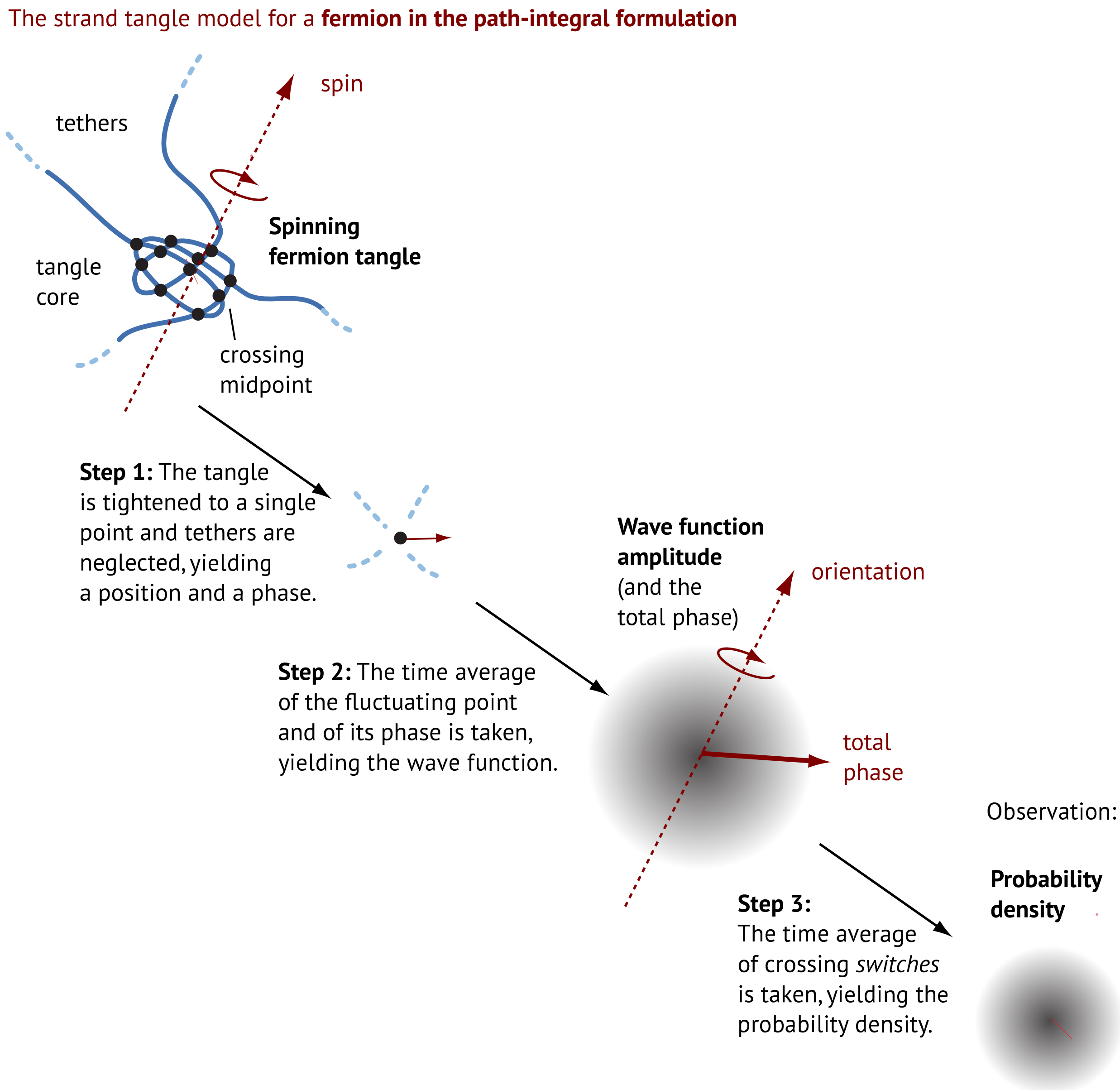

FIG. 11: The wave function and probability density in the *path integral formulation* are due to time-averaged fluctuating (almost) point-like particles. The figure illustrates how a tangle that is "pulled tight" defines the position and the local phase. The fluctuations of the point-like tangle core then lead to a wave function with phase. Again, the modulus of the wave function leads to a probability density. Again, the figure is simplified: in reality, the phase of the wave function of a localized particle *depends* on the position. In particular, an advancing particle is an advancing rotating arrow.

arrow attached to it. The second step is to imagine that the *tight tangle core as a whole*, which is continuously spinning, randomly changes its position and orientation. Averaging over all possible paths of the tight tangle core yields the wave function. In the third step, averaging all crossing switches yields the probability density.

The second averaging method was used already by Battey-Pratt and Racey [76]. Mathematically, it yields the same expressions for amplitude and phase that arise in the first method. However, the phase definition with a tight tangle corresponds more directly to the idea of the rotating arrow on a path used by Feynman. In addition, the appearance of half angles in the definition of the wave functions becomes more intuitive. In any case, the description of the quantum state using a *tight* tangle fluctuating as a whole is equivalent to the description with a *loose* tangle where each strand is fluctuating. When spin effects are neglected, both descriptions lead to the wave function described by an amplitude and a phase, that is, to a continuous complex function of space and time.

How can one be sure that the averaging process over time results in the correct value of the wave function $\psi(\mathbf{x}, t)$? The argument is the same as that used in the path integral formulation of quantum theory. Because an advancing rotating tight tangle reproduces the non-relativistic, spin-less fermion propagator, it also reproduces the wave function and, as will appear below, the Schrödinger equation.

In short, the strand tangle model strongly suggests that *wave functions are blurred crossing distributions.* Equivalently, *tangles are fluctuating skeletons of wave functions.* More precisely, the working hypothesis is as follows: a spin-less particle is described by its time-averaged spatial distribution of crossings generated by its fluctuating particle tangle. The *local amplitude* appears to describe the time-averaged *local density* of strand crossings and is a positive real number. The *local phase* appears to describe the time-averaged *local orientation* of the crossings. Consequently, a spin-less particle appears to be described by a complex-valued field that describes its crossing distribution. A spin-less particle also appears to yield a *probability density* that describes the *local crossing switch density*. Amplitude, phase and probability density appear to be due to tangle shape fluctuations.

The next two parts of this article will *prove* that crossing distributions do have all the known properties of wave functions and that crossing distributions yield the equations of Schrödinger, Klein-Gordon, Pauli, and Dirac. The fourth part will explain the origin of particle masses, the origin of elementary particles, and the origin of gauge interactions.

Part II: Superpositions, Hilbert space, Hilbert space and measurements

9 Wave function superpositions and tangle superpositions

Crossing distributions can only be models for wave functions if they form a *Hilbert space*. A Hilbert space is a vector space with an inner product (and several technicalities).

To show that crossing distribution from a vector space, *linear combinations* of two wave functions – called *superpositions* in physics – need to be defined. This requires the definition of two operations: scalar multiplication and addition. The first, simple way is to define the operations for wave functions, as in quantum mechanics:

▷ The *scalar multiplication* $a\psi$ and the *addition* $\psi_1 + \psi_2$ of wave functions ψ_i are defined by applying the respective operations on the complex numbers at each point in space, i.e., on the local values of the wave function. In the strand tangle model, this implies performing the operations on the corresponding crossing distributions.

This first definition is sufficient to show that crossing distributions form a vector space. The same

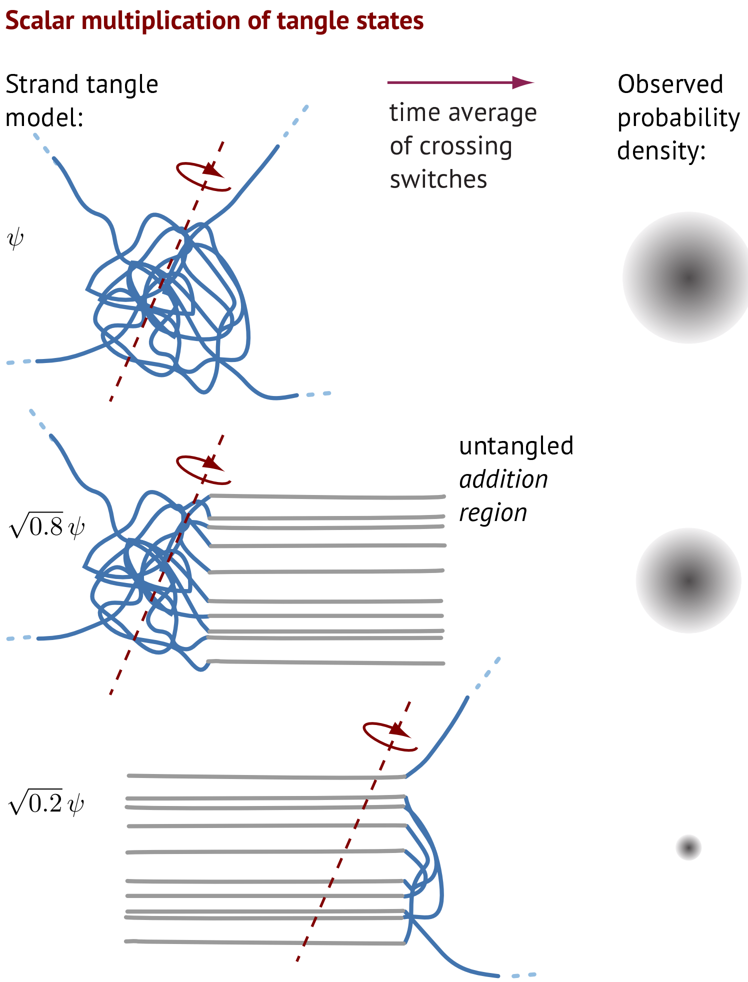

FIG. 12: The scalar multiplication of a localized tangle by two different *real* numbers smaller than 1 is illustrated. The illustration visualizes the resulting thinning of the corresponding wave function ψ.

approach can be used to define an inner product and then show that the crossing distributions form a Hilbert space, as expected. One can then continue directly with the next part of this article, the part on evolution equations, which starts with Section 19.

However, a second way to define a vector space for wave functions, equivalent to the first, is much more intuitive and striking. Addition and multiplication can be defined for the *tangle* describing a quantum system, and the short-time average – the blurring – can be taken *after* the operations on the underlying tangle are performed. This is done next, by using the *defining axioms* for vector spaces, for inner products and for Hilbert spaces.

▷ The *scalar multiplication* $a\psi$ of a state ψ by a complex number $a = r\mathrm{e}^{i\delta}$, with $r \leq 1 \in \mathbb{R}^+$ is formed by taking the underlying tangle, rotating its tangle core (i.e., each local crossing) by the angle 2δ, and then 'pushing' a *fraction* $1-r$ of the tangle to the cosmological horizon, thus keeping the fraction r of the original tangle at finite distances. Thus, scalar multiplication with $r \leq 1$ is a process of *tangle core thinning and rotation*. Time averaging then results in the wave function $a\psi = r\mathrm{e}^{i\delta}\psi$.

A simple case of scalar multiplication for a tangle representing a state ψ is illustrated in Figure 12. The figure illustrates the idea of *thinning* a tangle core. Pulling a tangle core apart in such a way that no crossings arise in between corresponds to dividing the tangle into two fractions. The relative size of the two fractions is determined by (the square root of) the relative volume integrals of the probability densities. The mentioned process of tangle core thinning is visualized by the untangled strand segments in the so-called *addition region*.

The tangle version of scalar multiplication is *unique*. Indeed, even though there is a choice about which specific fraction r of tangle crossings is kept and which specific fraction $1 - r$ of crossings is sent away, this choice is only apparent. The resulting crossing distribution, defined as the average over fluctuations, is independent of this choice because the tangle topology, which specifies the particle, remains intact.

The tangle version of scalar multiplication is *associative:* the relation $a(b\psi) = (ab)\psi$ holds by construction. The scalar multiplication of strands also behaves as expected for 1 and 0. Finally, strand multiplication by -1 is defined as the rotation of the full tangle core by 2π, as required by the belt trick. In other words, scalar multiplication can be modelled as tangle core thinning and rotating.

Also addition can be defined for tangles.

▷ The *addition* of two tangles $a_1\psi_1$ and $a_2\psi_2$, for which $|a_1|^2+|a_2|^2 = 1$ and for which ψ_1 and ψ_2 have the same topology, is defined by directly *connecting* the crossings not pushed far away during the scalar multiplication by a_1 and a_2. The connection of tangles must be performed in such a way as to maintain the topology of the original tangles; in particular, the connection occurring in the spatial *addition region* must not introduce any crossings. Time averaging then leads to the tangle function of the superposition $a_1\psi_1 + a_2\psi_2$.

An example of superposition for the case of two quantum states at different positions in space is shown in Figure 13. No strand is cut or re-glued during addition – although imagining doing so might help for visualizing the operation. The *addition region* with untangled strands shown in the figure will be of importance later, when entanglement is explored.

The definition of linear combination requires the final tangle $a_1\psi_1 + a_2\psi_2$ to have the same topology and the same norm as each of the two tangles ψ_1 and ψ_2 to be combined. Physically, this means that only states for the same type of particle can be added; this also means that particle number is preserved. Tangles thus automatically implement the corresponding *superselection rules* of quantum theory. This property is welcome because, in conventional quantum mechanics, the superselection rules need to be added by hand. In contrast, the strand tangle model contains them automatically.

One notes that the sum of two tangle functions is *unique*, for the same reasons given in the case of scalar multiplication. The addition is commutative and associative, and there is a zero state, or identity element, given by the trivial, i.e., the untangled tangle. The definition of addition also implies distributivity with respect to the addition of states and with respect to multiplication by scalars. Finally, the definition of addition can be extended to more than two terms and is associative.

In short, the definitions for the addition and for the scalar multiplication of crossing densities – including the one using tangle connections and tangle thinning – prove that crossing distributions form a *vector space*, as expected and required from any model for wave functions.

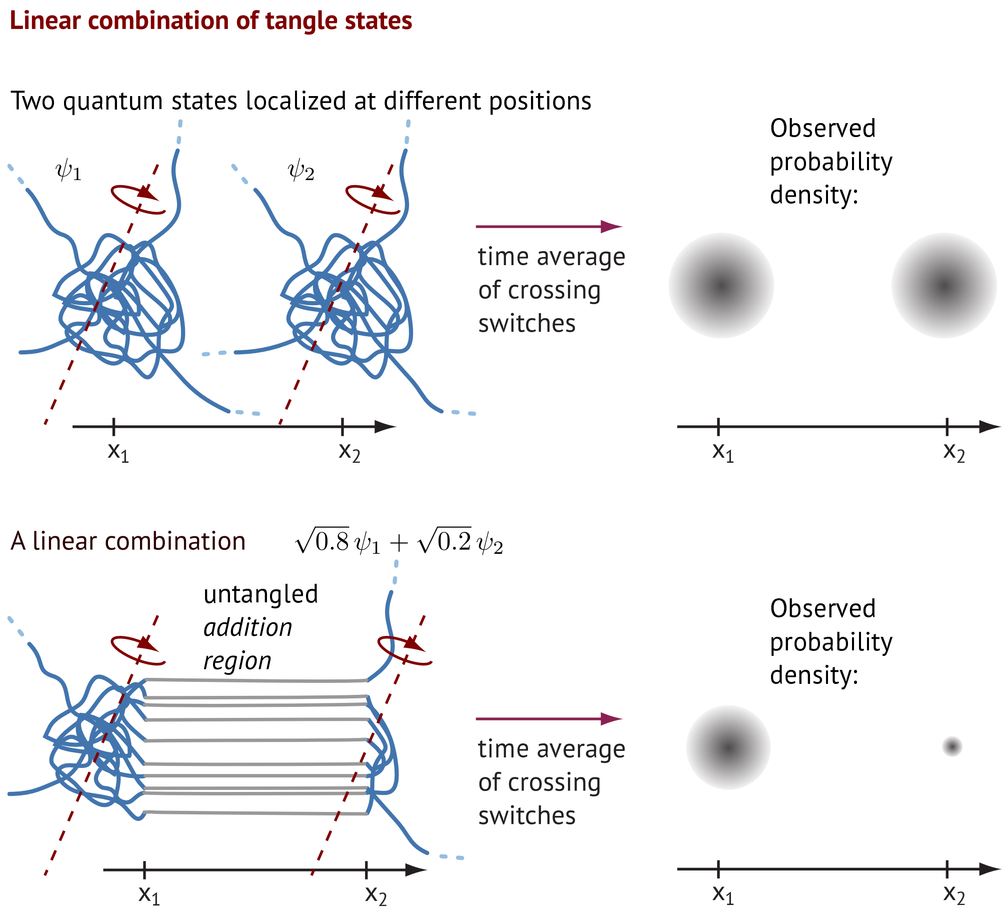

FIG. 13: A linear combination of two states representing a particle localized at two different positions, visualizes the superposition of wave functions in the strand tangle model.

10 From tangles to Hilbert spaces

To form a Hilbert space, crossing distributions must allow the definition of an inner (or scalar) product. In conventional quantum mechanics for a spin-less particle,

▷ The *inner (or scalar) product* between two states $\psi_1(\mathbf{x},t)$ and $\psi_2(\mathbf{x},t)$ is defined as $\langle\psi_1|\psi_2\rangle = \int\overline{\psi}_1(\mathbf{x},t)\psi_2(\mathbf{x},t)\,\mathrm{d}\mathbf{x}$.
▷ The *norm* (or *modulus*) of a state is $\|\psi\| = \sqrt{\langle\psi|\psi\rangle}$.
▷ The *probability density* ρ is $\rho(x,t) = \langle\psi|\psi\rangle = \|\psi\|^2$.

In the strand tangle model, the conjugate tangle $\overline{\psi}$ is formed from the tangle ψ by exchanging the sign of each crossing, i.e., by exchanging underpasses and overpasses. As a consequence,

▷ The inner product $\langle\psi_1|\psi_2\rangle$ is the (suitably normed) number of crossing switches required to transform the tangle $\overline{\psi}_1$ into the tangle ψ_2.

This inner product has all the required properties: it is Hermitian, sesquilinear, and positive definite. The required technicalities about the completeness of the norm are also realized by tangles of strands. As a consequence, wave functions defined with strand tangles form a *Hilbert space*.

Being related to crossing switches, the inner product is also a physical observable, in contrast to the states themselves. This property is as expected.

The inner product of wave functions allows defining the *norm* of a wave function. In conventional quantum mechanics, the norm of a wave function is the square root of the integral of $\overline{\psi}\psi$, taken over full space. ($\overline{\psi}\psi$ is equal to R^2 in the spin-less case of equation (8).) In the strand tangle model, the norm of the wave function is defined in the same manner. With the numerical factor defined for wave functions in equation (8), the resulting integral has the value 1 for every quantum state. Thus, one-particle wave functions are normalized in the strand tangle model.

It should be noted that an investigation of quantum electrodynamics [64] shows that the (appropriately signed) minimum number of crossings is the electric charge of a particle in units of $e/3$. The probability density reproduces the charge density. All movements, shape fluctuations, and strand exchanges of strand tangles yield charge conservation. Figure 23 and Figure 30, found below, show the tangles of all charged and neutral elementary particles. For neutrinos, the electric charge vanishes. In the strand tangle model, neutrino tangles with their 'apparent' or 'partial' crossings lead to a vanishing total charge.

As mentioned previously, the probability density is the crossing *switch* density. Because the definitions of the probability density and the inner product involve crossing switches, both quantities are physical *observables*. This is as expected from the fundamental principle of the strand tangle model.

Once states and their Hilbert spaces are defined, *operators* on that Hilbert space can be defined using strands. This topic is briefly explored in Appendices F and G. The fundamental reason that Hilbert spaces arise simply from $\hbar$ is summarized in H.

In short, it was proven that if strand tangles are used to define wave functions as crossing distributions, then they reproduce superpositions and form Hilbert spaces. In addition, the probability densities, defined as crossing switch densities, behave as expected from conventional quantum theory and observations. The next checks are whether strands reproduce the free motion of particles and whether crossing densities reproduce interference.

11 How can tethered particles pass each other?

In everyday life, we can imagine two wooden balls, each connected to the border of space by six mutually perpendicular steel tethers under tension. Two such steel balls cannot move past each other, because at a certain relative position, their tethers will touch and prevent the free movement of the two steel balls past each other.

In the strand tangle model, the situation for two tethered fermions differs markedly. Because tethers have no observable properties, they have no tension, are extremely flexible, have no fixed length, produce no forces, and have no mass. Therefore, tethers do not hinder each other in any way, nor do they have any effect on the motion of the tangled cores to which they are attached. The touching of distant, unobservable, extremely flexible, and extremely extendible tethers does not affect the motion of the particle cores to which they are connected.

In short, for all practical purposes, the touching of tethers far away from the cores can be ignored. Fluctuating tethers do not hinder the free motion of non-interacting particles across space. This result is needed to describe interference.

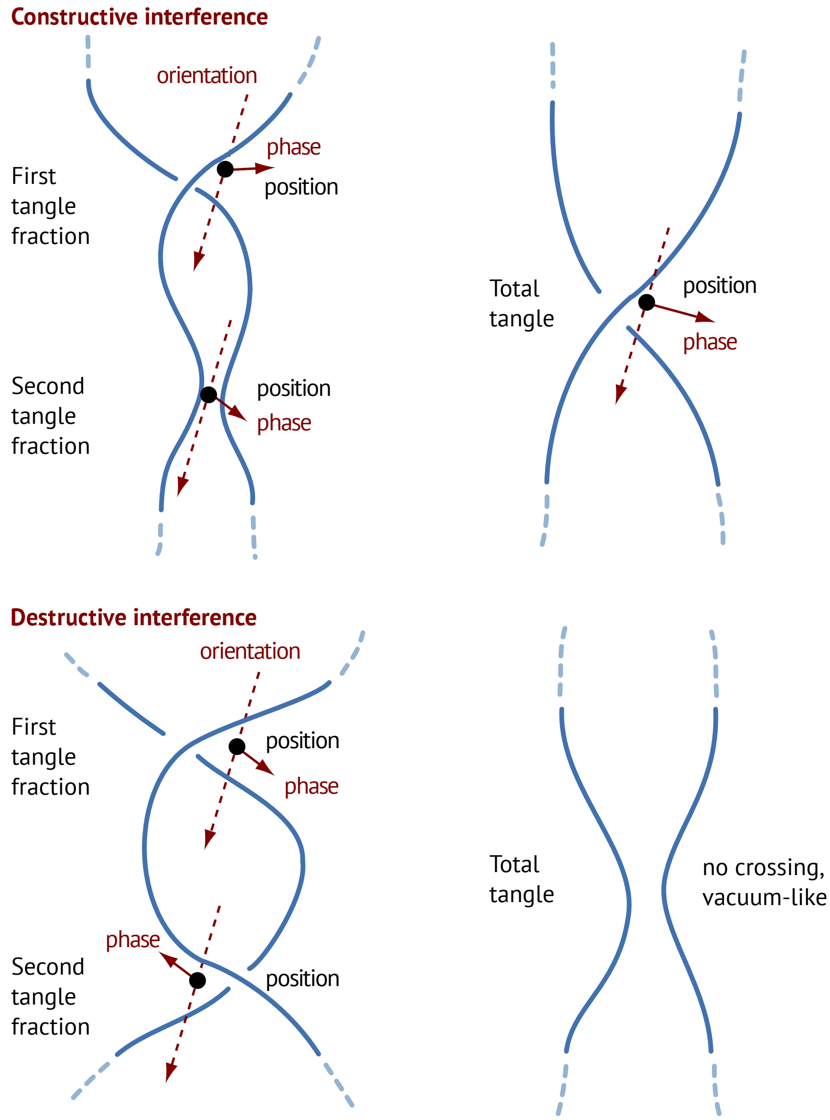

FIG. 14: The strand explanation at the basis of interference: two crossings connected by strands superpose constructively (top) and destructively (bottom).

12 Quantum interference of fermions

The observation of *interference* of quantum particles – such as electrons, neutrons, atoms and molecules – is due to the linear combination of quantum states with different phases at one position in space. Interference results from the linear combinations of wave functions. Such superpositions are a central feature of quantum physics. In particular, they are central to the description of wave-particle duality.

The strand tangle model reproduces interference. For example, using the above definition of superpositions, an equally weighted sum of a tangle and the same tangle with a phase rotated by $\pi/2$ (thus with a core rotated by π) results in a tangle whose phase is rotated by the intermediate angle, thus with a phase rotated by $\pi/4$. Strands thus describe interference.

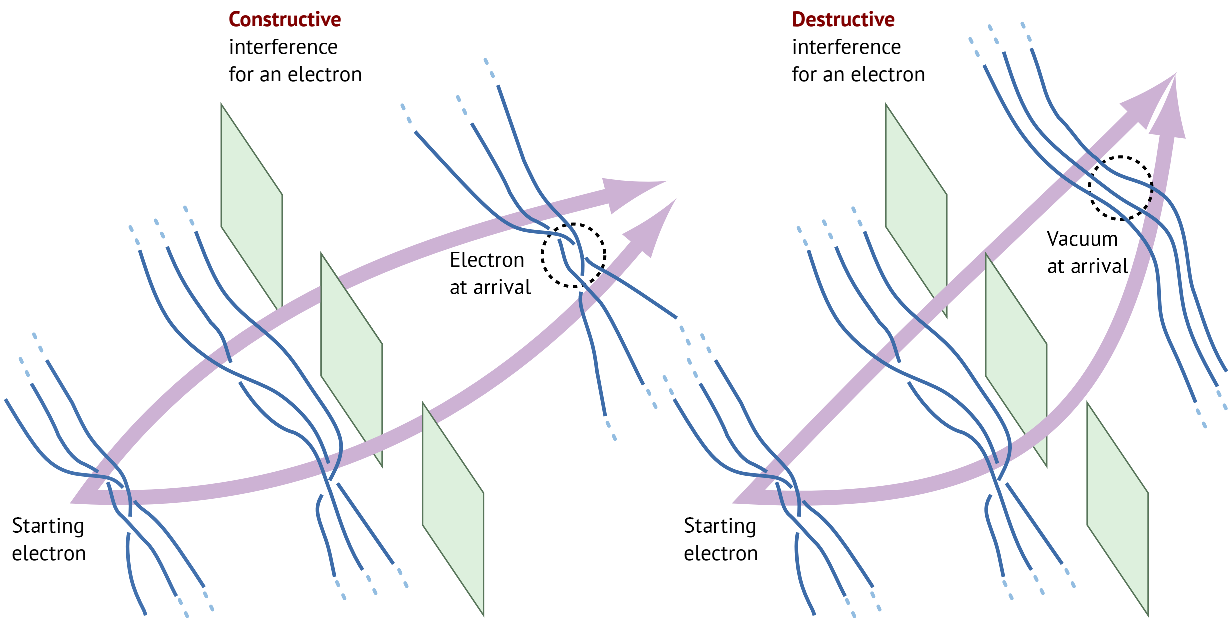

FIG. 15: The tangle explanation of interference is illustrated for a fermion tangle passing a double slit. Depending on the phase difference arising in the two paths, the fermion tangle shows constructive interference (left) or destructive interference (right). The tangles of the particles making up the screen can be ignored, as explained in Section 11.

The most interesting case of interference is *extinction*. In experiments, the multiplication of a wave function ψ by -1 gives the negative of the wave function, i.e., its additive inverse $-\psi$. The local sum of a wave function and its negative is zero, thus explaining extinction in conventional quantum theory.

In the strand tangle model, the negative of a tangle has a core rotated by 2π. Using the above definition of linear combinations, the sum of a quantum state with its *exact* negative requires rotating half of the core by 2π and then connecting it to the other, unrotated half, without crossings between them. This is impossible. Therefore, the resulting tangle has a *vanishing* crossing density in the spatial region where this operation is attempted, that is, in the spatial region where a state is added to its exact negative. Vanishing crossing density is the defining characteristic of the vacuum. Strand tangles thus explain extinction as a consequence of particle tangles.

As a further consequence, tangle addition allows describing and visualizing the double slit experiment. Because of its tethers, a matter tangle obeys fermion statistics and the rotation behaviour of spinors. This behaviour is visualized in Figure 15. As explained in the preceding section, the influence of the tethers of all particles making up the screen can be ignored.

In short, fluctuating strand tangles describe and explain both the constructive and the destructive quantum interference of matter particles. This result directly awakens the desire to explore the case of photons.

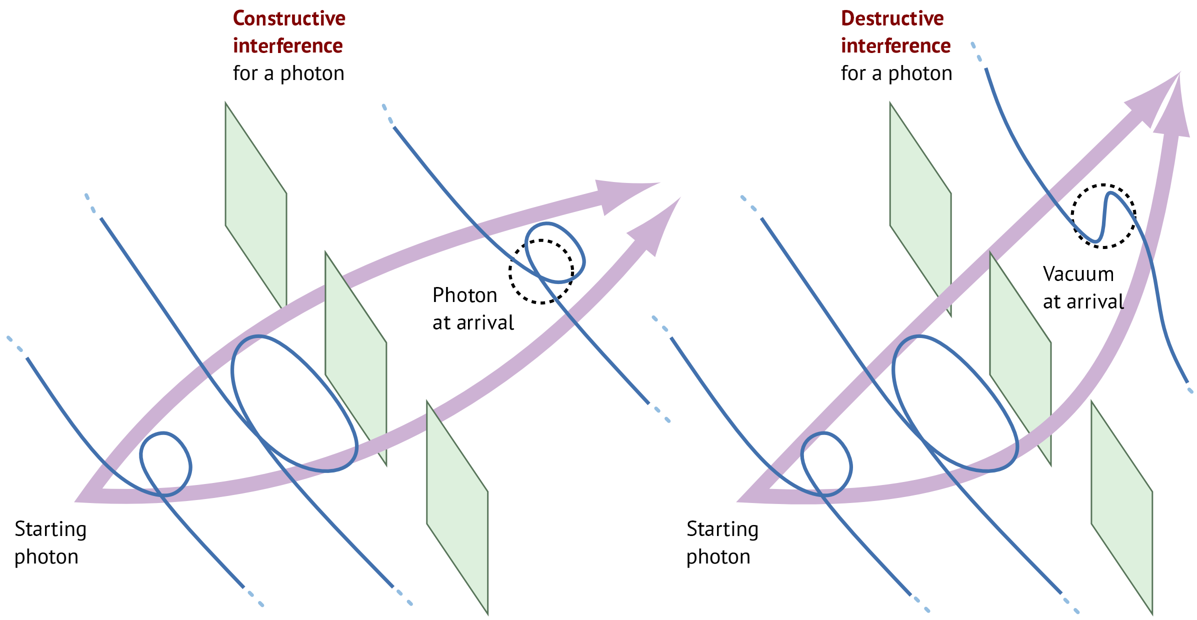

FIG. 16: The tangle explanation of photon interference is illustrated. Depending on the relative phase between the two paths, a photon passing a double slit shows constructive interference (left) or destructive interference (right).

13 An intermezzo: quantum interference of photons

In the strand tangle model, a photon is a single strand with a *twist*, as illustrated in Figure 16, Figure 17, and later on in Figure 30, which shows all boson tangles possible in nature. As expected and explained later on, a photon is a propagating first Reidemeister move; thus, it has spin 1 and a vanishing mass. The twist is the tangle core of the photon and its size is the photon wavelength. (One needs to recall that only crossing switches are observable in the strand tangle model; tethers are not observable.) When a photon advances, its core advances and rotates. Being a boson, the phase of a photon is determined by the pointing direction of its twist. (This is in contrast with elementary fermions, for which the phase is given by *half* the pointing direction of the core.) Also the photon, like the electron, is thus an advancing rotating arrow, as illustrated in Figure 17. Thus, the strand tangle model simply adds a twist and two tethers to the conventional description of the photon as a rotating arrow. For example, the strand tangle model shows that a photon whose core has rotated by 2π is equivalent to a photon without such a rotation: the tethers can fluctuate from one case to the other, showing the equivalence of two states. This is the – much simpler – boson version of the fermion belt trick. Also the 'boson trick' is illustrated in the top left of Figure 17. The description of electric and magnetic fields with strand twists is also possible. The way of visualizing Maxwell's equations with the help of strands was explored in detail in a previous paper [64].

Figure 16 visualizes the interference of photons in the case of a double-slit experiment. The 'negative' of a photon state is a strand whose looped twist points in the opposite direction. The addition of half a twist with its other negative half yields a strand without crossing – a vacuum strand. This situation visualizes extinction and is shown on the right-hand side of the figure.

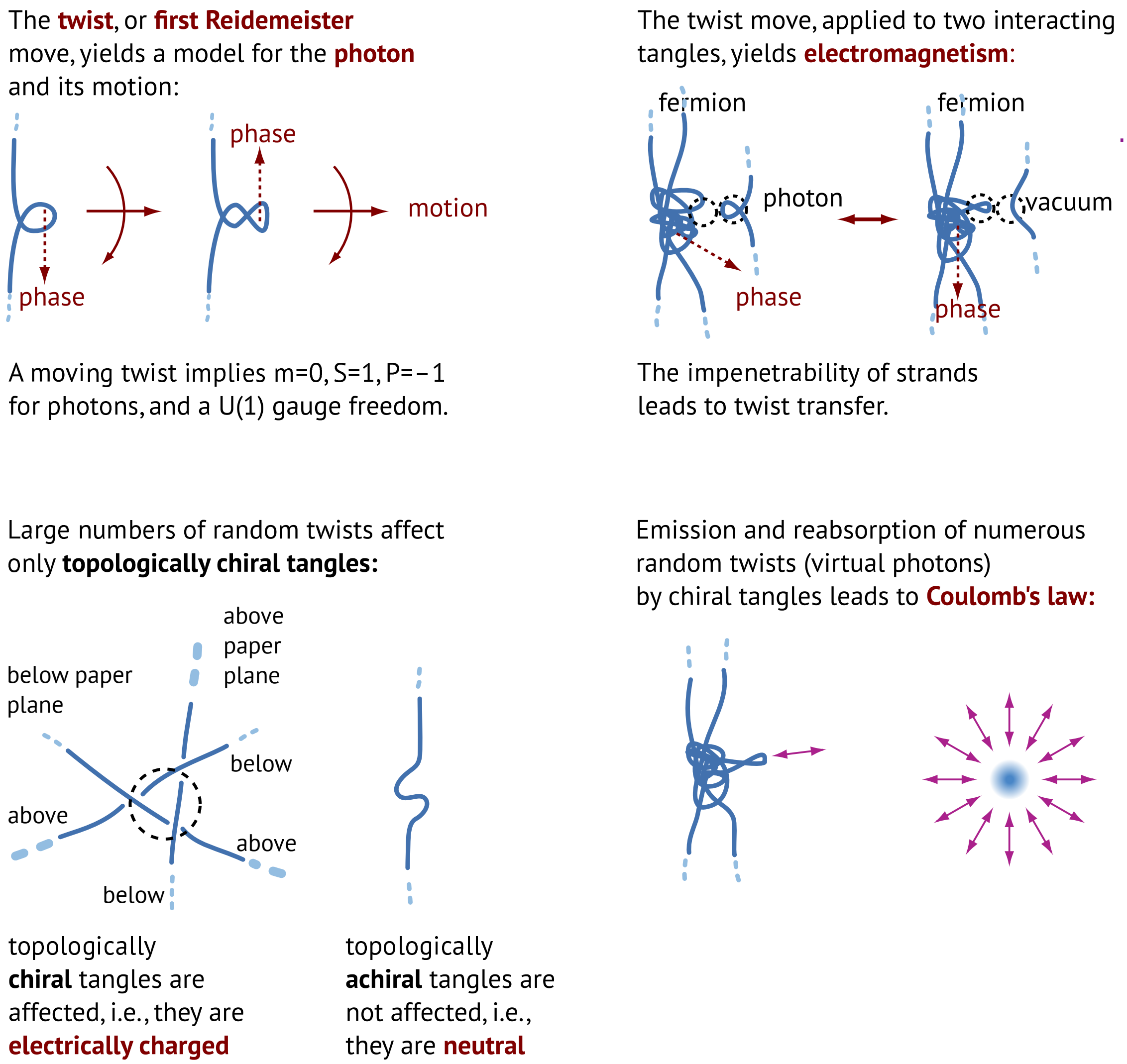

FIG. 17: The strand tangle model for electromagnetism is illustrated, as explored in detail in reference [64]. The motion of a photon with its rotating phase, the absorption of a photon, the origin of electric charge from tangle chirality, and the origin of Coulomb's law are visualized.

In short, strand tangles describe and explain the quantum interference of photons, if one recalls that only crossing switches are observable and that crossing switches can only appear where crossings exist. The simple strand tangle model of the photon, including the implied U (1) gauge symmetry, is confirmed in more detail below. A literature search shows that similar descriptions of single photons with a curved strand – such as the 'corkscrew model' of the photon – are part of physics lore [81], but have never been published.

14 Measurements, Born's rule, wave function collapse and decoherence

Experiments show that every measurement of an initial quantum state ψ has two effects. First, every measurement yields an *eigenvalue* a of the operator of the variable to be measured. Sec-

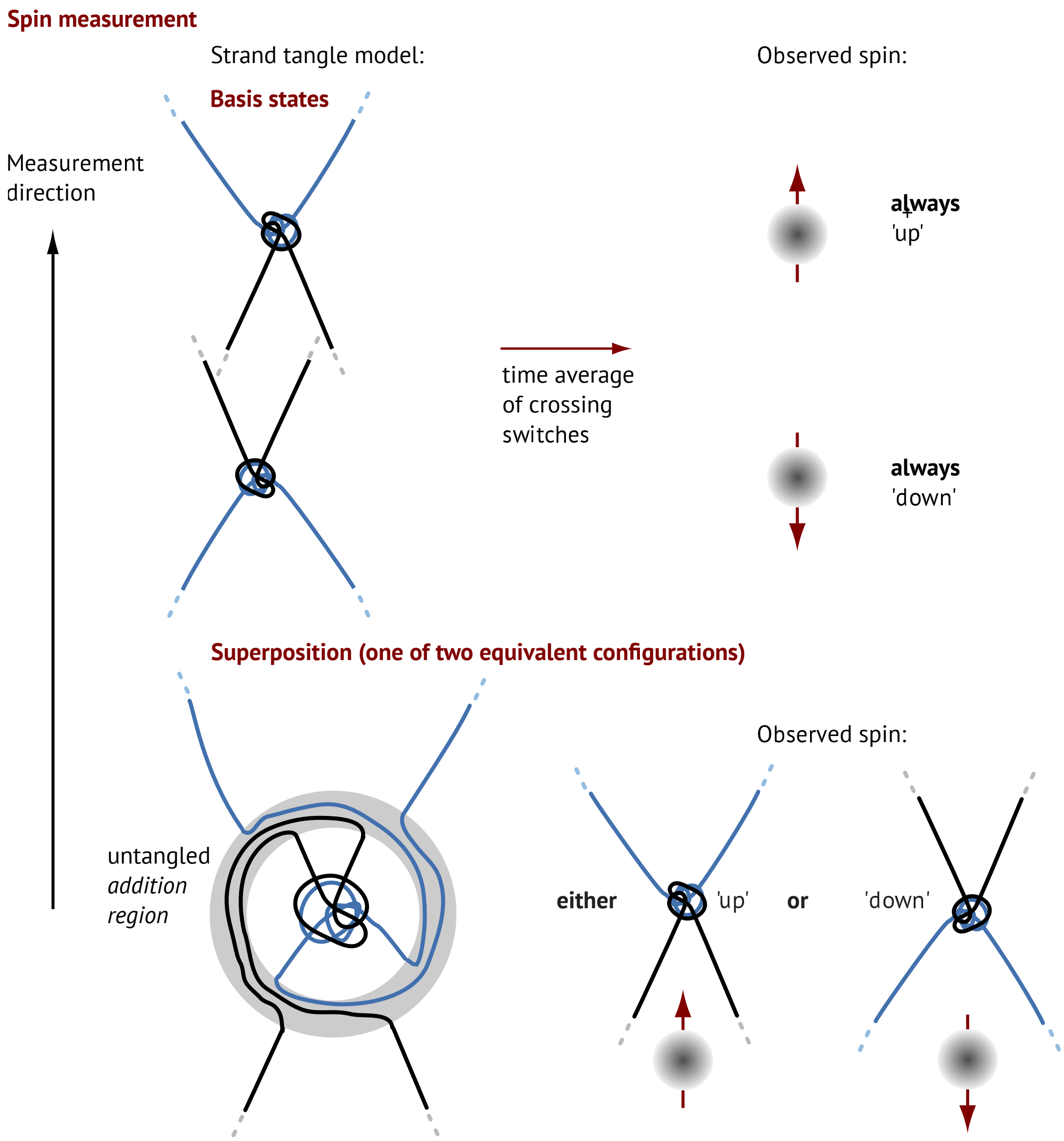

FIG. 18: The measurement of a spin superposition makes the addition region disappear either outwards or inwards.

ondly, every measurement changes, or projects, or collapses, the initial quantum state into the corresponding *eigenstate* ψ_a (in the most common situation without degeneracy). The collapse occurs with a probability given by $|\langle\psi_a|\psi\rangle|^2$, the squared inner product between the quantum state and the eigenstate. This is known as *Born's rule*.

In nature, every measurement apparatus is a device that stores and displays measurement results. This is possible because every measurement apparatus is a device with a memory, and thus is a *classical* device. All devices with memory contain at least one *bath*, i.e., a subsystem described by a temperature [117]. Thus, every measurement apparatus *couples* a bath to the system that it measures. The coupling depends on and defines the observable to be measured by the apparatus. Every coupling of a bath with a quantum system results in *decoherence*. Decoherence leads to wave function collapse and probabilities. This shows that collapse and measurement probabilities are necessary and automatic consequences of decoherence in conventional quantum theory.

The strand tangle model describes the measurement process in precisely the same way as conventional quantum theory [118, 119]. In addition, strands *visualize* the process.

- ▷ A *measurement* is modelled as the specific deformation, induced by the bath of measurement apparatus, that *deforms* the strands of a particle tangle into the resulting eigenstate.
- ▷ This deformation of the particle tangle is the *collapse* of the wave function.
- ▷ The storage of the result in the measurement apparatus, its memory, involves the tethers of the quantum system (after the collapse) and the tethers in the bath of the apparatus.

An example of measurement for the case of a spin superposition is illustrated in Figure 18. Tangles visualize the measurement process:

- ▷ When a measurement is performed on a superposition of two spin states, *the untangled 'addition region' is made to shrink or expand into disappearance by the bath.*

When this occurs, one of the underlying eigenstates 'gobbles up' the other eigenstate: the wave function *collapses*. This process is triggered by the strands in the bath in the measurement apparatus (not shown in the figure). The strands of the apparatus let the addition region disappear either towards the outside or the inside. The choice is determined by the details of the bath coupled to the system during measurement. Thus, bath fluctuations determine the outcome of measurements. Furthermore, the probability of measuring a particular eigenstate will depend on the (weighted) volume that the eigenstate takes up in the superposition because the bath will choose that eigenstate more often.

In the strand tangle model, when the bath in the measurement apparatus selects an outcome, all the bath strands change the original quantum state being measured into the eigenstate of the outcome. This change occurs by deforming all the strands of the quantum state being measured into the strand configuration of the eigenstate. In other words, a large number of bath strands causes the quantum state to collapse by pushing the strands of the quantum system. In any measurement, the initial quantum state and the final quantum eigenstate differ only in their strand shapes, and not in their topologies.

For example, when the position of a charged particle is measured, an initially fuzzy state, that is, a spread-out quantum state, is localized at the measured position. When the particle charge triggers the particle detector, the tangle crossings of the particle interact with the crossings of the charges inside the detector. This interaction localizes the quantum state of the particle, and thus the tangle core, at the spot where the detector is triggered because when charges interact, their cores are automatically localized.

Thus, the strand description of measurement is a specific realization of wave function collapse induced by the environment. This approach has been championed by Zeh and many after him. The details of contextual collapse are still being refined [120]. In simple words, strands state that there is a stochastic aspect in every environment. This appears to resolve the issues raised by Adler [121].

The strand visualization of the wave function collapse triggered by the bath also implies that the collapse *takes time*. The collapse time is the time taken by the bath to trigger the collapse. The strand visualization of the wave function collapse also clarifies that the collapse is *not limited by any speed limit*, as no energy and no information are transported, and thus no signal is transmitted. Indeed, the collapse occurs because the strands of the system are deformed by the strands from the bath.

In short, it was shown that the strand tangle model describes measurements in the same way as conventional quantum theory. A measurement apparatus – which by definition includes a bath – interacting with a quantum system has no other choice than to enforce Born's rule through the strands of its bath. In particular, strands visualize the collapse of the wave function as a shape deformation from a superposition tangle to an eigenstate tangle triggered by the large number of strands of the bath. This description agrees with the usual description that uses decoherence.

15 The decoherence time

In nature, the decoherence process takes time. This time is due to the interaction with the bath in the apparatus or the environment. Generally speaking, the denser and more energetic the microscopic degrees of freedom of the involved bath, the faster the triggering and decoherence.

The decoherence time can be estimated in various ways. A simple estimate arises for baths composed of many scattering particles. In this case, the decoherence time is smaller than the time between two scattering events due to the bath. The resulting localisation is of the order of the (thermal) de Broglie wavelength of a typical bath particle. The decoherence time is given by the particle flux and the interaction cross-section, as explained by Joos and Zeh [118] and by Tegmark [122]. In most practical situations, the resulting decoherence time is extremely short and results in localisation in an extremely small domain.

A second estimate of the decoherence time uses the relaxation time and temperature of the bath [119, 123]. Extremely small values for the decoherence time result in all everyday situations. Long decoherence times are possible if interactions with baths are minimized. This typically requires a careful experimental setup.

Also in the strand tangle model, the decoherence time is an interaction time; it is the time that the bath strands take to project the particle tangle onto the eigenfunction of the measured observable. Also in the strand tangle model, the effects of scattering processes occur. Also in the strand tangle model, the relaxation time at the origin of decoherence arises due to microscopic processes in the bath.

In short, strands reproduce decoherence in all its effects: decoherence takes time, destroys macroscopic superpositions, and is a result of fluctuations in the strands present in the environment, more specifically, in its baths.

16 Quantum entanglement and topological entanglement

In nature, two or more particles can be *entangled*. Entangled states are coherent many-particle states that are not separable, that is, they cannot be written as the product of single-particle states. Entangled states are a fascinating aspect of quantum physics, particularly in the case of entangled macroscopic states. Such states do not exist in classical physics but are observed and explored in many quantum experiments.

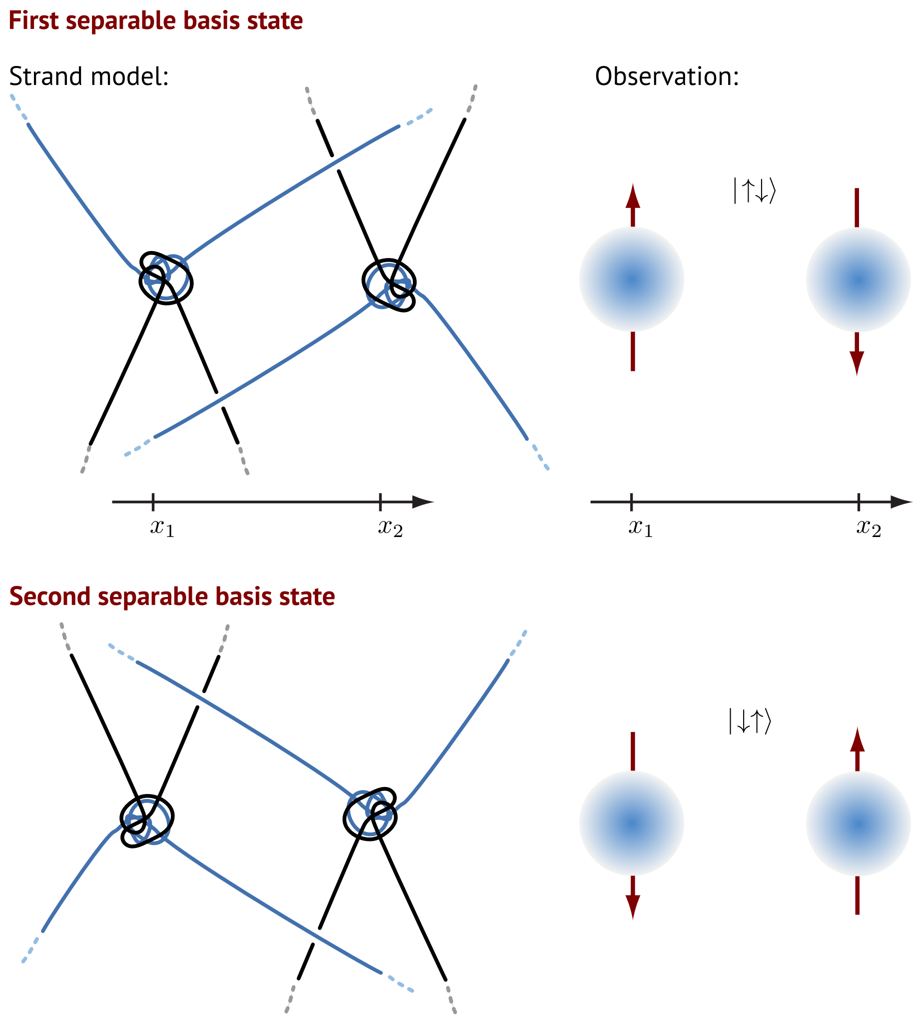

FIG. 19: Two examples of two distant particles with spin in *separable*, *unentangled* and *incoherent* states are illustrated, both in the strand tangle model and in the corresponding observations.

The strand tangle model describes N-particle systems in 3 spatial dimensions. In conventional quantum theory, an N-particle wave function is usually described by a single-valued function in $3N$ dimensions. It is less known that a single-valued N-particle wave function in $3N$ dimensions is mathematically equivalent to an N*-valued* wave function in three dimensions. Usually, N-valued functions are not discussed and lead to uneasiness. However, the strand tangle model naturally defines N wave function values at each point in space: each particle has its own tangle, and each tangle yields, via short-term averaging, its complex value(s) at each point in space. Due to this separation of particle states, the strand tangle model can describe the state of N particles in 3 spatial dimensions – despite the recurring prejudice that this is impossible. For details, see Appendices G and I.

In particular, the strand tangle model of many-particle states allows defining entangled states for two particles.

▷ An *entangled state* is a non-separable superposition of several particle states. Mathematically, an entangled state is *not* a product of separate particle states.

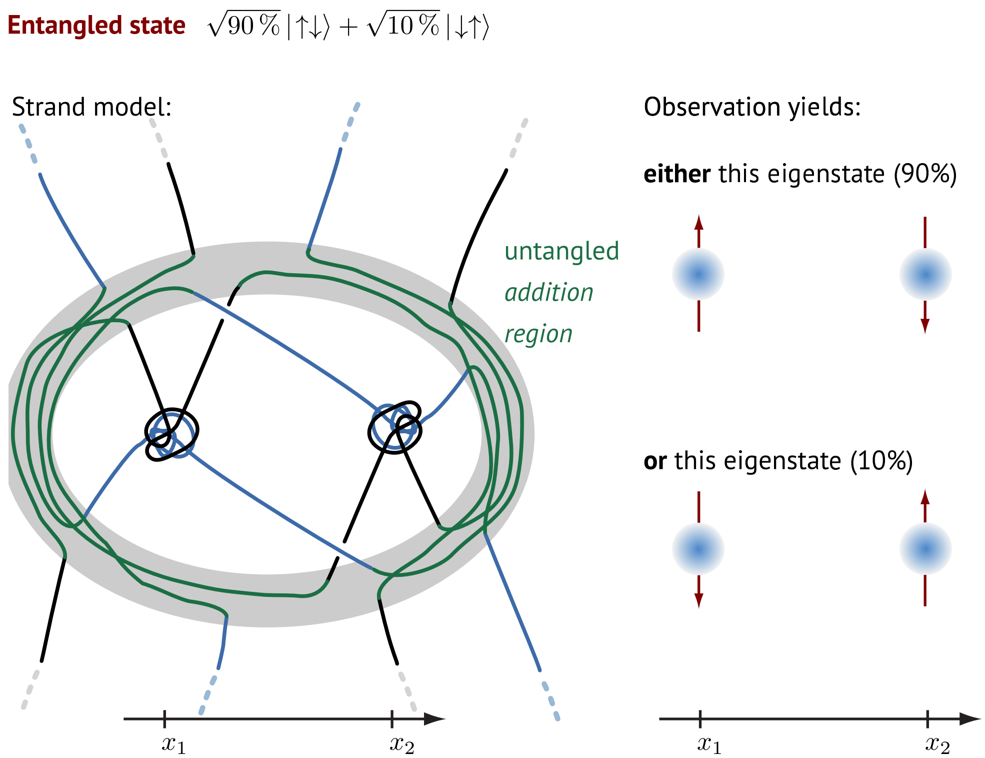

FIG. 20: An *entangled*, *inseparable* and *coherent* spin state of two distant particles is illustrated. The addition region around both particles prevents that either particle can be seen as a separate system that is independent of the rest of its environment.

In the strand tangle model, a state is *non-separable* whenever the tethers of the particles remain entangled even if the tangle cores are pulled apart. More precisely, as illustrated in Figure 20, two states are entangled if their strand addition region surrounds both states.

A well-known example of entanglement is the spin entanglement of two identical fermions in a spin 0 state that are separated. The example was introduced and discussed in detail by Bohm [124].

In the strand tangle model, two distant fermions in a *separable* state are modelled as two distant, separate tangles of identical topology. Figure 19 illustrates two such separable basis states in the strand tangle model, namely the two states with total spin 0, given by $|\uparrow\downarrow\rangle$ and by $|\downarrow\uparrow\rangle$. Such states (with their linked tethers) are typical outcomes of spin measurement experiments. However, other states are also of interest. Using the definition of tangle addition, a superposition such as $\sqrt{90\,\%}\,|\uparrow\downarrow\rangle + \sqrt{10\,\%}\,|\downarrow\uparrow\rangle$ of the two spin-0 basis states looks as illustrated in Figure 20. In conventional quantum mechanics, such a state is not a product state, and therefore it is *entangled.* In the strand tangle model, the entanglement of the state is visible in the addition region. When the spin orientation of one of the particles is measured, the untangled 'addition region' disappears. The result of the measurement will either be the state favoured by the inside of the addition region or the state favoured by the outside. Because the tethers of the two particles are linked, after the measurement, independently of the outcome, the spin of the two particles will always point in opposite directions. This happens for every particle distance. Despite this extremely rapid and apparently superluminal collapse, no energy travels faster than light. After the measurement, the state is separable. Thus, the strand tangle model exactly reproduces the behaviour of entangled spin $1/2$ states observed in experiments.

The similarity of quantum entanglement and topological entanglement has been noted for a long time [125–132]. Tethers provide a basis for this analogy. For example, suitably connecting the tethers at spatial infinity should recover the results of the classification of multi-particle entanglement given in reference [129].

In short, it was shown that strand tangles reproduce quantum entanglement through topological entanglement of tethers, using the definition of wave functions as tangle crossing distributions. Entanglement thus follows from the fundamental principle. This directly raises a question.

17 Hidden variables

At first sight, the strand tangle model seems to introduce hidden variables into quantum theory. One is tempted to argue that the fluctuating shapes of strands play the role of hidden variables. However, non-contextual hidden variables are impossible in quantum theory, as shown most famously by the Kochen–Specker theorem, which is valid for sufficiently high Hilbert space dimensions [133]. In real-life systems, the conditions of the theorem are always satisfied.

Despite the first impression, the strand tangle model does *not* contain hidden variables. First, strands and their shapes are neither observable nor measurable in any way. In particular, strand shapes are *not physical observables* and thus neither physical nor hidden variables. Appendices A and B describe this point in detail. Only crossing switches are observable.

Secondly, strand shapes evolve in a manner dictated by the influence of the environment, which consists of all other strands in nature, including those of empty space itself. Therefore, the evolution of strand shapes and crossing switches is *contextual*.

For the two reasons just given, the strand tangle model does not contradict the Kochen–Specker theorem. The strand tangle model provides *no* observables beyond those of quantum theory. As expected from any model that reproduces decoherence, the strand tangle model leads to a *contextual* and *probabilistic* description of nature. The strand model thus reproduces the approach to quantum theory as the theory of extrinsic properties presented by Kochen [134].

The results of the strand tangle model agree with the latest research on probabilities in quantum mechanics. This includes the investigations of Colbeck and Renner [135].

In short, it was shown that the strand tangle model does *not* make use of non-contextual hidden variables. Nevertheless, in the strand tangle model, quantum theory *emerges* from fluctuating tangle shapes. The emergence does not change the usual probabilistic description of quantum phenomena. Quantum theory remains fascinating.

This section concludes the part that showed that crossing densities of particle tangles are wave functions, form Hilbert spaces, and visualize interference, measurement, decoherence and entanglement. In simple language, *wave functions are blurred tangles*. Equivalently, *tangles are fluctuating skeletons of wave functions*.

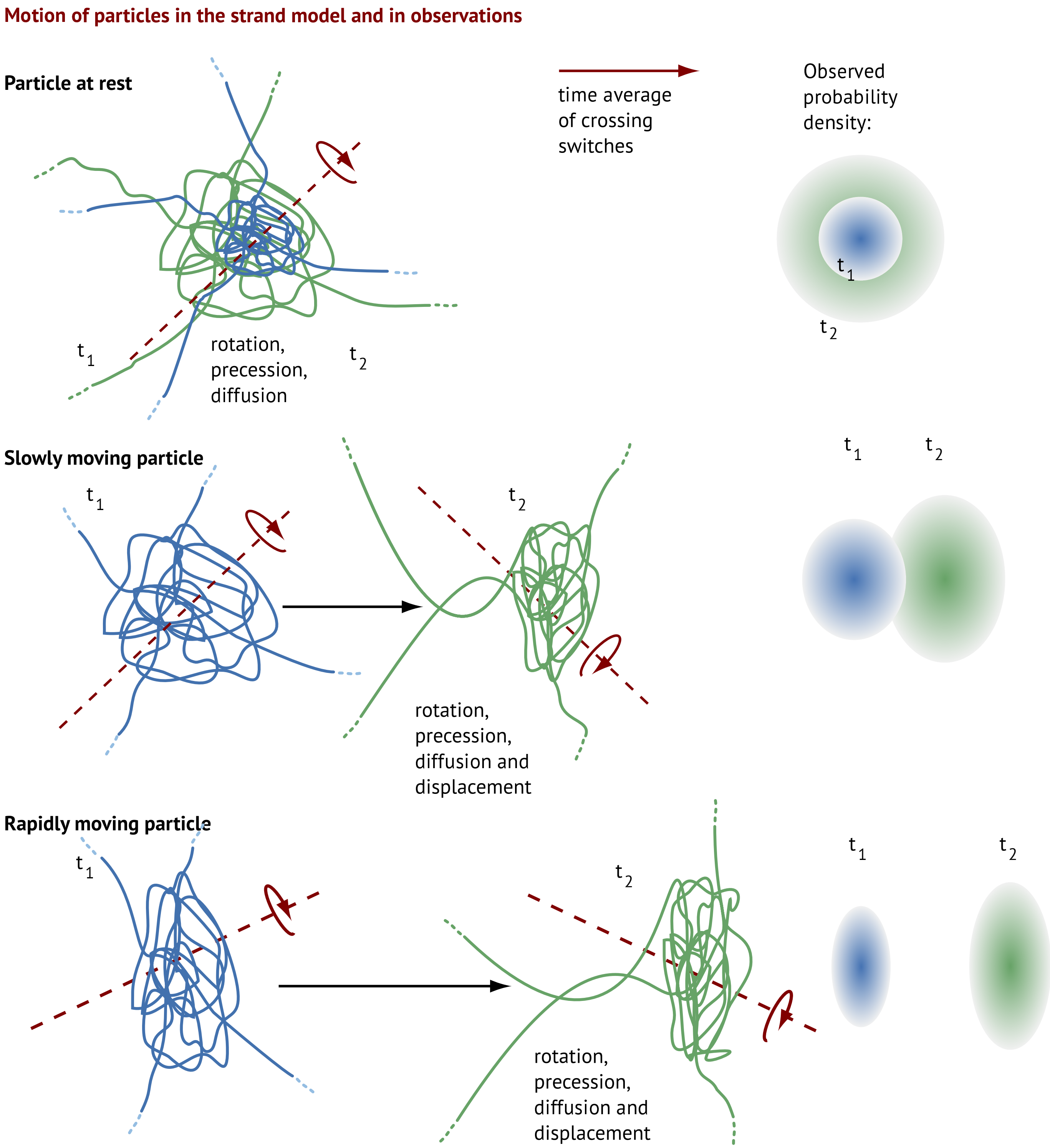

FIG. 21: Tangles of free particles: at rest (top), in slow motion (middle) and fast motion (bottom), illustrating how relativistic speeds lead to core flattening.

Part III: Dynamics of states

18 Spin of particles as the rotation of tangle cores

Part I has shown that a tethered object behaves like a massive spin 1/2 particle. The exploration has also shown that a tethered object can rotate continuously, without hindrance. Part II has shown that crossing distributions of tangles define wave functions. The present part explores the *time dependence* of wave functions. The basic ideas follow naturally from the exploration so far:

- ▷ A spinning particle is described as a continuously rotating tangle core. ▷ A moving particle is described as a spinning tangle core advancing in space.

In other words, the strand tangle model takes the term "spin" by the letter. Spin is core rotation. Thus, the tangle model implements the ideas of Schrödinger, Feynman, Hestenes, and many others: a moving quantum particle is described by a phase rotating around the particle path.

Why do advancing particle tangles rotate? This is the only situation in this text in which it is necessary to recall that the vacuum is also made of strands. These vacuum strands, in combination with the asymmetric tangle core, lead to rotation around the particle path, similar to a propeller moving through a fluid. This propeller motion is the origin of the rotating phase arrow for quantum particles. The exploration in Part I has shown that tethers do not hinder such a rotation. Dirac's belt trick allows continuous rotation.

In the strand tangle model, the propagation of tangles implies the rotation of the tangles. In fact, the analogy with propellers can be carried further. A propeller rotates in a fluid because it is achiral. Also the tangle cores of fermions are all achiral. Furthermore, the rotation speed of a propeller advancing through a fluid depends on the shape of the propeller, in particular on the angles of its fins. In the strand tangle model, the rotation speed of a core depends on its average shape, which in turn depends on its topological structure.

In short, in the strand tangle model, a moving quantum particle behaves like a tethered spinning propeller or a tethered spinning top. The spinning propeller or spinning top is the continuously rotating tangle core. Dirac's belt trick couples propagation and rotation.

19 Emergence of the Schrödinger equation

The fundamental principle of Figure 6 states that crossing switches define the quantum of action $\hbar$. As shown now, the fundamental principle explains how the crossing distribution of a particle tangle evolves in time.

Experiments show that quantum particles move, spin and diffuse. A qualitative visualization of the motion of a quantum particle follows from what was deduced from tangles so far and is illustrated in Figure 21. In the strand tangle model, a localized particle with constant speed is described by a localized tangle core that *advances*. While advancing, the tangle core *rotates* (and precesses). The rotation and precession are due to the belt trick around the advancing particle.

While advancing, the tangle core also *spreads out.* The spreading yields a diffusion of the probability density. The diffusion of a tangle is a consequence of the impenetrability of the strands: the various strand segments continuously push against each other, thus widening the core over time.

Every tangle rotation leads to crossing switches. A rapid tangle rotation results in many crossing switches per time, whereas a slow rotation results in few crossing switches per time. The fundamental principle states that the number of crossing switches per time yields a multiple of $\hbar$ per time. This quantity is commonly called *(kinetic) energy*. In other terms,

▷ Particles with *high* energy have *rapidly* rotating tangles; particles with *low* energy have *slowly* rotating tangles.

Using the fundamental principle that relates crossing switches to $\hbar$, the kinetic energy E of a tangle is related to the angular frequency ω of its core rotation by

$$E = \hbar\omega \quad . \tag{10}$$

Equivalently, the local phase of the wave function ψ changes during the rotation. This implies that

$$\omega = i\partial_t \quad , \tag{11}$$

a relation that will be used shortly. The linear motion of a tangle also requires looking at the number of crossing switches *per unit distance.*

▷ Rapidly moving tangles show many crossing switches per distance; slowly moving tangles show few crossing switches per distance.

The fundamental principle implies that the natural observable to measure crossing switches per distance is action per distance:

▷ The *(linear) momentum* of a moving tangle is the number of crossing switches per distance.

Using the fundamental principle, the momentum p is related to the wave number $k = 2\pi/\lambda$ of the core motion by

$$p = \hbar k \quad . \tag{12}$$

The local phase of the tangle core rotates during propagation. This implies

$$k = -i\partial_x \quad . \tag{13}$$

The appearance of the imaginary unit i describes the rotation of the phase during propagation. This second relation completes the description of matter wave functions without spin.

An advancing particle tangle implies a continuously repeated belt trick. The greater the momentum of the particle, the more its rotation axis must align with its direction of motion. This effect is illustrated in Figure 21. In the non-relativistic case, this alignment leads to a *quadratic* increase in the number of crossing switches with momentum p: one factor p is due to the increase in the speed of rotation, and the other factor is due to the increase in the alignment. This yields

$$E = \frac{p^2}{2m} \quad \text{and} \quad \omega = \frac{\hbar}{2m}k^2 \quad . \tag{14}$$

The constant m is a proportionality factor that depends only on the details of the tangle core structure; it is called the *mass* of the particle. This dispersion relation agrees with the measurements, as long as $p/m \ll c$. In the tangle image, the dispersion relation is valid as long as the speeds are so low that the fluctuating core retains its spherical shape. The complexity of the belt trick implies that spherical tangle cores – massive particles – must move more slowly than light. (In contrast, in relativity, the fluctuating core is a deformed sphere with an ellipsoidal shape. Figure 21 illustrates that situation.)

At this stage of the exploration, Schrödinger's argument from 1926 can be repeated. Substituting the differential relations into the dispersion relation, the evolution equation for the wave function ψ is

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\partial_{xx}\psi \quad . \tag{15}$$

This is the Schrödinger equation for a free particle (written for only one space dimension for simplicity).

In short, it was proven that in the strand tangle model, the tangle core of a matter particle is localized. The belt trick implies that the core rotation and core displacement are related, and this relation allows the definition of an inertial mass value. As a consequence, the wave function, or crossing density, of a non-relativistic spin-less particle obeys the Schrödinger equation. In contrast to usual quantum mechanics, the mass value is *not a free parameter* but uniquely determined by the tangle structure. Several tasks remain: checking indeterminacy, deducing the relativistic description, including spin, and calculating inertial mass.

20 Emergence of indeterminacy

The Schrödinger equation implies Heisenberg's *indeterminacy relations* or *uncertainty relations*. They have been confirmed in every experiment to date.

Also in the strand tangle model, the indeterminacy relations can be deduced from the Schrödinger equation. In addition, the indeterminacy relations are a direct consequence of the fundamental principle illustrated in Figure 6. Indeed, the smallest indeterminacy of every action measurement is half a crossing switch. When a strand configuration corresponds to the middle case of Figure 6, it is not clear to which of the two outer configurations it belongs. Therefore, for any tangle configuration

$$\Delta W \leq \hbar/2 \text{ and thus } \Delta x \Delta p \leq \hbar/2 \quad . \tag{16}$$

For the same reason, the indeterminacy of the measurement of any two observables whose product is an action value is given by half a crossing switch.

In short, in the strand tangle model, quantum particles naturally obey the usual indeterminacy relation.

21 Rotating arrows and path integrals

A simple procedure allows visualizing the equivalence between the strand tangle model and the formulation of quantum theory with path integrals. In the strand tangle model, the tethers are not observable, and the fluctuating tangle core defines the fuzzy position and phase of a quantum particle. If the tangle core is approximated to be of *vanishing* size, thus 'point-like', then the motion of the core describes the ‘path’ of the quantum particle. A simple visualization of this approximation is to imagine that the tangle consists of actual ropes that are pulled outwards. The tangle core then becomes as small as possible; one obtains a *tight* tangle of approximately negligible size.

In his popular book on QED, Feynman described the motion of a quantum particle as an advancing and rotating arrow. In the strand tangle model, the continuous rotation of the core visualizes Feynman’s rotating arrow. The various possible motions of the ‘point-like’ core correspond to the different paths in Feynman’s description. Wave functions appear when the effects of all possible paths are superposed. In particular, the phase and amplitude for each path must be added like small vectors. In the strand tangle model, the effects of all possible paths are added automatically, through the fluctuations of the tangle motion. With the definition of tangle addition given above, path addition occurs in exactly the manner described by Feynman. Point-like tangles of electrons and photons reproduce all the chapters of Feynman’s book. The differences between the path integral and the strand tangle model are explored in Part IV.

In short, the strand tangle model of quantum particles reproduces the *path integral formulation* of quantum mechanics when particle tangles are approximated as *tight* tangles of vanishing (or Planck) size. Tight tangles arise if one *imagines* tangles to consist of actual ropes that are pulled outwards. Tight tangles that fluctuate over space yield, when averaged over time, the usual wave function.

22 Emergence of the Klein-Gordon equation

In 1980, Battey-Pratt and Racey [76] showed that tethered, relativistic and spin-less particles are described by the Klein-Gordon equation. Their approach can be adopted in the strand tangle model. The spin-less case assumes a constant spin orientation. The only difference to the derivation of the spin-less Schrödinger equation is the relativistic behaviour. Time dilation, combined with the belt trick, leads to the following relation, which includes the core speed v and half the core spinning frequency ω:

$$\nabla^2\psi = \frac{\omega^2 v^2}{c^4(1 - v^2/c^2)}\psi \quad . \tag{17}$$

Inserting the relativistic expression for the observed angular rotation frequency of the tangle phase $\omega/\sqrt{1 - v^2/c^2}$ (the core rotates with twice that frequency) yields

$$\nabla^2\psi - \frac{1}{c^2}\partial_{tt}\psi = \frac{\omega^2}{c^2}\psi = \frac{m^2c^2}{\hbar^2}\psi \quad , \tag{18}$$

where mass m is again introduced as a constant that relates the translation speed and angular frequency of the belt trick. This is the well-known *Klein-Gordon equation.* The calculation confirms that in the strand tangle model, the core spinning frequency $\omega = mc^2/\hbar$ due to the belt trick reproduces what Schrödinger called the *Zitterbewegung*.

The differences between the ideas of Battey-Pratt and Racey and the strand tangle model are minimal. First, the fluctuations inherent in the strand tangle model – and the definition of the quantum phase with crossings – explain that the phase is defined and varies all over three-dimensional space, and is not only defined at the location of the rotating core. Secondly, the crossing density yields an amplitude that is a function of position. In this way, the strand tangle model solves the issues mentioned by Battey-Pratt and Racey in their paper. In short, a fluctuating strand tangle yields a full description of its state with a wave function $\psi(x,t)$ that fully behaves as expected.

In short, the strand tangle model *reproduces* the result by Battey-Pratt and Racey: for relativistic spin-less matter particles, tethers imply the Klein-Gordon equation. The next task is to add spin.

23 Deducing Pauli spinors and the Pauli equation from tangles

An important requirement for any model of wave functions is that it includes *spin*, and in particular the variation of spin orientation over space and time. The results derived so far can be used to cover this case. At a given position in space, a fluctuating tangle core has a local average density of crossings, local average phase, and local average orientation of the rotation axis. As before, it is assumed that the tangle core rotates rigidly. The simplest case is the non-relativistic one.

To extend the Schrödinger equation to describe the rotation axis and its orientation, it is most practical to use the *Euler angles* α, β and γ [112, 113]. They allow a description of the crossing distribution as

$$\Psi(x,t) = \sqrt{\rho}\,\mathrm{e}^{i\alpha/2} \begin{pmatrix} \cos(\beta/2)\mathrm{e}^{i\gamma/2} \\ i\sin(\beta/2)\mathrm{e}^{-i\gamma/2} \end{pmatrix} \quad , \tag{19}$$

which is the natural description of a wave function that includes the orientation of the axis of rotation. Similar to the spin-less case, the crossing density is the square root of the probability density $\rho(x,t)$. Again, the angle $\alpha(x,t)/2$ describes the phase, i.e., (one half of) the rotation *around* the axis. The newly added local orientation of the axis is described by a two-component matrix that uses the two angles $\beta(x,t)$ and $\gamma(x,t)$. Because of the belt trick, the expression for the crossing distribution contains only *half* angles. Due to the half angles, the two-component matrix is not a vector, but a *spinor*, as Paul Ehrenfest called it, in analogy to 'vector' and 'tensor'. For $\beta=\gamma=0$ at all times and positions, the wave function ψ used in the Schrödinger equation is recovered.

As described by Payne [136], by Penrose and Rindler [137], or by Steane [138], a Pauli spinor is best visualized as a flagpole and a sign. The direction of the flagpole is described by two angles. These two angles describe the spin orientation and are new; they did not appear in the case of spin zero particles. The direction of the flag around the pole is described by a third angle. The length of the flagpole is described by a positive real number $\sqrt{\rho}$. The last two parameters are the same as those for the spin zero case. The flag angle around the pole is the *phase* of the wave function; it also corresponds to the rotating arrow used by Feynman in his book on QED [116]. The length of the flagpole is the *amplitude* of the wave function. Finally, the sign of the spinor indicates whether the flag has been rotated by an even or uneven multiple of 2π. (In relativistic case that includes antiparticles, the handedness of the chiral core distinguishes particles and antiparticles.)

The strand tangle model for a fermion reproduces the flagpole visualization of spinors in all its details. The orientation of the tangle core is described by three angles. The crossing density of the tangle core – the "size" of the core – reproduces the positive real number $\sqrt{\rho}$. The twistedness of the tethers reproduces the sign of the spinor.

The other ingredient required for the description of the spin is a description of its *motion*. A moving spinning tangle representing a propagating wave function implies that propagation is described by the wave vector $\boldsymbol{k}$ multiplied by the spin operator $\boldsymbol{\sigma}$. The *spin operator* $\boldsymbol{\sigma}$, for a spin $1/2$ matter particle, is defined as the vector built from the three specific matrices

$$\boldsymbol{\sigma} = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) \quad . \tag{20}$$

These three matrices are called the *Pauli matrices*.

The description of the spin axis and the description of the spinning motion can be inserted into the non-relativistic dispersion relation $\hbar\omega = E = p^2/2m = \hbar^2 k^2/2m$. This yields the wave equation

$$i\hbar\partial_t \Psi = -\frac{\hbar^2}{2m}(\boldsymbol{\sigma}\boldsymbol{\nabla})^2 \Psi \quad . \tag{21}$$

This is *Pauli's equation* for the evolution of a free, non-relativistic quantum particle with spin $1/2$.

Anticipating the inclusion of electrodynamics [64], it is also possible to include the electric and the magnetic potentials, using *minimal coupling* to the electromagnetic field. Minimal coupling implies substituting $i\hbar\partial_t$ with $i\hbar\partial_t - qV$ and substituting $-i\hbar\boldsymbol{\nabla}$ with $-i\hbar\boldsymbol{\nabla} - q\boldsymbol{A}$. This introduces the electric charge q and electromagnetic potentials V and $\boldsymbol{A}$. A bit of algebra involving the spin operator then leads to the Pauli equation for a charged particle

$$(i\hbar\partial_t - qV)\,\Psi = \frac{1}{2m}(-i\hbar\boldsymbol{\nabla} - q\boldsymbol{A})^2\Psi - \frac{q\hbar}{2m}\boldsymbol{\sigma}\boldsymbol{B}\Psi \quad , \tag{22}$$

where the magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ appears explicitly. This equation is famous for describing the motion of silver atoms, which have spin $1/2$, in the Stern-Gerlach experiment. The magnetic field splits a beam of silver atoms into two beams. This effect is due to the new, last term on the right-hand side, which does not appear in the Schrödinger equation. The term is a pure spin effect and implies a g-factor of 2. Depending on the spin orientation, the sign of the last term is either positive or negative. Thus, it acts as a spin-dependent potential. The two options for spin orientation then produce the upper and lower beams of silver atoms that are observed in the Stern–Gerlach experiment.

In short, a non-relativistic tangle core that rotates continuously and rigidly *reproduces* the Pauli equation for non-relativistic particles with spin 1/2. The next step is to explore the relativistic case, which also includes antiparticles.

24 Antiparticles – and rational tangles

The explanatory power of the strand tangle model is impressively confirmed for the case of antiparticles. In the model, *antiparticles* are mirror tangles that rotate in the opposite direction. All particle tangles are also chiral. Thus, the strand tangle model correctly models the opposite handedness of particles and antiparticles.

Figure 22 illustrates the situation using the rational tangles for the electron and the positron. Their tangles will be introduced below, in Section 31. The figure allows deducing that the two tangles can be continuously transformed into each other. This possibility explains how rational tangles can reproduce Dirac spinors, for which the particle–antiparticle content usually varies from one place to the next. Such a continuously varying content is observed in experiments and

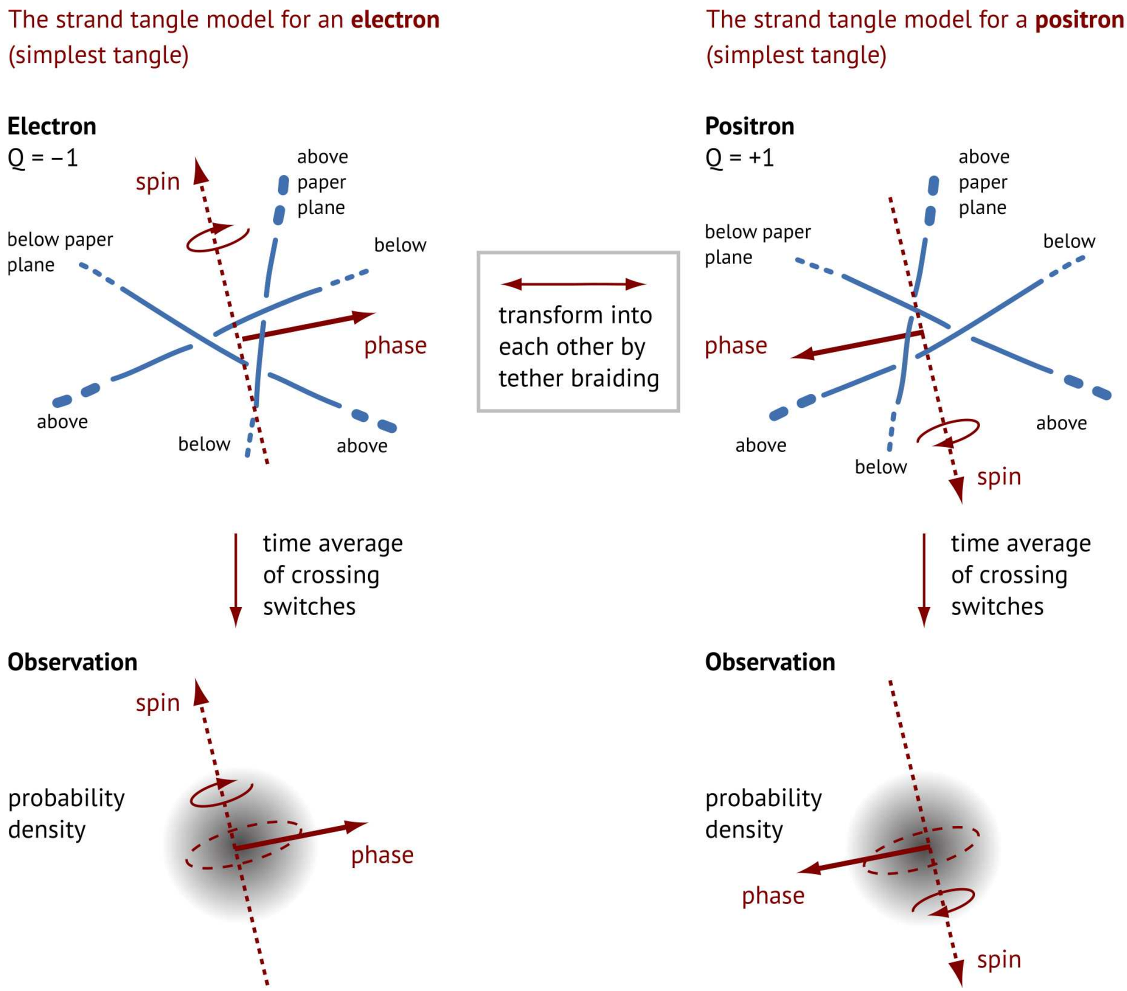

FIG. 22: The simplest tangle for an electron (left) can be continuously deformed to the simplest tangle for a positron (right). One way to perform the deformation is to deform two neighbouring tethers, shift the third strand, and then to properly untwist the four tethers of the first two strands. Another way to perform the deformation is to bring all tethers into one plane, then rotate three tethers together against the other three, and then bend all six tethers again out of the paper plane.

is an essential aspect of the Dirac equation. For example, the transformation of particles into antiparticles is at the basis of Klein's paradox [139]. The ability to reproduce this transformation is a further indication that the strand tangle model of particles agrees with reality.

A *continuous* transformation between an electron and positron can only be reproduced using *rational* tangles. As explained in Figure 1, a tangle is called *rational* if it is unknotted and arises purely through the braiding of the tethers. The transition between particles and antiparticles is a change in tether braiding. Open knots, prime tangles, and all other knotted tangles do not allow a smooth transition between particles and antiparticles. Only rational tangles, the simplest of all tangles, are natural candidates for describing elementary particles.

In short, only rational tangles of strands allow visualizing the continuous transition between antiparticles and particles using mirror tangles and tether deformations.

25 From Dirac spinors to the free Dirac equation

The free Dirac equation, describing spin as well as relativistic effects, follows from strand tangles. This section summarizes an older published argument and provides two qualitative arguments.

The first argument begins with a paper by Battey-Pratt and Racey from 1980. They showed [76] that every *tethered* massive quantum particle is described by the Dirac equation if the tethers are unobservable. In the strand tangle model, the approach of Battey-Pratt and Racey is taken over, with the only extension being that the quantum particle itself, that is, its core, is also made of strands. Each particle tangle defines a 4-component *Dirac spinor* $\psi(x)$, as follows:

- ▷ Averaged over a few Planck times, the density of the strand crossings yields the *amplitude* of the wave function, and its square the probability density.
- ▷ Averaged over a few Planck times, the position of the *centre* of the core yields the *maximum* of the probability density.
- ▷ Averaged over a few Planck times, the orientation of the core yields the *spin orientation*.
- ▷ At each position x, the upper two components of the spinor $\psi(x)$ are defined by the local average of finding, at that position, the (tight) tangle, with a given orientation and phase.
- ▷ At each position x, the *lower* two components of the spinor $\psi(x)$ are defined by the local average of finding, at that position, the (tight) *mirror* tangle, i.e., the antitangle or antiparticle, with a given orientation and phase.

In the strand tangle model, a free, non-interacting fermion advancing through space is described by a constantly rotating rational tangle core, whose central position moves through space along a straight line. This is, as mentioned, the tangle description of Feynman's description of a quantum particle as an *advancing* and *rotating* arrow [116]. When the tangle is imagined to have a negligible core size, it visualizes and reproduces the Feynman propagator. Simultaneously, a rotating tangle visualizes the description given by Hestenes [140–142] of the free Dirac equation. In the strand tangle model, particles are tangles and thus tethered cores; they behave as assumed by Battey-Pratt and Racey and therefore follow the Dirac equation.

Using the fundamental principle of the strand tangle model, the conclusion of Battey-Pratt and Racey can be rephrased in the following concise way:

- ▷ The free Dirac equation is the *differential* version of Dirac's trick on a fluctuating rational tangle.

A more precise formulation is as follows:

- ▷ The belt trick implies the γ^μ matrices and their Clifford algebra, i.e., their geometric algebra properties [140–142].
- ▷ The first two components of the γ^μ matrices describe the rational tangle core, i.e., the particle, whereas the last two components of the γ^μ matrices describe the rational mirror tangle, i.e., the antiparticle.

In summary, using the results of Battey-Pratt and Racey [76], the relativistic description of rational tangles yields the free Dirac equation.

A second qualitative argument helps to understand the appearance of the free Dirac equation from strands. The free Dirac equation

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi = mc\psi \tag{23}$$

is due to five basic properties of nature:

1. The action limit given by $\hbar$, which yields wave functions ψ.
2. The speed limit for massive particles given by c, which yields Lorentz transformations and Lorentz invariance.
3. The spin $1/2$ properties in Minkowski space-time.
4. Particle–antiparticle symmetry, where this and the previous point are described by Dirac's γ^{μ} matrices.
5. A particle mass value m that connects phase rotation frequency and wavelength using the imaginary unit i.

These five properties are necessary and sufficient to yield the free Dirac equation. (The connection between the γ^{μ} matrices and the geometry of spin was first worked out about a century ago by Fock and Iwanenko [143].) The strand tangle model reproduces these five properties in the following way:

1. All physical observables are due to crossing switches, which imply a minimum observable action $\hbar$ and thus the existence of wave functions.
2. Tethers constrain the tangle cores to advance less than one Planck length per Planck time, which is slower than c (see Figure 2).
3. Tethers connect tangle core rotation and tangle core displacement and lead to a finite mass value m much smaller than the Planck mass, because of the low probability of the belt trick.
4. Tethers reproduce the spin $1/2$ properties for rotation, exchange and boosts, and thus yield the γ^{μ} matrices, with tangles and mirror tangles corresponding to particles and antiparticles.
5. Tangle rotation through the belt trick results in particle propagation.

In other words, both in nature and in the strand tangle model, the inability to observe action values below $\hbar$ leads to wave functions and probability densities. Both in nature and in the strand tangle model, the inability to observe speed values larger than c leads to Lorentz invariance and to the relativistic energy-momentum relation. Both in nature and in the strand tangle model, the mass, the spin $1/2$ properties, and the γ^{μ} matrices arise. This implies the Dirac equation for free particles. For conventional quantum theory, this argument was made by Simulik [144–146]. In the strand tangle model, all of these properties are due to the tethers.

The strand tangle model also explains quantum motion in a third way. In nature, all motion, also quantum motion, can be described with the principle of least action: motion minimizes action. The Dirac Lagrangian specifies how to determine and how to minimize the value of the action of a relativistic fermion. In the strand tangle model, *action* denotes the number of crossing switches. The principle of least action then becomes the *principle of fewest crossing switches.* In the strand tangle model, motion minimizes crossing switches. After spatial averaging, this relation for spinning and advancing fermion tangles leads to the free Dirac Lagrangian and the free Dirac equation.

In short, using three different arguments, the fundamental principle strongly suggests that strand tangles representing relativistic fermions follow the free Dirac equation.

26 Quantitative derivation of the Dirac equation

The first quantitative derivation of the Dirac equation from the tethering of particles was given by Battey-Pratt and Racey [76]. They showed that in relativistic particle motion, the average size of the belt trick is contracted by the Lorentz factor. When this effect is taken into account, the Dirac equation appears.

Using the research results of the last decades, various other derivations of the Dirac equation can be reproduced with strands. A simple quantitative derivation is based on the typical notation of a Dirac spinor with four complex components. The correpsonding eight real parameters in each Dirac spinor lead to the description of a Dirac spinor wave function given by Loinger and Sparzani [147] as

$$\Psi = \sqrt{\rho}\, \mathrm{e}^{i\delta}\, L(\boldsymbol{v})\, R(\alpha/2, \beta/2, \gamma/2) \quad , \tag{24}$$

where $\sqrt{\rho}$ is the amplitude, L is a matrix with three real parameters describing the boost transformation, R is a matrix with three real parameters describing the orientation and phase of the spin, and δ is the fraction of particle and antiparticle. All parameters depend on space and time.

Most parts of a Dirac spinor wave function can be visualized using a relativistic spinning top. In particular, it was shown in the 1960s that the amplitude and six parameters follow from spinning tops [147]. In the strand tangle model, the tangle core acts as the spinning top.

In the strand tangle model, a Dirac spinor is described by the geometry of its rotating tangle core, which can be imagined as a rotating tethered top. The geometry of a rotating top suggests describing a Dirac spinor – the rotating top or tangle core – using the following quantities derived from a spinning tangle core: *one* strand density, *three* angles specifying rotation axis (i.e., spin axis) and phase of the core, *three* parameters that describe the contracted shape of the tangle core and thus specify the boost direction and magnitude, and *one* angle that specifies the relative weight of particle and antiparticle.

The density, rotation, and orientation of the tangle core are defined and visualized in Part I. These parameters, including the contraction due to a boost in *one* direction, are illustrated in Figure 21. The relation between rotation frequency and displacement reproduces the mass of the particle. In addition to the visualization that uses spinning tops, a spinning *rational* strand tangle explains the last phase δ. This phase describes the relative fraction of particle and anti-particle, i.e., of core and mirror core. This phase is visualized in Figure 22. Thus, the geometry of strand tangles yields eight real parameters that correspond to the eight real parameters that describe Dirac spinors. The eight real parameters describing tangle cores correspond to the four complex numbers used by Dirac when he wrote down his equation [147–149].

Thus, spinning rational strand tangles naturally reproduce Dirac spinors. This is possible because free quantum particles behave as tethered spinning tops. Or:

▷ Free fermions are rotating tangle cores.

Once Dirac spinors are defined, the simplest derivation of the Dirac equation might well be the one given by Lerner in 1996 [150]. The derivation is based on two properties: conservation of the spin current and Lorentz covariance. Lerner showed that together, these two properties completely and uniquely imply the Dirac equation.

In the strand tangle model, an advancing particle in vacuum is visualized either as a tethered propeller or more precisely, as a tethered spinning top. The definition of spin with tethers implies that particles move at a constant speed, that is, tangle cores move at constant speed and keep rotating with constant rotation frequency. In other words, strand tangles imply that *spin current is conserved* over space and time. (Indeed, there is no friction or any other mechanism in the strand model that can destroy spin current conservation. The strands that make up the vacuum yield a constant effect on particle tangles: their rotation is conserved.) Thus, the first property used by Lerner is fulfilled. In addition, the definition of spin using tethers implies the *Lorentz covariance* of spin, i.e., the proper behaviour under rotations and boosts. This property was already confirmed by Battey-Pratt and Racey [76] in 1980. In particular, Lorentz covariance arises because under boosts, tangle cores change shape (in a way that resembles the change of a sphere to an ellipsoid), and the belt trick therefore changes accordingly. Thus, both properties used by Lerner are reproduced by the strand tangle model of relativistic fermions. Therefore, strands imply the free Dirac equation.

The quantitative argument by Lerner, like the three arguments in the previous section, implies that the strand tangle model agrees with the free Dirac equation.

Test 1: Tangles of strands predict the lack of the tiniest measurable deviation from the free Dirac equation.

Possible deviations from the free Dirac equation have been explored in detail, for example in reference [1]. None has ever been found. The strand tangle model predicts the complete lack of deviations from observations for all measurable energies and length scales, in agreement with all data. (This prediction is not in contrast with the Planck limits, as explained below.)

In short, it was proven that the relativistic behaviour and the conservation of spin density in the belt trick imply the free Dirac equation. In simple language, *Dirac's equation is due to Dirac's trick.* As a consequence, the strand tangle model with its automatic appearance of antiparticles allows visualizing all relativistic quantum effects. They include Zitterbewegung, which is caused by core rotation. They also include Klein's paradox once potentials are introduced. Therefore, the strand tangle model completely agrees with the free Dirac equation for all measurements. As a further consequence, the strand description of wave functions yields specific experimental predictions.

27 A limit on probability density – and on locality

The fundamental principle of the strand tangle model, with its (corrected) Planck limits and definition of wave functions as crossing densities, implies a specific Planck limit: a wave function *cannot* have an arbitrary high amplitude or modulus. Equivalently, no quantum system can have an arbitrary high probability density.

Test 2: Nature limits the value of probability density:

$$||\psi(\mathbf{x},t)||^2 \leq \left(\frac{c^3}{4G\hbar}\right)^{3/2} \approx 3 \cdot 10^{103}\,\mathrm{m}^{-3} \quad . \tag{25}$$

The strand limit value is the inverse of the smallest possible volume in nature. So far, no observation contradicts the (corrected) Planck limit for probability density and the corresponding Planck limit for the modulus of wave functions. Finding an exception would falsify the strand tangle model. In particular, the density limit prevents the existence of Dirac's δ distribution.

Note that there is no corresponding Planck limit for the phase or phase difference. The lack of such a limit is due to the definition of the phase angle as a length ratio. Length ratio values are not limited by Planck limits.

The two Planck limits for wave function amplitude and probability density do not appear to have been discussed in the research literature. The main reason for this is that the limits are extremely large. It seems unlikely that the limits for wave functions can ever be approached in an experiment. Any such experiment would require probes of Planck energy that are neither available nor possible. If at all, the two limits only play a role in quantum gravity, including the big bang itself. However, so far, no observation of any quantum gravity limit – any limit that contains $\hbar$, c and G – has been performed for any physical observable.

The strand limit for the probability density implies and confirms a basic property of nature:

Test 3: The strand tangle model predicts the lack of any point-like physical observable.

Point-like quantities cannot be detected, measured, or play a role in nature. They do not exist in nature. Point particles, δ functions (δ distributions), and singularities of any type cannot arise in nature. Once again, the fundamental principle challenges our habits of thought. In experiments, no point-like observable has ever been found.

Scientists took two millennia to define space as a continuous set of points and to get used to working with real numbers as coordinates for describing points. It is common to think that between two points, there is always room for a third, however small their distance may be. Poking gentle fun at thinkers who challenged continuity, such as Zeno of Elea, is a part of modern science. Despite this long history and common habits of thought, quantum gravity requires a different approach. In nature, action W, length l and force F are related by $W/l = Fl/c$. Inserting the force limit $F \leq c^4/4G$ and the action limit $W \geq \hbar$ implies

Test 4: Nature limits length intervals and length measurement errors:

$$l \geq \left(\frac{4G\hbar}{c^3}\right)^{1/2} \approx 3 \cdot 10^{-35}\,\mathrm{m} \quad . \tag{26}$$

In other words, nature limits locality to one corrected Planck length (see, e.g., reference [28]) and *every point-like concept is only an approximation.* Physical quantities are never more localized than one corrected Planck length, one corrected Planck area, or one corrected Planck volume. This result of modern physics is reproduced by strands and agrees with all experiments. The limit for probability density is consistent with these limits of locality.

In short, strands imply that the probability density, like any other density, is limited by the inverse of the smallest volume in nature. Strands further imply that this limit cannot be approached in any experiment.

28 Predictions about relativistic quantum theory and trans-Planckian effects

The derivation of the free Dirac equation from tangles of strands implies the lack of *any* observable deviations from relativistic quantum theory.

Test 5: The strand tangle model predicts that not only the Dirac Lagrangian for free particles, but also the Lagrangian for *interacting particles* is valid in *all* measurable situations and at *all* measurable energy and distance scales.

To date, no deviations have been observed. As argued below, the prediction remains valid when interactions are included, and the quantum properties of the gauge fields are also taken into account.

In apparent contrast, the strand tangle model predicts the absence of any trans-Planckian effect in nature. In particular,

Test 6: Observing an elementary particle with energy larger than $\sqrt{\hbar c^5/4G} \approx 6{\cdot}10^{18}\,\mathrm{GeV}/c^2 \approx 1\,\mathrm{GJ}$ would falsify the strand tangle model. A similarly corrected upper Planck limit also applies to the linear momentum, and the mass of every elementary particle.

Test 7: Observing any system or particle with frequency, acceleration density, pressure, area, volume, or temperature, length or time exceeding the Planck limits would falsify the strand tangle model.

These limits are quantum gravity limits. They are included in the fundamental principle or follow from it. So far, no quantum gravity limit – containing all three constants c, G and $\hbar$ – has ever been exceeded. The limits have not even been approached, by several orders of magnitude. As a consequence, in all observations, the Planck limits are not in contrast with the Dirac equation. Also all other Planck limits – those containing only one or two of the constants c, G and $\hbar$ – have never been exceeded. However, these limits can be approached and can be reached.

In short, strands predict the lack of any trans-Planckian effects, in agreement with all observations.

29 A prediction about other models for wave functions

Strands reproduce quantum theory. Wave functions are blurred tangles, and tangles are the skeletons of wave functions. These statements were deduced and tested in the previous sections. However, step, by step, without noticing, strands made an even stronger statement.

Test 8: The strand tangle model predicts that no non-equivalent alternative microscopic model of wave functions – that also describes gauge interactions and gravity – is possible. If any alternative is found, the strand model is falsified.

Because of the explanation of spin $1/2$, the explanation of fermion behaviour, and the explanation of the free Dirac equation, strands claim to be the unique description of emergent wave functions. This *uniqueness test* has been put into words in the most provocative manner, for two reasons.

First, the provocative prediction aims to motivate the search for alternative and *non-equivalent* explanations of wave functions and their emergence. The uniqueness test contains strong theoretical and mathematical predictions: no other model for spin $1/2$ and for wave functions is predicted to be possible. The lack of such descriptions in the past should not stop the search for alternatives in the future.

The uniqueness of strands was already claimed implicitly in Section 2 above when it was suggested that all nature can be described by unobservable tethers, observable crossing switches, and invariant (corrected) Planck limits. It should be checked in detail whether each step taken in this study is logically unavoidable, without any alternative.

Secondly, the uniqueness test contains precise experimental predictions: if another model of wave functions differs in its results or its predictions from the strand tangle model, observations are predicted to confirm the fundamental principle of the strand model and falsify the other model. This equally provocative prediction aims to motivate precision experiments of every possible type.

Strands explain wave functions and their emergence. To date, no other model has achieved this goal. To date, this uniqueness test has not been falsified. However, the situation can change at any time. And changing the situation should be attempted with intense efforts.

As mentioned in the introduction, the literature on emergent quantum theory is vast. Exploring the other approaches, including those by Adler [151], 't Hooft [152], Elze [153], de la Peña et al. [154], Blasone et al. [155], Grössing [156, 157], Acosta et al. [12], Hollowood [158] and Torromé [159], two conclusions appear to arise: first, so far, other approaches do not appear to contradict the uniqueness test; second, those approaches appear to be compatible with the strand tangle model. However, these aspects merit a more detailed exploration.

In passing, strands also yield a specific view of quantum field theory and axiomatic quantum field theory. Strands produce a specific definition of the concept of *quantum field,* as explored in Appendix I. The issue of an axiomatic description covering all physics is also clarified by strands, as explained in Appendix J. Also in this domain, all possible alternatives should be explored.

In short, the conventional description of relativistic quantum theory is *reproduced* by the strand tangle model. All experiments performed so far are reproduced by strands, despite the predicted lack of trans-Planckian effects. In addition, strands predict to be the unique possible description of emerging quantum theory. However, despite the agreement between conventional quantum theory and the strand tangle model shown so far, several differences arise; they are explored next.

Part IV: Differences to conventional quantum theory

Thus far, the strand tangle model deduced from the fundamental principle has simply *reproduced* conventional quantum theory. If this would be the only result of the strand tangle model, the model would be unnecessary because Occam's razor would speak against it.

The connection between wave functions and blurred crossing distributions yields the real reason for exploring tangles: the strand tangle model and quantum theory *differ*. All the differences are due to the possibility to *classify* particle tangles and their deformations.

It will turn out that *elementary* particles are the *simplest possible* tangles of strands:

▷ Elementary particles are *rational* tangles of one, two or three strands.

The concept of rational tangle was introduced in Figure 1; rational tangles arise by moving tethers around in space, in the same way that braids are formed. Therefore, rational tangles do not contain knotted regions. Rational tangles are three-dimensional generalizations of braids. Modelling elementary particles as rational tangles implies that their spectrum, their interactions, their quantum numbers, their masses and all their other particle properties are not free, but are *fixed* by their tangle structure. It will appear that rational tangles of strands *explain* the spectrum and properties of elementary particles.

In addition, the interactions of particle tangles can be classified. The arguments for the case of gauge interactions will be summarized below. Rational tangles *explain* and *fix* the gauge interactions and their Lie groups.

The explanation of particles and interactions provided by strands is also *hard to vary*, thus realizing Deutsch's famous requirement for a good explanation [160]. Tangles of strands provide an explanatory power that continuous wave functions do not. This promise does not arise in any other proposal in the research literature and is the real reason for exploring the strand tangle model. Checking the promise includes clarifying, as done below, how an *infinite* number of tangles yields a *finite* number of elementary particles in three particle generations and, likewise, a *finite* number of just three gauge interactions.

In short, the strand tangle model promises to restrict quantum theory to a specific set of allowed particles and allowed interactions. The strand tangle model can be tested by comparing the allowed particle properties, including mass, and the allowed interactions, including the gauge groups, with experiments. This is done in the following.

30 Particle mass from tangles

According to textbook quantum theory, when a particle advances, the quantum phase rotates.

▷ The (inertial) mass value m describes the coupling between translation and phase rotation.

A *large* mass value implies, for a given momentum value, a *slow* translation. A *large* mass value implies, for a given energy value, *frequent* crossing switches. Therefore, a complex tangle core has a large mass.

In the strand tangle model, particle translation and rotation are modelled by the translation and rotation of the tangle core. Figure 2 illustrates that tethers explain *why* core translation and rotation are coupled. When the core moves through the vacuum, the vacuum strands and the core touch and move each other because of their impenetrability. This results in a core motion that resembles that of a tethered asymmetrical body – a propeller or spinning top – moving slowly through a fluid.

When an asymmetrical body (without tethers) moves through a (viscous) fluid, it starts to rotate. For example, this occurs when a pebble falls through water, or when a metal object falls through honey. This rotation is due to the asymmetrical shape of the body [161–164]. Almost all of the tangle cores of the elementary particles are asymmetrical. (This also applies to the d-quark once Higgs braids are taken into account. The same occurs for the Higgs boson itself.) Tethers do not prevent the rotation of asymmetrical bodies, as shown in Part I of this text. However, tethers change the rotation frequency. In other words, the strand tangle model predicts that tangle cores will rotate when they move through the vacuum. The strand tangle model yields a coupling between translation and rotation. In other words, the strand tangle model predicts

Test 9: Tethered asymmetric tangle cores, such as localized cores composed of two, three, or more strands, are predicted to be *massive particles*. A tangle is *localized* if the core collapses to a tight tangle when the tethers are imagined to be ropes that are 'pulled outwards'.

Test 10: The mass value for tangles made of a few strands is *not arbitrary*, but is uniquely determined by the tangle structure, and by the resulting average core shape.

Test 11: Particle masses are thus *calculable* – if the tangle topology is known.

Test 12: The *more complex* the tangle core – for the same number of tethers – the slower the translation per rotation, and thus the *larger* the predicted mass value.

Test 13: *Unlocalized* tangles – which disappear if their ends are pulled outwards, such as a bend in a single strand – are predicted to be massless, even if they are helical.

These predictions are consistent with the fermion and boson tangles of Figure 23 and Figure 30. Above all, these predictions agree with observations – in particular for the quark model of mesons and hadrons [97]. But there is more. In the strand tangle model, the highest possible energy is one crossing switch per corrected Planck time. The spontaneous belt trick of a tangle core is less probable by far.

Test 14: The low probability for the belt trick for strand tangles implies that the mass m of *elementary* particles – thus of particles made of one, two or three strands – is much smaller than the Planck mass:

$$m \ll \sqrt{\hbar c^5/4G} = 6.1 \cdot 10^{27}\,\mathrm{eV}/c^2 \ . \tag{27}$$

Thus, the strand tangle model solves the *mass hierarchy problem* in particle physics. The mass inequality also agrees with the *maximon* concept introduced long ago by Markov [165]. Above all, the mass inequality agrees with experiments.

In short, the strand tangle model solves the hierarchy problem and predicts that particle masses can be calculated with the help of the probability of the belt trick for particle tangles. Thus, calculations of mass values require the following three steps: tangles must be classified, the assignment of tangles to the known elementary particles must be clarified, and a precise calculation method for the probability of the belt trick for a given tangle must be developed.

31 From tangle classification to the spectrum of elementary fermions

This and the following sections *summarize* how classifying strand tangles leads to the observed spectrum and to the observed quantum numbers of elementary fermions and bosons [63, 64, 96].

In the strand tangle model, *elementary particles* can consist of only one, two or three strands. More than three strands imply *composite* particles, such as protons, or more complex systems, such as atoms, molecules or solids.

The proof of the composition statement is based on the exploration of tangle cores. A simple confirmation results from the overview of elementary fermion tangles in Figure 23 and of the elementary boson tangles in Figure 30. No *simpler* rational tangles are missing. Any *more complex* rational tangle that has, compared with the tangles in those figures, an *additional* strand falls into one of the following two cases:

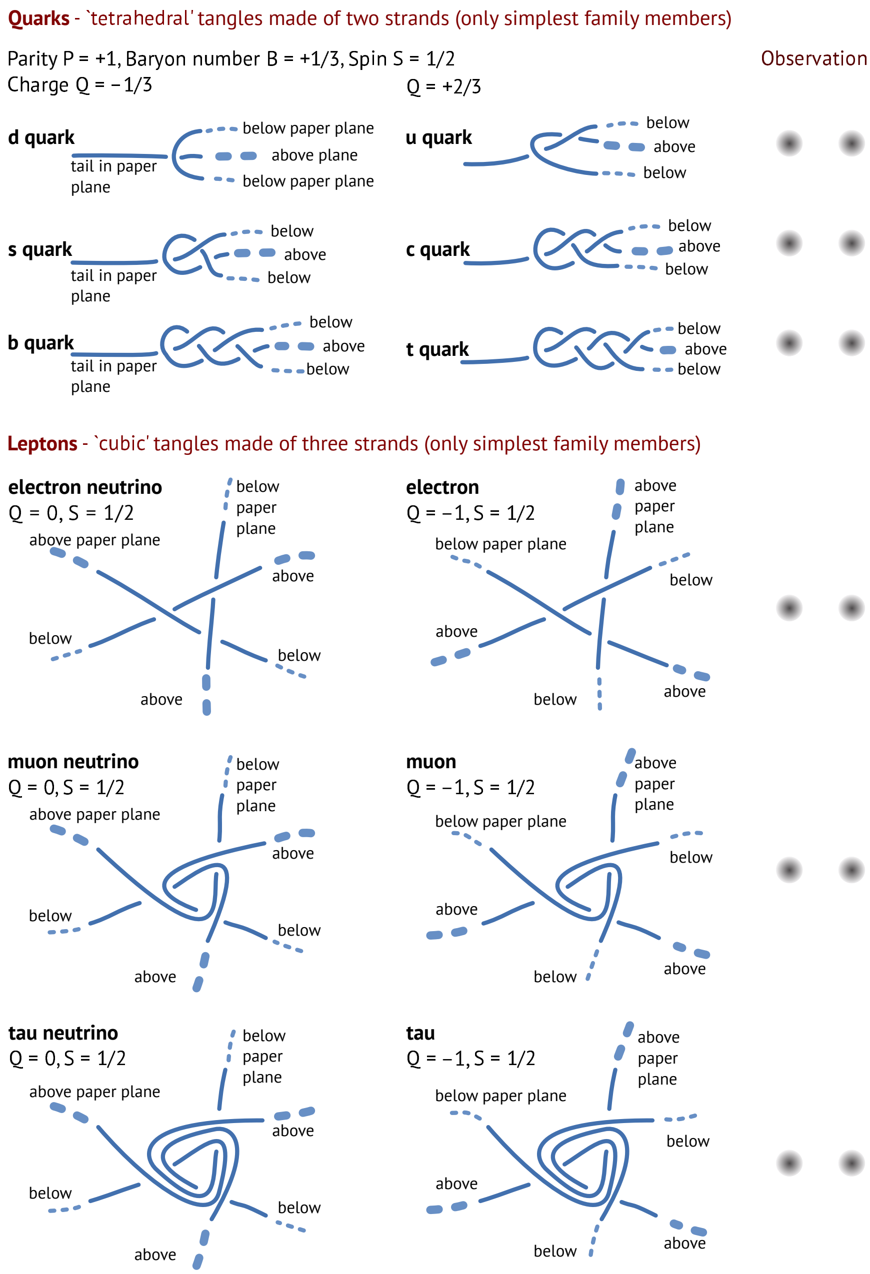

FIG. 23: The figure shows the simplest conjectured tangles for each elementary fermion. Elementary fermions are rational tangles that are formed by braiding tethers. All fermion tangles are made of two or three strands. The tangles yield spin $1/2$ and fermion behaviour. The cores are localized, realize the belt trick, and thus yield positive mass values. The localized cores lead to Higgs coupling, as illustrated in Figure 34. At large distances from the tangle core, the four tethers of the quarks follow the axes of a tetrahedron, and the six tethers of the leptons follow the coordinate axes. The neutrino cores are simpler when observed in three dimensions: they are twisted strand triplets. The tangles of the electron, muon, and tau are topologically chiral, and thus electrically charged. Neutrino cores are geometrically chiral, but not topologically chiral; thus, they are electrically neutral. All massive particles have additional, more complex tangles in addition to those shown here (see Figure 24 and Figure 25) and form three generations. No additional elementary fermions appear.

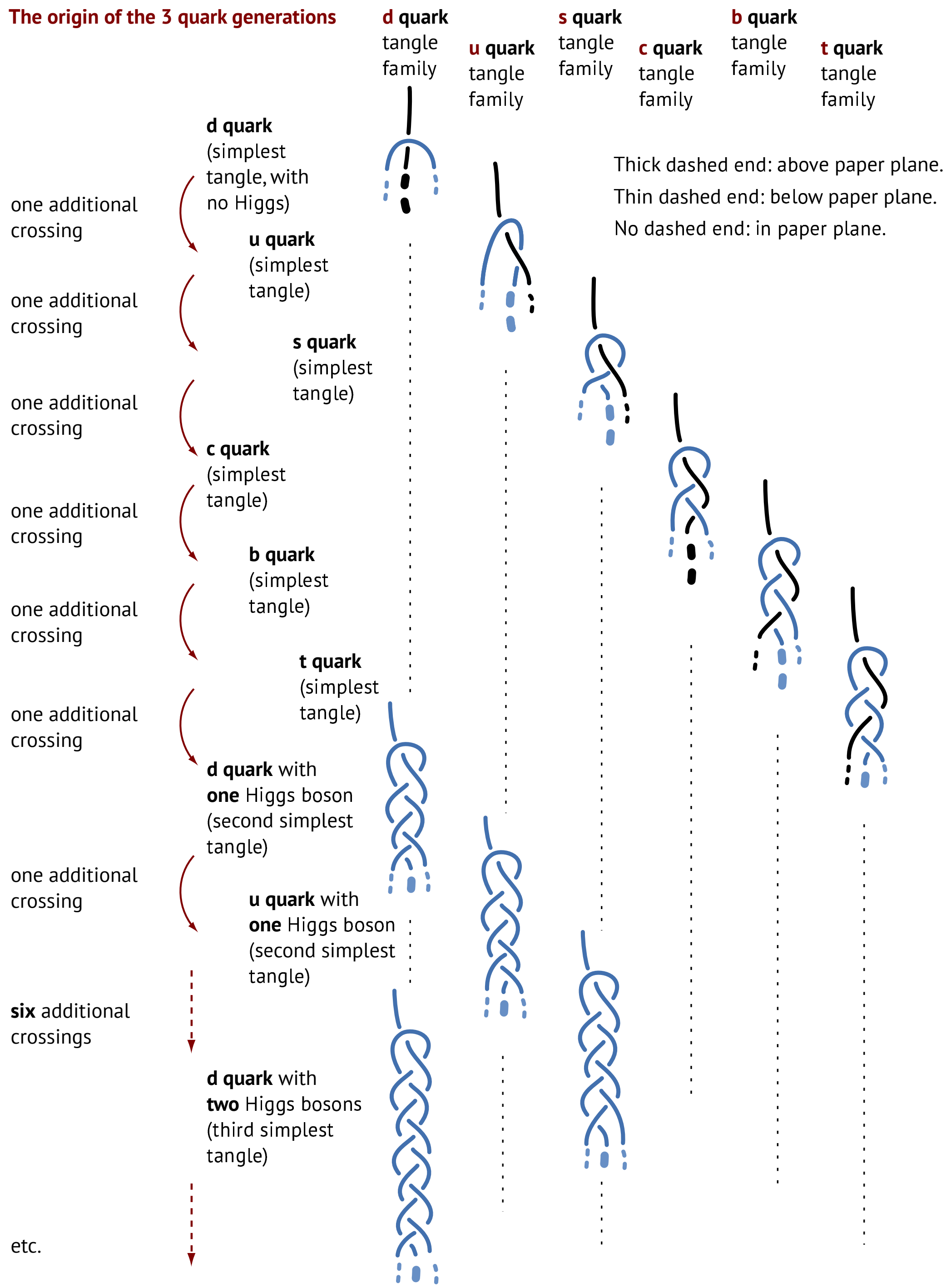

FIG. 24: Quarks lead to six tangle families of two strands, each with an infinite number of tangles. The six families define the six quark types and the three generations. The same classification arises for anti-quarks, which are represented by the respective mirror tangles; they are not shown here. In the strand tangle model, the number of generations is thus a result of the structure of the Higgs braid, which is itself a result of the three dimensions of space. (Figure improved from reference [97].)

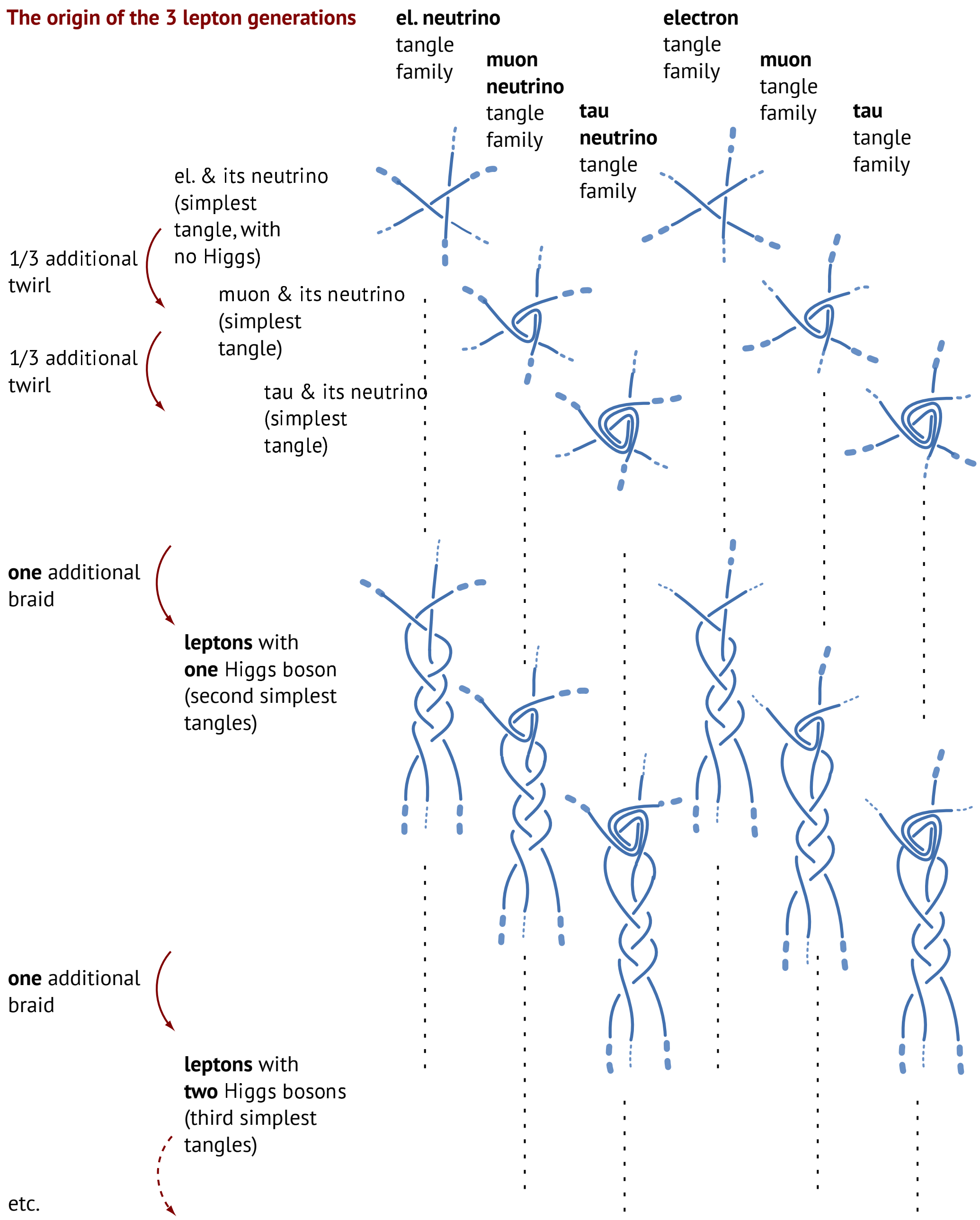

FIG. 25: Leptons lead to six tangle families of three strands, each with an infinite number of tangles. The simplest tangles for each lepton are shown at the top of the figure. The tangles due to one added Higgs boson are shown further down. The six families define the six lepton types and the three generations. Anti-leptons are represented by the corresponding mirror tangles; they are not shown.

- The more complex tangle can be a tangle already found in the two figures. For example, a photon tangle that is extended with an additional strand, can be, depending on the details, a graviton tangle, a quark tangle or an (unbroken) W_i tangle. Similarly, a strand added to a quark tangle can lead to a lepton tangle. These cases thus lead from one elementary particle to another.
- The more complex tangle can be taken apart into simpler tangles. For example, this occurs if a strand is added to a gluon tangle or to a W tangle. The resulting four-stranded core consists of two cores of two strands each. The same occurs if a strand is added to a lepton; such a four-strand tangle can also be separated into two simple cores. Likewise, adding a strand to a graviton leads to the possibility of splitting the resulting tangle into several elementary bosons.

As a result, all elementary tangles have at most three strands.

Specifically, *elementary fermions* consist of either two or three strands. One-stranded rational particle tangles can neither have spin $1/2$ nor have mass, because the belt trick does not apply to them, as argued at the beginning of this article. It turns out that

▷ Two-stranded fermions are *quarks*; three-stranded fermions are *leptons*.

The simplest elementary fermion tangles are shown in Figure 23. All massive elementary particles also have additional tangles, as illustrated in Figure 24 and in Figure 25. In fact, each massive elementary particle is described by an infinite *family* of tangles that contains the simplest possible core, the simplest core plus one Higgs braid, the simplest core plus two Higgs braids, etc. (Also the simplest tangles of the fermions can be imagined as made from the vacuum plus a partial Higgs braid.) Thus, the strand tangle model reproduces Yukawa coupling as the origin of particle mass.

The specific structure of the Higgs braid limits both quarks and leptons to three generations. Figure 24 shows how the infinite class of quark-like tangles is split into six infinite families, corresponding to three generations with two quarks each. In other words, the infinitely many possible quark and antiquark tangles consist of $6+6$ separate infinite tangle families. Each family is an infinite series of tangles. In a family, each tangle differs from the next by the Higgs braid. A similar structure of six tangle families arises for the leptons. The origin of the three lepton generations is illustrated in Figure 25.

An important check for the tangle–particle assignments are the resulting quantum numbers. In nature, quantum numbers and their conservation laws restrict the possible particle reactions.

Spin was already discussed several times above, especially in Part I. Because elementary fermions are composed of only two or three strands, one obtains:

Test 15: The strand tangle model implies that all fermions are massive.

Test 16: The strand tangle model implies that there is no elementary fermion with spin $3/2$ or larger.

The first consequence implies that neutrinos are massive Dirac fermions. This agrees with all experiments so far. Also the second consequence, which contradicts supersymmetry, agrees with all experiments so far. In the strand tangle model, all fermions with high spin values are *composed* and all *elementary* fermion tangles have spin $1/2$.

Flavour quantum numbers count the number of quark tangles. Quark tangles automatically provide the correct values by counting the respective cores. Quark flavour change is achieved by tether braiding. This process only occurs in the weak interaction, because only the weak interaction moves tethers against each other. Only the weak interaction braids and unbraids tethers. The strand tangle model thus reproduces the observation that only the weak interaction can change quark flavours, and thus lead to quark mixing. Similarly, the strand tangle model reproduces the observation that only the weak interaction leads to neutrino mixing.

Electric charge is the number of crossings involved in the chirality of the tangle [64]. This assignment reproduces all observed electric charge values of the elementary particles. Electric charge is automatically conserved in the strand tangle model.

Charge parity C is the topological chirality of a tangle. This assignment reproduces all the observed C parities of the elementary particles.

Parity P describes the behaviour of a tangle under spatial reflections. This assignment reproduces all the observed P parities of the elementary particles.

Time reversal T changes the spin, i.e., the rotation direction of a tangle. This assignment reproduces the observed behaviour under time reversal of all elementary particles. The CPT theorem is automatically satisfied, as observed.

Weak and strong charge are also explained for each fermion and boson, as explored in [63, 97]. The charges explain the interactions to which the corresponding particles are subjected. All observations are reproduced.

The topological basis for quantum numbers is the general reason that they are integers. Quantum numbers are *topological invariants,* and thus are not influenced by strand fluctuations. The strand tangle model also explains why certain quantum numbers are additive, whereas others are multiplicative.

Using the tangle model of the strong interaction, the quark–tangle assignments in Figure 23 fully reproduce the quark model of hadrons, as shown in references [63] and [97]. The allowed and forbidden meson quantum numbers are explained. The meson tangle structures also provide natural and correct retrodictions of which mesons violate CP symmetry. The hadron tangles also reproduce all meson and baryon mass sequences. Also the existence of glueballs is deduced.

As just mentioned, the lepton tangle assignments and the quark tangle assignments also reproduce the weak interaction. The braiding of tethers explains particle mixing [96]. The similarity between the tangle of the electron neutrino and any section of the vacuum tangle has important consequences for dark energy.

Test 17: Strands predict the lack of contradictions between tangle properties and observed particle properties, such as forbidden values of quantum numbers or new quantum numbers – also for composed particles. Any contradiction would falsify the strand tangle model. For example, millicharged particles or weakly charged particles without mass cannot exist in the standard model.

Test 18: Strands predict the lack of any unknown energy scale in high-energy physics. Any deviation from the high-energy desert would falsify the strand tangle model.

Test 19: Strands predict the lack of any substructure in elementary particles that *differs* from tangles of strands. Any observed substructure – such as preons, rishons, ribbons, Möbius bands, prequarks, knots, tori, or any other localised or extended substructure – would falsify the strand tangle model.

Test 20: Strands predict the lack of any new elementary fermion of any kind would. Any such discovery would falsify the tangle model.

In particular, the last prediction implies that *dark matter*, if it exists, is *not* composed of unknown elementary particles.

It should be noted that the strand tangle particle model of fermions is compatible with several particle models deduced from general relativity with and without torsion [166, 167]. In such models, an elementary fermion is assumed to be a Planck-scale *torus*. In the tangle model, Figure 22 shows that the electron can be seen – when the unobservable tethers are neglected, and only crossings are kept – as three spinning crossings. In practice, this yields a fuzzy torus that is always larger than a few Planck lengths. Essentially the same conclusion is reached in the cited papers on torus models.

The tangle particle model of fermions can also be compared to the fermion structure proposed by Bilson-Thompson [168–171]. The only reason to pursue the strand tangle model is the possibility of deducing particle dynamics: wave functions, the Dirac equation, and the gauge interactions do not appear to arise in that model. The same difficulty arises in other proposed topological particle models [172–176].

In short, in the strand tangle model, the classification of elementary fermion tangles leads to the three generations and to the fermion spectrum observed in nature. No additional elementary fermion appears possible. Every observed quantum number is due to a *topological* property of particle tangles. In contrast, the fundamental constants – mass values, mixing angles and coupling constants – are due to *geometric* properties, namely to the (average) *shape* of tangles.

32 From tangles to interactions and gauge groups

Thus far, only *free* particles have been described with tangles. For example, it was shown above that a free, relativistic spin-less tangle distribution ψ or a free relativistic spinor Ψ of an elementary fermion, deduced from a tangle, is described by the Klein-Gordon and Dirac equations, respectively. Therefore, the evolution of the free fields ψ and Ψ are described by their corresponding free Lagrangian. Free particle motion and free Lagrangians arise when tangle cores rotate and move as *rigid* structures. The next step is to explore the interactions and determine their properties [63, 64, 95–97].

Consistently with the geometric effects of Hermitian operators (briefly discussed in Appendix F), gauge interactions arise when the tangle cores do not behave like rigid structures, but instead when the tangle cores change their shape:

▷ Gauge interactions are tangle core deformations.

This statement is interesting because tangle core deformations can be classified. Classification is possible with the help of the moves published by Reidemeister in 1927 [108]. There are only three possible Reidemeister moves: *twists*, *pokes*, and *slides*. These three moves are illustrated on the left side of Figure 26. Exploring the three Reidemeister moves yields the gauge groups U(1), (unbroken) SU(2) and SU(3) [63, 95, 96]. Figure 26 shows how an approaching gauge boson core deforms a fermion core. The gauge boson is absorbed and disappears, and the induced fermion core deformation leads to a phase change in the fermion core. Exploring the details yields complete agreement with experiments. In particular, modelling interactions as core deformations leads to minimal coupling and the usual Lagrangians of QED, of QCD, of the weak interaction, its symmetry breaking, and to the standard model with massive neutrinos.

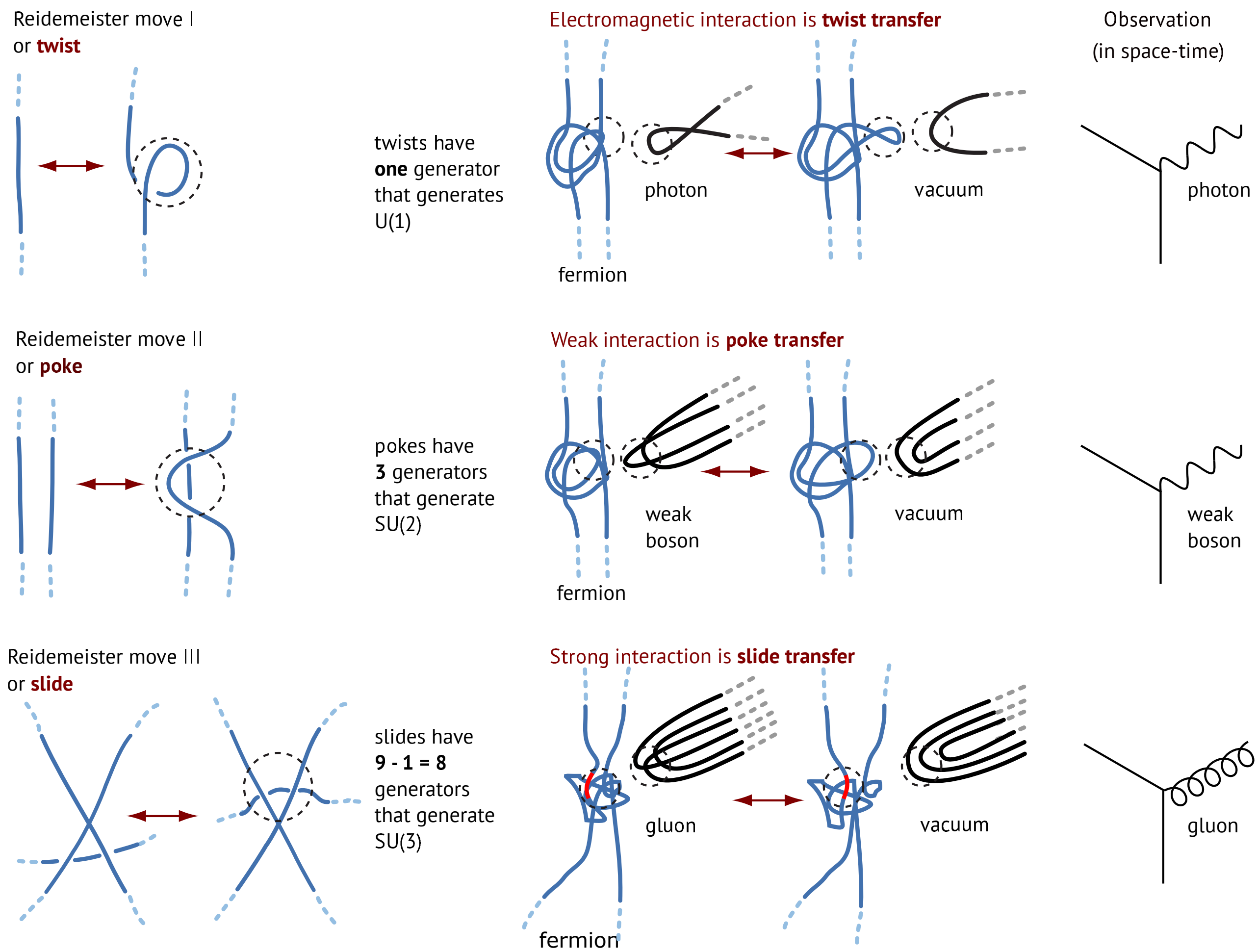

FIG. 26: The possible observable deformations of tangle cores are classified by the three *Reidemeister moves*. The three Reidemeister moves also specify the generators of the three observed gauge interactions and determine the generator algebras. Each generator rotates the bent strand segment enclosed by a dotted circle by the angle π. The full gauge group arises when these rotations are generalized to arbitrary angles. As a result, the three Reidemeister moves determine the three gauge groups [96]. (Figure improved from reference [63].)

Test 21: The tiniest observed deviation from minimal coupling, for any gauge interaction, at *any* measurable energy or scale, would falsify the strand tangle model.

In the strand tangle model, the gauge group U(1) arises because it describes how twists – photons – behave under localized core rotations: rotating a photon core by 2π, i.e., performing a double twist, is equivalent to no rotation. This is visualized in Figure 27. General twists, or first Reidemeister moves, can be seen as rotations by an arbitrary angle of the strand segment enclosed by the dotted circle. Twists can be visualized by imagining that the dotted circle encloses a circular transparent plastic board that contains the strand segment. It is straightforward to check that the set of rotations of the transparent board fulfils all axioms of the Lie group U(1).

Electric fields are volume densities of virtual photons, i.e., of (bound) twists. *Magnetic fields* are flow densities of (bound) twists. In other words, photon exchange or twist exchange im-

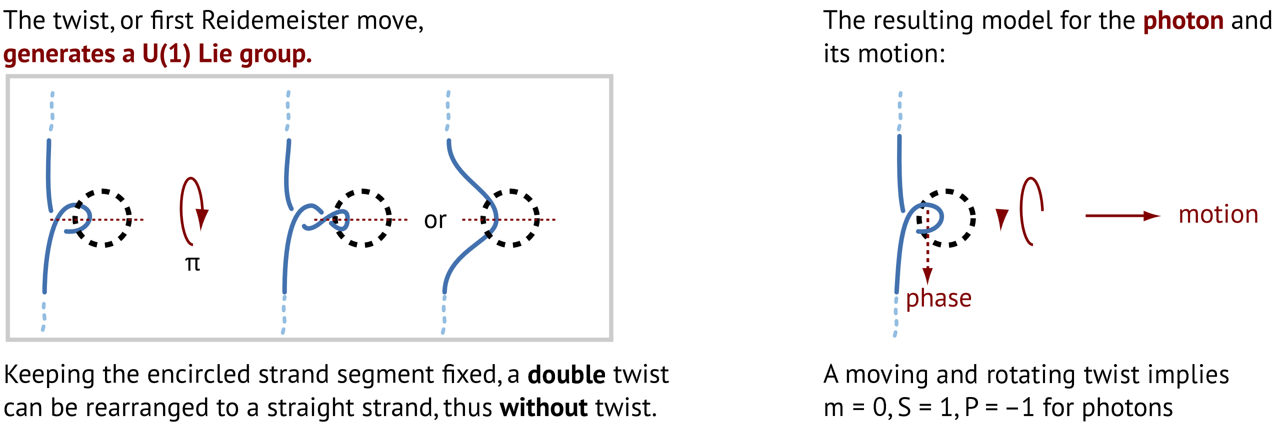

FIG. 27: The *twist,* the first Reidemeister move, generates the Lie group U(1). When twists – rotations of the region inside the dotted circle by π – are generalized to arbitrary angles, they yield the group elements. The twist also implies the strand tangle model for the photon.

plies that the *electromagnetic field* is defined by the space-time rotation rate that it induces on an electric charge. This visualizes the descriptions by Feynman, Hestenes, and Baylis [116, 140–142, 148, 149] and leads to minimal coupling, as shown in [64]. *Electric charge* results from topological tangle chirality. Because of the definition of electric charge, only massive tangles can be electrically charged.

Test 22: Discovering an electrically charged but massless elementary particle would falsify the strand tangle model.

Test 23: Observing the slightest deviation from U(1) or from quantum electrodynamics, at *any* measurable energy, would falsify the strand tangle model.

Thus far, all observations have confirmed the strand tangle model of electrodynamics, which fully reproduces quantum electrodynamics.

In the strand model, the gauge group SU(2) arises from *pokes*, that is, from the second Reidemeister move. Pokes can be seen as localized rotations by the angle π around any of the three coordinate axes. The three possible pokes are illustrated in Figure 28. Again one can visualize pokes by imagining that the dotted circle encloses a circular transparent plastic board that contains, in this case, two parallel strand segments. Exploring pokes, it turns out that they form an SU(2) algebra, which is generated by the three possible rotations by π of the dotted circle region [95, 96]. Figure 28 can be used to prove that the three pokes behave under concatenation in the same way as i times the Pauli spin matrices under multiplication. General pokes, or generalized second Reidemeister moves, are defined as rotations by an arbitrary angle of the two strand segments enclosed by the plastic board. It is straightforward to verify that general pokes realize the group axioms and the Lie group axioms. In total, generalized pokes thus form the full Lie group SU(2).

Strands imply that only massive fermions can exchange weak bosons: only massive particles interact weakly. Thus, strands imply and predict massive neutrinos. Due to the tangle structure of particles, *maximal parity violation* arises: parity violation occurs because core rotations due to spin $1/2$ resemble the core deformations due to the group SU(2) of the weak interactions [63, 96].

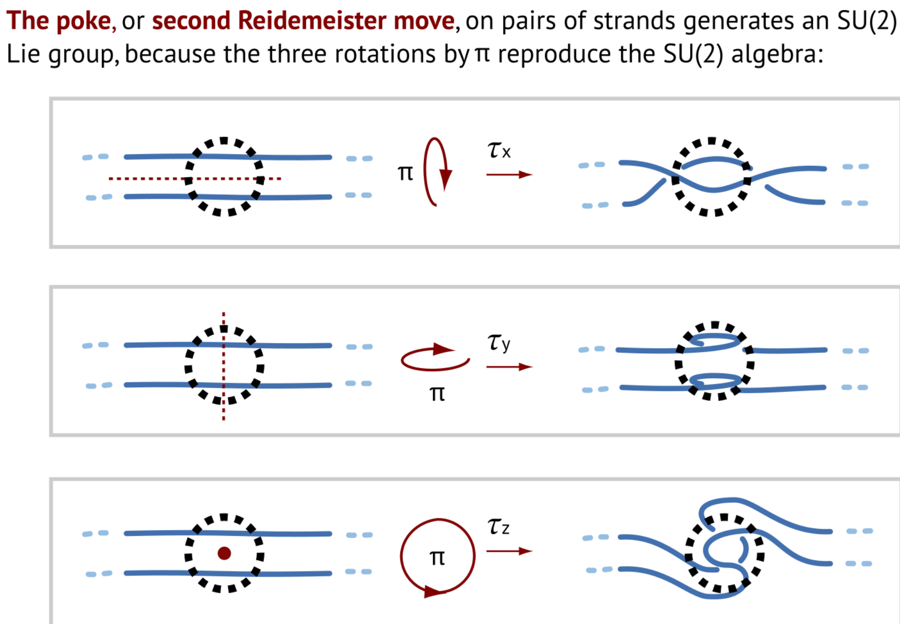

FIG. 28: The three types of *pokes*, second Reidemeister moves, are visualized. Pokes can be modelled by rotating a region – inside the dotted circle – containing *both* strands by the angle π. (Other visualizations are also possible, e.g., by deforming only one strand.) The three pokes generate the Lie group SU(2) and imply a model for (unbroken) weak bosons.

In addition, SU(2) *breaking* arises: when a vacuum strand is included in the massless weak bosons, it leads to the W and Z boson tangles [63, 96]. Also particle *mixing* is a consequence of the strand tangle model of the weak interaction: it arises from tether braiding, which itself is a specific type of poke deformation.

Test 24: Discovering the smallest deviation from the known weak interaction properties of the standard model with massive Dirac neutrinos and PMNS mixing, at *any* measurable energy, would falsify the strand tangle model.

So far, all observations confirm the strand tangle model of the weak interaction, which completely reproduces the conventional description.

In the strand model, the gauge group SU(3) arises from *slides*. Slides, or third Reidemeister moves, reproduce the algebra of the eight generators of SU(3). This is the main result of previous publications [63, 95, 96]. The full gauge group SU(3) arises once slides are seen to be local rotations by π that generate the SU(3) Lie algebra. Slides are illustrated in Figure 29. Slides are best visualized by imagining that the dotted circle encloses a circular transparent plastic board that contains, in this case, two crossing strand segments. (Other visualizations are also possible.) Exploring the algebra of the rotations by π numbered from 1 to 8 in the figure, one finds that they behave like i times the Gell-Mann matrices. (Slides 3, 9, and 10 are linearly dependent; it is conventional to use only slides 3 and 8 instead.) In other words, the first eight slides generate the Lie algebra of SU(3). General slides, or generalized third Reidemeister moves, can be seen as rotations by an arbitrary angle of the two crossing strand segments enclosed by the plastic board. These generalized slides realize all the required axioms and yield the full SU(3) group.

Slides, or **third Reidemeister moves**, can be defined on strand pairs in three-strand structures. They can be generalized to yield the generators of the Lie group SU(3).

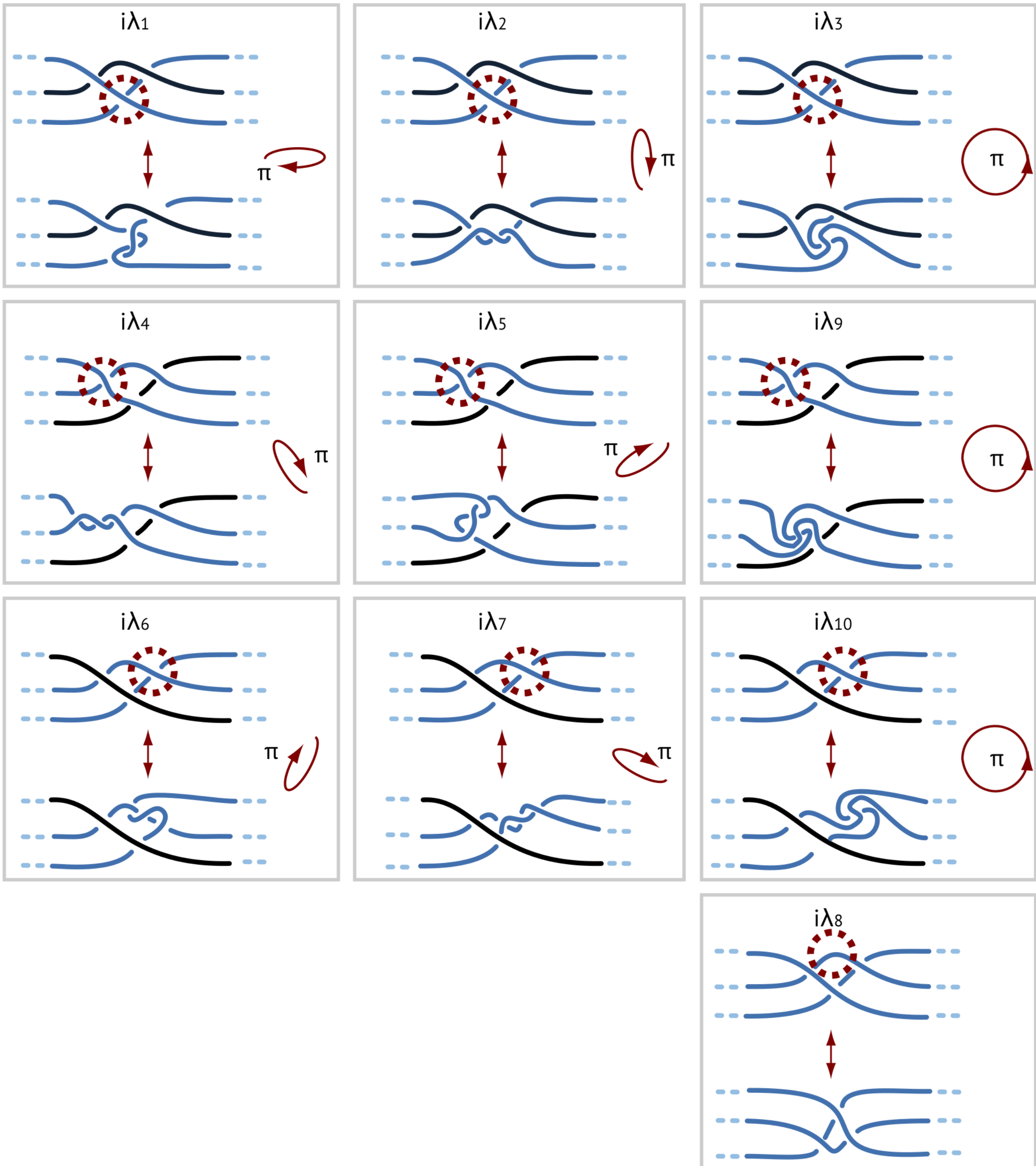

FIG. 29: The ten important deformations deduced from the *slide*, the third Reidemeister move, are visualized. Each triplet row defines a SU(2) subgroup. Slides can be modelled by rotating the region inside the dotted circle by the angle π. (Other visualizations are also possible, e.g., by deforming only one strand.) With the definition of λ_8, the eight slides $i\lambda_1, \ldots, i\lambda_8$ (thus without $i\lambda_9$ and $i\lambda_{10}$) generate the Lie group SU(3) and imply a model for gluons. Each of these eight deformations corresponds to i times a Gell-Mann matrix.

As shown in reference [97], the strand tangle model for the strong interaction implies the quark model for hadrons, in all its details. Strands also yield glueballs, with their correct quantum numbers. Also all composed particles formed by the strong interaction have the correct quantum numbers. The strand tangle model also implies that the strong interaction cannot produce CP violation. Furthermore, the colour charge of a quark tangle is given by the orientation of the three-ended side in space. Colour fields are densities of virtual gluons. Due to their structure, gluons carry a double colour charge.

Test 25: Observing the smallest deviation from quantum chromodynamics or from the usual strong interaction properties, at *any* measurable energy, would falsify the strand tangle model.

Thus far, all observations have confirmed the strand tangle model of the strong interaction [97], which fully reproduces quantum chromodynamics.

Because only three Reidemeister moves exist, the strand tangle model implies:

Test 26: Observing any additional gauge group, such as SU(10), E8 or any gauge supergroup, at any energy, would falsify the strand model.

This prediction is wide-ranging, because it eliminates many unification attempts.

One notes that the visualization of the three gauge interactions with strands agrees with and extends the ideas of Boudet [177]. He describes gauge theory using geometric concepts, in parallel to the approach by Hestenes [140–142]. Strands can be seen as simplifying these descriptions.

In short, when gauge interactions are modelled as *deformations* of elementary particle tangle cores, classifying these deformations yields only three possible types: each of the three Reidemeister moves leads to one gauge interaction. The three possible Reidemeister moves – called twists, pokes and slides – determine the gauge groups U(1), SU(2) and SU(3), as implied by Figure 26 and as explored in references [63, 64, 95–97]. The resulting charges, conservation properties and parities agree with observations. The calculations of the coupling constants that were started in references [64, 97] need to be improved.

33 From tangle classification to the spectrum of elementary bosons

In the strand tangle model, *elementary bosons* consist of one, two, or three strands. More strands imply groups or composites of *several* bosons. The Reidemeister moves, illustrated in Figure 26, directly suggest that one-stranded bosons correspond to photons, three-stranded bosons to gluons, and two-stranded bosons to the weak unbroken W_1, W_2 or W_3 bosons. This holds before symmetry breaking; symmetry breaking is the process in which two-stranded boson tangles incorporate a vacuum strand, yielding the three-stranded, massive W and Z bosons. A complete overview of all the possible boson tangles is given in Figure 30.

All (unbroken) elementary gauge boson tangles are trivial, that is, topologically untangled, as Figure 30 shows. Their trivial tangles can be described as one bent strand among straight strands. The bends in the tangles can *transfer* or *hop* from one strand to the next, yielding boson behaviour. In particular, the one bent strand implies spin 1.

The W and Z bosons, also shown in Figure 30, form a special case. They are composed of three strands that (asymptotically) lie in a plane. This symmetry plane – which distinguishes their

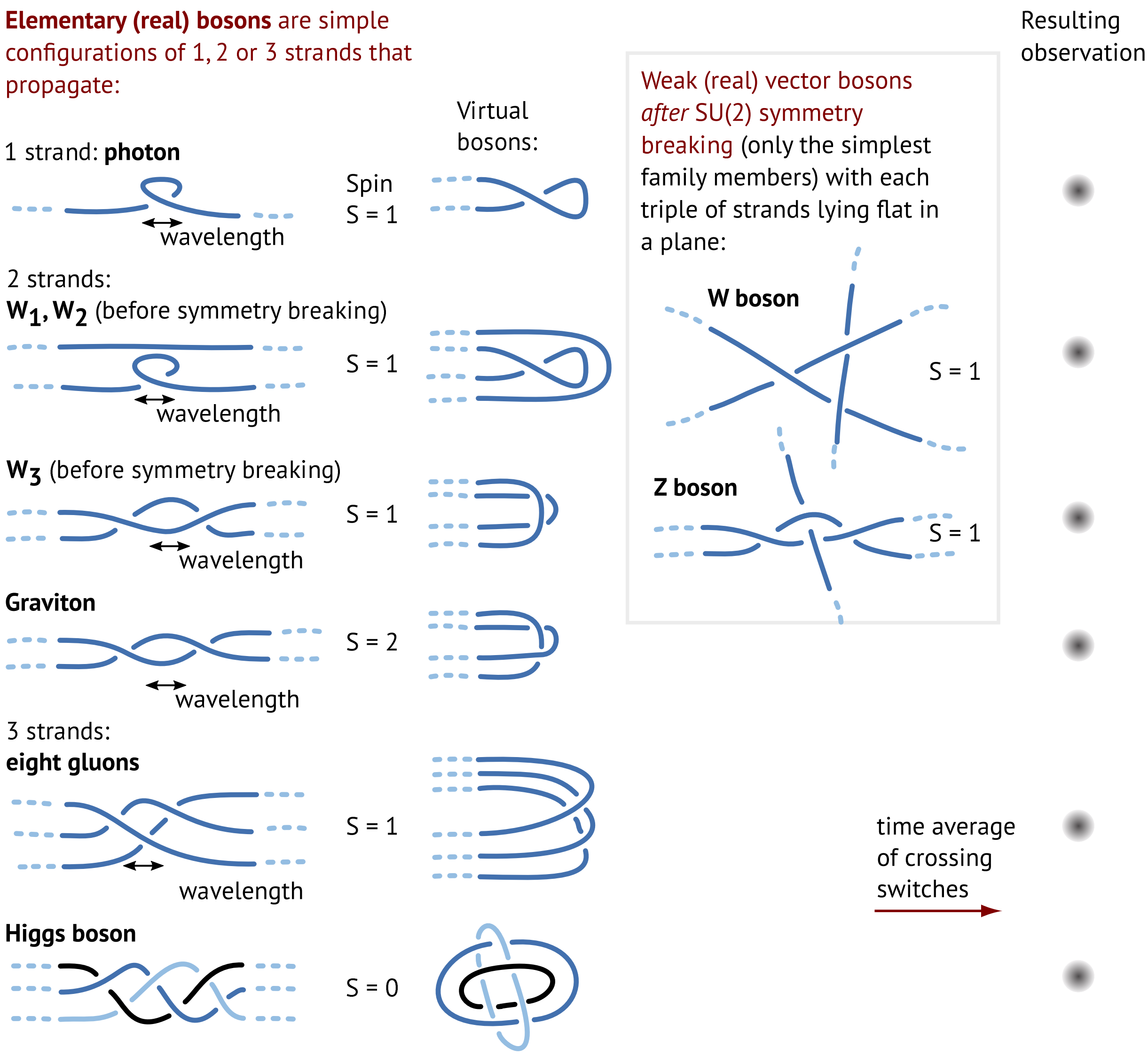

FIG. 30: The proposed rational tangles for each elementary boson. The tangles are composed of one, two, or three strands. For each gauge boson, the tangle structure determines the spin value of 1 because any curved strand can rotate by 2π and return to its original shape. The graviton has spin 2 because each of its curved strands can rotate by π and return to the original shape. All boson tangle cores rotate during propagation. Photons and gluons are massless and both are described by a single tangle. The W and Z tangles are asymptotically planar. The W, Z and Higgs are localizable and thus have mass; therefore they have also more complex tangles in addition to the simplest ones shown. No additional elementary bosons are predicted to exist. The W tangle is the only topologically chiral core; thus, it is the only electrically charged elementary boson.

tangle from electrons and from electron neutrinos – leads to spin 1 (and to non-vanishing mass). Their specific tangle structure leads to boson behaviour.

Coupling constants also follow from the strand tangle model. Interactions of fermions – when a gauge boson is absorbed or emitted – are due to deformations of tangle cores. The probability of such deformations determines the coupling constant uniquely [63, 64, 96, 97]. The challenge of calculating the coupling constants with high precision and comparing them with the data remains open.

In the strand tangle model, the Higgs boson is a braid composed of three strands. The spin of the Higgs boson is less evident. Exploring the Higgs braid shows that it is deduced from the Borromean rings. Random fluctuations in its tangle do not lead to its rotation. For this reason, the Higgs tangle has spin 0. In summary, among rational tangles, only a standard braid has spin 0, and only such a braid reproduces the observed parity values. Only the standard braid has a vanishing electric charge and a vanishing strong charge. Finally, only a standard braid reproduces the mass-producing property, as shown now.

For every massive particle, Higgs braids can be added to the simplest tangle core. This does not change any quantum number because the Higgs has the same quantum numbers as the vacuum. Therefore, every massive particle – fermion or boson – is described by an *infinite set* of tangles. The set contains the simplest possible core, the core plus one Higgs braid, the core plus two braids, etc. These options are illustrated in Figure 24 and Figure 25. The precise particle mass value is influenced by this single or multiple Higgs boson addition.

The processes illustrated in Figure 24 and Figure 25 also imply that the Higgs boson couples to itself. The self-coupling, illustrated in Appendix K, implies that the Higgs boson itself is massive as well. This is observed.

In contrast, no addition of a Higgs braid to cores of massless elementary particles is possible. Therefore, massless elementary particles are described by a *single* tangle. The mentioned strand properties of the Higgs braid also imply

Test 27: Strands imply the lack of any *additional Higgs boson*. Observing one would falsify the strand tangle model.

The graviton, the last elementary boson, has an untangled core that returns to itself after a rotation by π; thus it has a vanishing mass and spin 2. With the help of the graviton, strands describe general relativity and its Lagrangian, including the cosmological constant [80, 96, 98]. Black holes, quantum gravity and cosmology are reproduced as well. This topic is not explored further in the present text.

As a further consequence,

Test 28: The strand tangle model predicts that there is no elementary boson with spin 3 or larger.

Thus far, this deduction agrees with observations: all known particles with such high spin values are composed. In addition, the trivial tangles explain why all elementary particles with integer spin are bosons, and vice versa. The case of composed particles was explored above, in Section 7.

No additional elementary gauge boson appears possible: neither is a higher number of strands possible in an elementary particle nor is a more complex tangling of strands with boson properties possible. The boson tangles for photons, gluons and gravitons imply that these particles are massless; their tangle cores can rotate unhindered by tethers, in contrast to the W, Z and Higgs bosons, which cannot; they have to perform a kind of belt trick to rotate and therefore have mass.

Test 29: Strands predict the lack of any *additional elementary gauge boson*. Discovering one would falsify the tangle model.

Once tangles are assigned to the gauge bosons, the corresponding charges can be defined from the topological properties of their tangles. *Electric charge* is related to the topological chirality of a tangle [64]. Consequently, the electric charge is conserved in interactions. *Colour charge* is related to the threefold spatial symmetry of quark and gluon tethers [97]. Consequently, the colour charge is conserved in interactions. The *weak charge* is due to a combination of the topology and geometry of the tangles.

Strands also naturally imply a lack of localized composites made of photons. The possibility of extremely short-lived ZZ-composites or W^+-W^- composites is minimal. In contrast, composites of gluons are allowed by the strand tangle model; so are many other boson composites, from meson molecules to organic molecules. But several boson composites are forbidden:

Test 30: If 'photonballs' are discovered, the strand tangle model is falsified.

Test 31: If glueballs with incorrect quantum numbers are discovered [97], the strand tangle model is falsified.

In contrast, strands reproduce the existence of entangled bosons, such as biphotons. These are regularly observed [178].

The classification of elementary bosons implies

Test 32: If any *further elementary boson*, of any spin, is discovered, the strand tangle model is falsified.

Thus, the strand tangle model implies and predicts the absence of supersymmetric particles. In combination with the previous section on fermions, the prediction becomes even more general:

Test 33: If any new *elementary* particle is found – including anyons, axions, supersymmetric partners, or any elementary dark matter particle – the strand tangle model is falsified.

This prediction thus follows from the strand tangle model of particles and its classification of the possible rational tangles – assuming no mistakes or oversights.

In short, classifying elementary boson tangles reproduces the observed gauge bosons with the help of trivial tangles of one, two or three strands, reproduces the observed mass-producing Higgs boson with a braid, implies the doubly twisted two-stranded spin-2 graviton, and leaves no room for other elementary bosons, or other elementary particles of any kind.

34 Predictions about elementary particle mass values

In the strand tangle model, the mass value of every elementary particle is *emergent*. The mass value of a fermion is due to the frequency of the belt trick that appears because of spontaneous strand fluctuations when a tangle moves [96]. The mass is also due to the Higgs mechanism. These connections imply several predictions that can be tested before calculating any mass value.

Test 34: In the strand tangle model, only localized particle tangles have mass. A tangle is *localized* if the core collapses to a tight tangle when the tethers are imagined to be ropes being 'pulled outwards'. If the tangle disappears, it is *unlocalized*. Only localized tangles have inertial mass. In particular, in the strand tangle model, only fermions, W, Z and the Higgs boson have positive mass. Strands predict that only *localized* tangles interact with the Higgs field, and thus have Yukawa couplings.

Test 35: Because particle masses are due to the belt trick frequency, massive particles are surrounded by a cloud of virtual gravitons; these are the two-stranded twists arising in their tethers, illustrated in Figure 2. Virtual gravitons have spin 2 and mass zero, and therefore induce a $1/r^2$ dependency of gravity in flat space. Only localized tangles have gravitational mass.

Test 36: The mass values of all elementary particles are *positive*, *fixed*, *unique* and *constant* in time and space, across the universe.

Test 37: Mass values for particles and antiparticles, i.e., for tangles and mirror tangles, are predicted to be *equal*.

All of these predictions agree with all observations [20].

The strand tangle model allows comparison of the *inertial* and *gravitational* mass values. On the one hand, the belt trick generates a displacement and thus relates rotation and displacement. This relation, illustrated in Figure 2, defines the inertial mass. On the other hand, the tether twists generated by the belt trick correspond to virtual gravitons. Thus, the belt trick also defines gravitational mass. Because both mass values are due to the same process, the strand tangle model implies

Test 38: Strands predict that the inertial and gravitational mass of all particles are intrinsically *equal*, at all times and places.

Thus far, the prediction agrees with all the observations. This prediction is of interest because some versions of modified Newtonian dynamics (MOND) suggest a difference, although only for large masses and curved space [179, 180].

Strands imply that particle mass values depend on the tangle structure of the particles. For all hadrons, the prediction that more complex tangles have larger mass values (for the same number of tethers) is valid in all cases, as shown in [63, 96]. For elementary particles, the most important prediction is about leptons.

Test 39: The neutrino tangle assignments explain their extremely small mass values, especially for the electron neutrino. For this particle, the simplest tangle is similar to a vacuum configuration. Strands predict that the electron neutrino mass arises only through the terms due to the Higgs mechanism, and thus is the smallest of all elementary particles.

Test 40: Strands imply

$$m_e < m_\mu < m_\tau \quad \text{and} \quad m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau} \quad . \tag{28}$$

Test 41: Strands predict that neutrino masses and mixings can be calculated before they are measured with precision.

The prediction of the normal ordering of neutrino masses should be testable in experiments in the coming years.

In the strand tangle model, all particle tangle cores get *flatter* at higher energy – or higher four-momentum. This change will influence the frequency of the belt trick. As a result,

Test 42: Strands predict the *running* of elementary particle masses with energy.

The prediction qualitatively agrees with the observations. A quantitative test is a research topic.

Thus far, only rough calculations of particle masses from belt trick probabilities have been achieved [64, 97]. However, no precise method has been developed yet. It is expected that exploring the behaviour of asymmetric bodies and their rotation in a viscous (Stokes) flow might lead to better approximations [181]. However, this field of research is in its infancyl young.

In short, calculating particle masses – and their running with energy – with the help of computer simulations that determine the relationship between the structure of a tangle core, the belt trick and the motion of a tangle will allow independent *testing* the strand tangle model. Already now, additional tests of the strand tangle model are possible.

35 Least action and the Lagrangian of the standard model

Also this section provides a pedagogical presentation of previously published papers [63, 64, 96, 97]. According to the fundamental principle of the strand tangle model, the action of a physical process is the number of crossing switches that occur. Each crossing switch yields an action increment of $\hbar$. Therefore,

▷ Least action is the smallest number of crossing switches.

In other terms, the strand tangle model implies

▷ Every motion and every process in nature minimizes the number of crossing switches.

This is consistent with the fundamental principle because a crossing switch is less probable than other strand deformations. A crossing switch is a form of change; a crossing switch requires action, and thus it requires a 'cost', whereas a simple strand shape deformation does not.

Strands thus suggest an underlying reason – i.e., an explanation – for the principle of least action. This may be the only explanation in the literature.

Test 43: Strands predict the lack of the slightest deviation from the least action principle and from the description of motion with Lagrangians. Discovering such a deviation would falsify the strand tangle model.

It was argued above that the Dirac equation for *free* particles is the differential version of Dirac's trick. Alternatively, the free particle Dirac equation is due to the minimization of the crossing switch number. Therefore, strands imply the Lagrangian of the free Dirac equation. Based on the results of the above sections, this argument can be extended to include interactions and particle reactions. But it is more interesting to look at the standard model.

The strand tangle model reproduces the appearance and disappearance of particles with the tangling and untangling of strands. The involved strands are either constituents of reacting particles or of the vacuum. Gauge interactions are due to tangle core deformations that occur by transfer of twists, pokes or slides, as illustrated in Figure 26. The figure also shows that as a result, particle reactions and decays are described by Feynman diagrams. A complete list of Feynman diagrams that can arise from strand tangles is presented in Appendix K. They correspond exactly to those of the standard model [182, 183]. In other words, classifying untangling and tangling options leads to the existing Feynman diagrams, *without additions or omissions*, as described in detail in [63, 64, 96, 97].

▷ The standard model with massive Dirac neutrinos and PMNS mixing results from rational tangles.

The connection between tangles and quantum field theory leaves no freedom of choice; it is unique.

Test 44: The strand tangle model predicts that the standard model with massive Dirac neutrinos is valid at all measurable energies, without any modification.

Test 45: Strands predict the *lack* of any measurable effects beyond the standard model.

Test 46: Strands predict that there is only one possible quantum field theory: the standard model with massive Dirac neutrinos that mix. (See also Appendix M.)

Strands appear not to allow any modification of the standard model whatsoever. So far, this conclusion agrees with all observations.

The strand tangle model implies that quantum fields have a *Lagrangian* and follow the principle of least action. The strand tangle model implies the same perturbation expansions as conventional quantum field theory. The perturbation expansions are due to more complex tangles arising at smaller scales. (The strand tangle model also implies general relativity and its Lagrangian, as shown elsewhere [80, 96].)

The strand tangle model also implies that

Test 47: Physical space has *three dimensions* – at all measurable scales.

Test 48: There is *only one vacuum state* in nature – in flat space. There are no domains or domain walls between 'different' vacuum states.

As previously mentioned, the strand tangle model implies that there is a natural *cut-off* at the Planck scale and a well-defined *continuum limit* at the same time. The strand tangle model also implies that all observables are *approximately local* – within Planck dimensions.

Test 49: If the standard model with massive Dirac neutrinos turns out to be an incomplete description of physics, the strand tangle model is *falsified*.

Test 50: The strand tangle model predicts that additional dimensions, additional elementary particles, additional interactions, additional symmetries or additional energy scales are *not possible* in nature.

Test 51: The strand tangle model predicts that the deduced values for the fundamental constants – elementary particle masses, mixing angles and coupling constants – *agree with experiment*.

Test 52: The strand tangle model predicts that values for the fundamental constants that differ from the observed ones are *impossible*.

Test 53: In other words, the strand tangle model predicts the *lack of generalizations* of any kind.

The lack of generalizations is unusual but is required and expected from any unified description of nature: any unified description must be *unique*. It appears that the strand tangle model fulfils this requirement.

Test 54: Strands predict the lack of any alternative, non-equivalent explanation for elementary particles, gauge interactions, and fundamental constants. Finding one invalidates the standard model.

Thus far, all the conclusions have been confirmed.

Appendix L mentions differences and overlaps between strands and strings. Appendix M explains the consequences of strands for the Yang-Mills millennium problem. Appendix N looks at the Nielsen-Ninimiya theorem from the perspective of strands.

In short, it was shown that *rational tangles* yield the standard model with massive Dirac neutrinos, without modification, because tangles limit the possible fermions, the possible bosons, the possible gauge interactions, the possible interaction vertices, and the possible particle mixings. All terms in the standard model Lagrangian follow from tangles and their deformations. In addition, strands imply the possibility of calculating fundamental constants of nature. In addition to the particle masses discussed above, the coupling constants and mixing angles can also be calculated. Some of these constants have been investigated and estimated in [96], [63], [64] and [97]. Strands imply that no alternatives to the observed universe and to the standard model are possible. Thus far, this agrees with all experimental data. In addition, the strand tangle model provides a new view on several topics and yields several additional tests.

36 Electromagnetism, measurements and minimum time

The fundamental principle illustrated in Figure 6 defines all observations, all measurements and all physical observables as consequences of crossing switches. The details of the electromagnetic interaction – briefly summarized in Figure 17 as being due to twists – imply that *crossing switches are observable because they couple to electromagnetic fields*.

In nature, every measurement process and every measurement device makes use of electromagnetism. For example, all human senses, even hearing or touch, are electromagnetic. Scales and all devices that measure mass, weight, or force rely on electromagnetism. All seven base units of the International System of Units (SI) are defined and applied using electromagnetic means of observation. Also every comparison with a standard or unit uses electromagnetism.

Strands confirm at the most fundamental level that without electromagnetism, there are *no* observations and *no* measurements.

Test 55: Performing any non-electromagnetic observation or measurement is impossible. Performing one would falsify the strand tangle model.

Strands thus explain why crossing switches are observable: crossing switches couple to electromagnetism. Instead, simple strand deformations do not couple to electromagnetism and are not observable. Therefore, strands *explain* the fundamental principle.

The relation between crossing switches and electromagnetism also explains how the minimum time $\sqrt{4\hbar G/c^5}$ arises in the fundamental principle. A crossing switch could, in principle, take an arbitrarily short time. However, such an ultra-rapid crossing switch cannot couple to the electromagnetic field; a photon wavelength shorter than a (corrected) Planck length is impossible. Such an ultra-rapid crossing switch would not have any physical effects, and would not be observable.

Test 56: No effects in nature whatsoever are due to time or length intervals shorter than the corrected Planck limits. Observing any such effect would falsify the strand tangle model.

In short, strands imply that all measurements are electromagnetic. Strands also imply that only crossing switches that take longer than the corrected Planck time are relevant for observations. Trans-Planckian effects are predicted not to be measurable and thus not to occur in nature. These conclusions agree with all observations.

37 Further experimental tests

Can the strand tangle model be tested in experiments? The obvious tests are the comparison of calculations for neutrino masses, other elementary particle masses, mixings, CP phases and coupling constants with experimental observations. These are the fundamental constants of the standard model. Other possibilities are rare.

The lack of trans-Planckian effects predicted by strands includes a lower limit on the effective elementary particle diameter:

Test 57: No elementary particle has an intrinsic diameter smaller than twice the Planck length.

The predicted size of elementary particles also implies a limit on the locality of interactions. However, such tiny deviations from locality are not expected to produce any measurable effect.

In a remote future, a Planck-scale deviation from locality *might* be observable in measurements of the electric dipole moment. Possibly, the electric dipole moment of elementary particles could be shown to correspond to the tangle assignments. The measurements of the electric dipole moment of the electron are closest to achieving this sensitivity [184]. However, the strand tangle model predicts no dipole moment for the electron, to first order.

For other elementary particles, the dipole moment values should be a few times the smallest length times the unit charge. But high precision implies long measurement times. Due to their short lifetime, measuring the electric dipole moment of the W and Z bosons or the Higgs seems not possible. Measuring the dipole moment of a single quark also seems out of reach. It has to be seen whether the measurements of the dipole moment of the neutron will ever reach the sensitivity of the measurements of the electron.

Deviations from point-like behaviour can also be searched for in other settings. It is impossible that electron-electron scattering will reach the required distances, both for the electromagnetic or the weak interaction. Also decay processes do not provide the required sensitivity. In the strong interaction, the tangle structure may lead to experimental effects, maybe even non-perturbative effect. However, the probability remains low [97].

In short, apart from the calculation of the fundamental constants of the standard model, the feasibility of any direct experimental test for particle tangles seems low. In any case, strands predict that observations will never detect new physics or trans-Planckian physics.

38 Open issues: tangle questions and gravitational decoherence

The above presentation of the strand tangle model may contain inconsistencies or errors. First, it could be that the tangle assignments for the quarks, leptons, or Higgs boson need corrections.

For example, a smooth transition between particles and antiparticles is difficult to visualize for massive quarks.

▷ Even if some tangle corrections will be necessary, the strand tangle model is predicted to remain valid.

A second open point concerns the completeness of the classification of the rational tangles given above. For example, when a braid is formed from three sets of two strands each, to which set of elementary particles does the resulting configuration correspond? This and similar questions about more complex tangles are yet to be answered.

The third open topic is the relationship between entanglement, decoherence, and gravity. Many scholars have explored this topic, and are still doing so [185, 186]. In the strand tangle model, these three processes exhibit both differences and similarities. On the one hand, gravity is due to the exchange of virtual spin 2 particles made of two strands, whereas entanglement and decoherence are not due to particle exchange. On the other hand, in the strand tangle model, all physical processes are built from crossing switches, which implies that all processes share fundamental similarity.

In short, it is possible that investigations will derive the necessity for certain corrections to the strand tangle model. In addition, the discovery of new effects in the relativistic quantum gravity domain cannot be completely excluded. However, such effects are predicted not to imply any new physics or any trans-Planckian physics.

39 Conclusions and outlook

Ciò che per l'universo si squaderna:
...
La forma universal di questo nodo
...
Dante, *Paradiso*, Canto XXXIII, 87, 91.

Using a single fundamental principle, modelling fermions as fluctuating rational tangles of strands with Planck radius allows reproducing all properties of wave functions and particles. The wave function is the density of crossings due to fluctuating strands. The fundamental principle states that crossing switches describes all observables.

Evolving strand tangles describe the motion of all quantum particles. Propagating matter particles are modelled as advancing and rotating tangle cores. The probabilistic behaviour of quantum theory is reproduced in all its details. The relativistic motion of fermion tangles is described by the Dirac equation.

In addition and in contrast to usual quantum theory, classifying tangles explains the observed elementary particle spectrum, including the quantum numbers of all particles. Classifying tangle deformations with the Reidemeister moves explains the observed gauge interaction spectrum, including the gauge Lie groups U(1), SU(2) and SU(3). Classifying tangling and untangling processes explains the possible Feynman vertices. As a result, quantum field theory, including quantum electrodynamics, quantum chromodynamics, the weak interaction, the Higgs mechanism, and the standard model of particle physics with massive Dirac neutrinos, emerge and arise from tangles of strands without any modifications. Tangles also explain that the fundamental constants – particle masses, coupling constants and mixing angles – have unique values, without alternatives. Numerous experimental predictions of high precision have been obtained. So far, all agree with the observations.

Conceptually, the strand tangle model is simple, unique, consistent, complete, and predictive. In particular, the strand tangle model implies that the standard model of particle physics – with massive neutrinos and PMNS mixing – is both final and beautiful.

The first challenge for future research is the numerical simulation of the strand processes presented above. Animations of particle propagators and vertices in Feynman diagrams will also be useful. Exploring the Heisenberg picture of quantum theory will deepen understanding. The relation to Hestenes' geometric algebra can be investigated further. Deducing axiomatic quantum field theory from tangles is worthwhile. The relation between crossings, qubits and entanglement should be explored. The study of the rotation of asymmetric and tethered bodies in viscous flows should allow better approximations for the mass values of elementary particles.

In the experimental domain, the strand tangle model yields over 50 precise predictions and tests. If only one of the listed tests does not agree with observations, the strand tangle model is falsified. In particular, strands predict the lack of physics beyond the standard model with massive Dirac neutrinos, predict the lack of corrections to general relativity, predict the lack of trans-Planckian effects, and predict the normal ordering of neutrino masses. In the domain of numerical simulation, efficient and precise calculation methods for the fundamental constants still need to be developed. All calculations are predicted to agree with the measurements. If they do not, the strand tangle model is falsified. With a finite effort, neutrino masses and mixing angles can be calculated before they are measured.

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Appendix

A Strands are unobservable

The strand tangle model states that single tethers and single strands *cannot* be observed. This unobservability can be shown to follow from nature's properties and is explained in detail in reference [65].

The speed limit c is observable because some systems realize it, such as light and gravitational waves, and because other systems approach it quite closely, such as particles in accelerators or in cosmic rays. The force limit $c^4/4G$ is observable because some systems realize it, such as black holes or gravitational horizons, and because other systems approach it quite closely, such as colliding black holes. The quantum of action $\hbar$ is observable because some systems realize it, such as trapped electrons, and because other systems approach it quite closely, as observed in the photoelectric effect.

In contrast, the limits of relativistic quantum gravity, such as the corrected Planck length, the Planck time or, for single elementary particles, the Planck energy, are neither realized nor approached by any system in nature. The observation of single gravitons or of single microscopic, Planck-scale constituents (degrees of freedom) of space is not achievable. All experiments and observations are several orders of magnitude away from the Planck limits, and will never approach them. For example, measurements of the electron dipole moment are a factor 10^3 away from the corrected Planck length. Measurements of time intervals are at least a factor 10^{20} away from the corrected Planck time. Measurements of elementary particle energy are more than a factor 10^7 away from the corrected Planck energy. Measured temperatures are more than a factor 10^{12} away from the Planck temperature. Regardless of the advances in measurement techniques, the limits provided by relativistic quantum gravity will remain distant and unattained.

As a consequence of the unattainability of relativistic quantum gravity limits, single strands or single constituents are *not observable.* The argument can be stated in general form:

▷ Any Planck-scale constituent or degree of freedom of nature – whatever it may be – is unobservable.

This is the main reason that makes it difficult to *prove* that the Planck-scale constituents or degrees of freedom are strands. The best that can be achieved is to measure relativistic quantum gravity effects for a large number of strands. In other words, relativistic quantum gravity is possible only *statistically*. For example, gravitons are only detectable in large numbers in the form of gravitational waves. The Planck-scale constituents or degrees of freedom in space can only be detected in large numbers, in the form of curvature, or through their thermodynamic effects in black holes and horizons. The Planck-scale constituents can also be detected through the wave functions of quantum particles, or through the gauge field intensities, or through the mass values and coupling constants that they create for elementary particles. In all these cases, *many* constituents are involved. Indeed, so far, all potentially testable expressions that contain the three constants c, G and $\hbar$ also contain the Boltzmann constant k. The entropy and the temperature of black holes are examples.

In short, single strands – single Planck-scale constituents or degrees of freedom – are not observable. In the strand tangle model, a *photon* or a *single-particle wave function* is the closest one can get to the observation of one or a few strands. However, even in these cases, the strands themselves are not observable. In the strand tangle model, as in nature, only the statistical effects of *many* strands are observable.

B Fluctuations rule out strand equations of motion

Can an equation of motion or a Lagrangian for strands be given? No. Some reasons are given in [65]. Several additional reasons can be added.

First, strands have no mass, and thus no inertia. In other words, there is *no* free motion of the strands, in contrast to water hoses. Strands only 'move' when pushed by another strand. Thus, there is no method to deduce the future shape of a strand when the present shape is given. Such a prediction is only possible *statistically*, due to the touching of all the neighbouring strands, and due to the lack of knowledge of the shape of any particular strand. In the present study, the strands that make up the vacuum were not considered. Vacuum strands fluctuate and push each other, but also push all particle strands, randomly and without rest. The vacuum fluctuations are the reason that particle strand shapes fluctuate. Vacuum fluctuations make strand equations of motion impossible.

Secondly, strands have *no* measurable properties: they have no mass, no tension, no momentum, no energy, etc. Therefore, it is impossible to define a measurable *position* for strands or for strand segments.

Thirdly, strands have no observable properties. In addition, their cross-section is unobservably small, as explained in Appendix A. Relativistic quantum gravity prevents measuring the position over time of any *single* microscopic, Planck-scale constituent. Such a description would require measuring space and time with a precision *better* than the Planck limits. But this is impossible. No measurement system is sufficiently precise to allow for such position or time measurements. The hope for an equation of motion for single constituents is unrealistic. In nature, equations of motion are only possible for *large numbers* of constituents. These equations are then called 'Einstein's field equations' in the case of empty space or the 'standard model Lagrangian' in the case of particles. Nature and relativistic quantum gravity are fundamentally probabilistic.

Fourthly, the wish for a strand equation of motion is, in fact, a wish for *hidden variables* at the Planck scale. The arguments just given confirm the result of Section 17: strands are not hidden variables and do not allow the definition of hidden variables, even at the Planck scale.

Fifthly, any unified equation of motion or any unified Lagrangian would contain certain physical fields. The origin of these fields would have to be defined. However, a unified theory must explain the origin of all quantities it uses. This is impossible in a description that uses Lagrangians or equations of motion.

The final result is valid generally, for any Planck-scale constituents:

▷ No unified theory of physics can be based on equations of motion or on a Lagrangian.

This applies not only to strands but also to any type of constituent that makes up black holes and thus makes up both space and particles. The description of strands is statistical and always involves many strands.

In the statistical description of strands, a practical challenge arises: strand shape fluctuations are difficult to describe. Given that strands have no constant length, again in contrast to a typical water hose, the description of shape fluctuations must take into account length fluctuations. How can one simulate the strand tangle model using a computer? Again, the answer lies in the domain that has been neglected in this text: the strand tangle model for the vacuum. In this text, especially in the illustrations, vacuum strands were not drawn or considered. Neglecting vacuum strands allowed deducing the main results of quantum theory. However, the vacuum is *full* of strands. There is almost no volume in space that lacks a strand segment. All these vacuum strands continuously push against each other and against the particle tangle strands, providing the source of fluctuations mentioned in the properties of strands. Thus, a computer simulation of the strand tangle model requires a simulation of vacuum strand fluctuations. The necessary ingredients are a high volume density of strands, resulting in fluctuations from impenetrable strand segments touching and pushing each other, time averaging over a few Planck times, and the ability to handle many strands. The development of practical calculation tools will be a task for the future.

In short, providing an equation of motion or a Lagrangian for single strands is impossible. Nevertheless, a computer simulation of the statistics of strands is possible. The most efficient way to simulate the strand tangle model numerically is a topic of research.

C Strands visualize qubits

Qubits play an important role in two research fields: quantum computing and quantum gravity. A 'qubit', a contraction of 'quantum bit', is the modern expression for the 'Ur' introduced by Weizsäcker in the 1970s [187, 188]. In modern quantum computing research, physical qubits are most commonly realized as trapped ions, quantum dots, nitrogen-vacancy diamonds, or superconducting loops. The state of a qubit is usually represented by the Bloch sphere.

The strand tangle model for one or two spins – as illustrated in Figure 18, Figure 19 and Figure 20 – is also useful for visualizing qubits. *In the strand tangle model, the Bloch sphere describes the orientation of the tangle core combined with the twist state of its tethers.* In quantum computing, quantum gates are represented by unitary operators. The different types of quantum gates can be represented as different operations on the Bloch sphere. In the strand tangle model, unitary operators are represented by strand deformations that keep the average shape of the tangle core untouched, but change its orientation in space. In this way, strands visualize single-qubit states as well as multi-qubit states.

The other use of qubits is in quantum gravity and related research fields of fundamental physics. The main research questions are as follows: How can one imagine a region of space-time that includes vacuum, matter, and particles in terms of qubits? In particular, do qubits form lattices, random structures, or other types of constructions? Is the number of qubits finite, countably infinite or uncountably infinite? In fact, all these questions ask about the microscopic, Planck-scale constituents or degrees of freedom of nature as a whole [189]. In 2001, Zizzi formulated this quest using the expression 'it from qubit' [190].

Given the possibility to describe quantum theory, gravity, and quantum field theory with strand tangles [63, 64, 80, 96], relations between tangle crossings and qubits arise almost naturally. If space, curvature, and particles are due to strands and their tangling, one might interpret the fundamental principle of Figure 6 as a kind of *simplified qubit*. In this interpretation, strands indeed suggest that nature is made of qubits, but also clarify that their number is countable in principle, but not fixed. Also, the fluctuations of the strands change the structure and interdependencies of the simplified qubits over time. Apart from the local three-dimensionality of space, no further structure appears to be fixed or required.

In short, strand tangles can be useful tools for teaching and visualizing quantum computing and qubits. In the field of quantum gravity, the fundamental principle can be seen as a specific version of 'it from qubit'.

D All of nature from strands

In the strand conjecture, all structures observed in the universe – space, horizons, and particles – are composed of strands. The corresponding strand structures are illustrated in Figure 31. In fact,

▷ The whole universe is expected to be one long closed strand – i.e., a strand with the

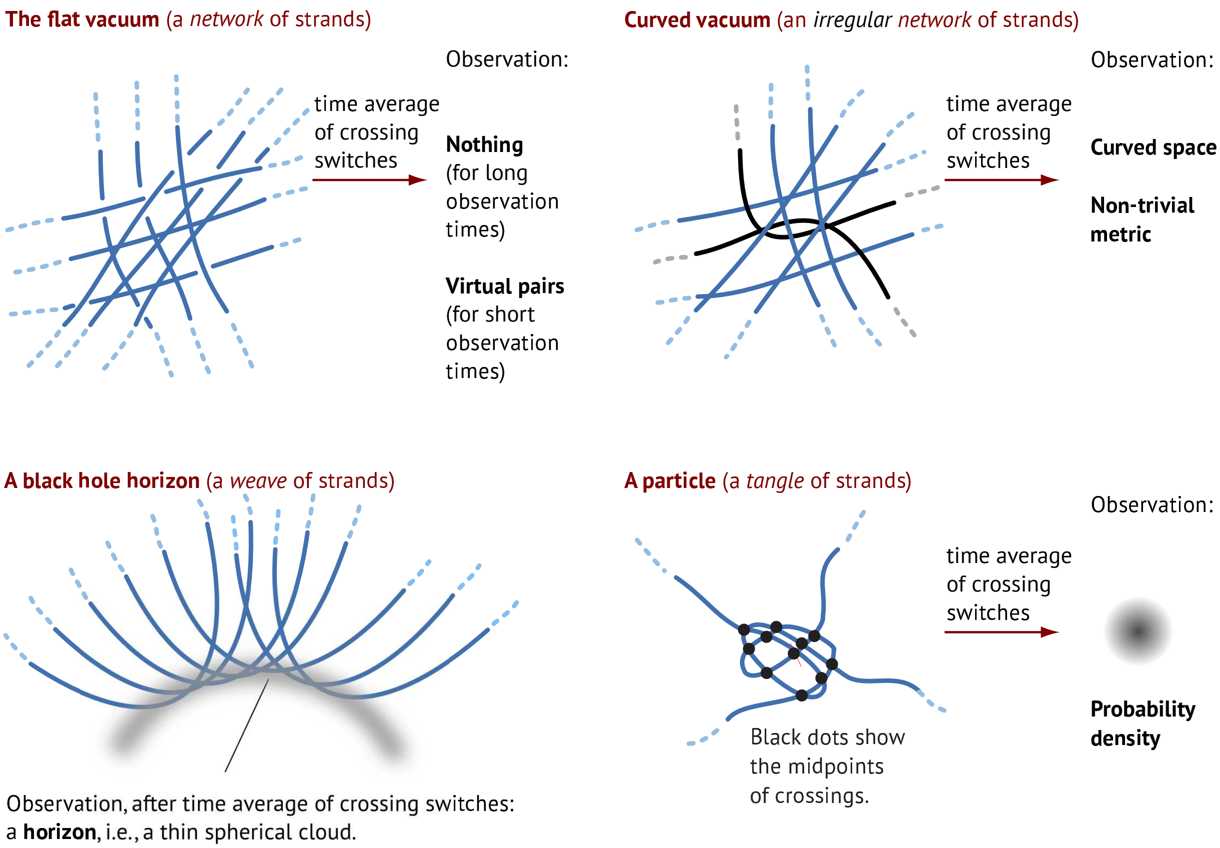

FIG. 31: Strands can form networks (space), weaves (horizons), and tangles (particles). The present text concentrates on particles. For the exploration of space, curvature and black hole horizons, see references [80] and [98].

topology of a ring – that is crisscrossing all of nature, forming space, particles and horizons. This strand became increasingly tangled over time. The corresponding model of cosmology is illustrated in Figure 32.

Strands thus realize the ancient idea that everything in nature is connected to everything else.

In the strand tangle model, flat empty space is realized by *untangled strands*, packed as densely as possible. Curved space is a more irregular network of strands, in which some strand pairs are twisted, representing gravitons. Black hole horizons are *weaves* of strands. How the strand tangle model implies general relativity has been explored separately [80, 96]. The present article focuses only on quantum particles. Thus, particles, modelled as tangles of strands, are effectively *defects* in space. To simplify the understanding, in the present study, only particle strands were drawn in the illustrations. Vacuum strands are typically not drawn.

In short, in the strand tangle model, everything in nature is made of strands. The present article concentrates on particles and wave functions in a flat space only.

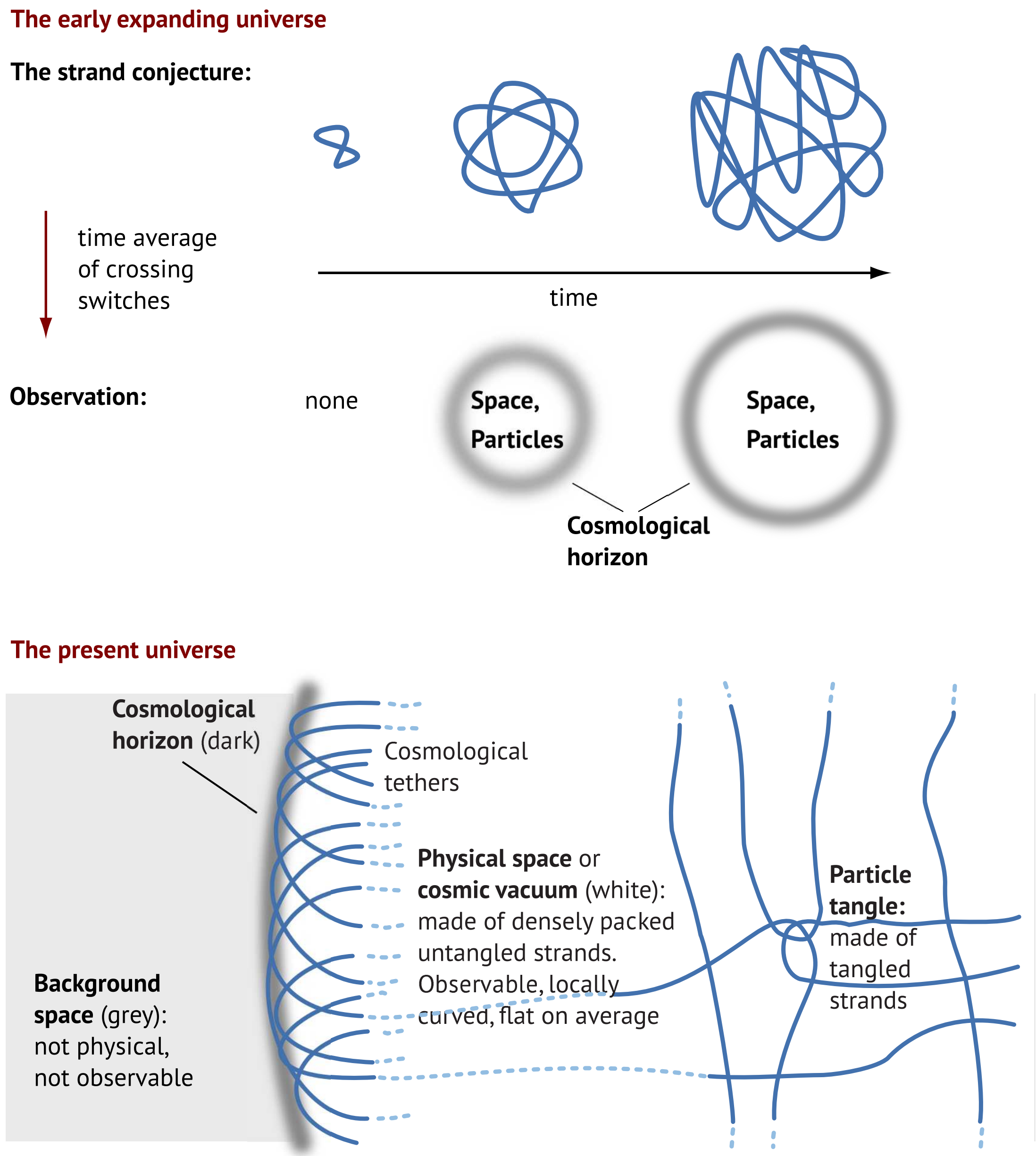

FIG. 32: The whole universe is expected to consist of a single strand, without ends. The figure illustrates its history (top) and a part of its present state (bottom).

E Strand radius and relativity

At first sight, an invariant strand radius seems in contrast to special relativity. Any finite length should contract for observers in relative motion – in the case of continuous space. In apparent contrast, the corrected Planck length that defines the strand diameter is an invariant quantity and does not Lorentz contract. A strand thus differs from a flexible tube or cylinder of everyday life because space is not continuous at Planck distances, and Lorentz contraction does not apply at those scales. This result is expected in quantum gravity.

To cater for the habit of thinking with continuous space and time, one can try to adapt or vary the details of a strand. An alternative model for a strand might be that of a flexible long *cloud*. The cloud would still be 'hard' and impenetrable, similar to the electron cloud of atoms. Another option for a model of a strand might be a line of *zero* radius, fluctuating, but with the property that its radius of curvature is never smaller than the Planck length. Such a line also realizes the minimum distance. Additional models of strands might also be possible.

In short, a constant Planck strand radius does not contradict relativity and provides the simplest visualization of strands.

F Unitary and Hermitian operators

In quantum theory, *unitary* operators describe the evolution of observables; they preserve inner products. In the strand tangle model, unitary operators deform tangles such that the corresponding wave function retains its norm, that is, in such a way that the corresponding tangle retains both its topology and *retains* the shape of its core.

In quantum theory, self-adjoint or *Hermitian* operators are important because they conserve probabilities, have a real spectrum, and describe physical observables. In the strand tangle model, Hermitian operators leave the tangle topology invariant but *change* the shape of the core.

Among the Hermitian operators, position and momentum are basic, as they are used to construct many other operators. In the strand tangle model, the position operator x changes the state ψ by making it vanish at all positions that differ from x. In the strand tangle model, the position operator x localizes the tangle at position x. The momentum operator p is defined as $p = \hbar k$, and the wave vector operator k is defined as $k = -i\partial_x$. In the strand tangle model, a multiplication by i corresponds to the rotation of a state, i.e., of a tangle core, by an angle π. The spatial derivative has the same meaning as usual.

One can combine the momentum operator p, the local rotation, with the position operator x, the localization. In the strand tangle model, the product xp differs from px, because of their different effects on tangles. Their difference is realized and visualized by the fundamental principle. In other terms, strands, like conventional quantum theory, lead to the operator equation

$$px - xp = i\hbar \quad . \tag{F1}$$

This well-known *commutation relation* was first deduced by Born and Jordan in 1925. In 1987, Kauffman suggested that the commutation relation is due to a crossing switch [69, 70]. However, at that time, no one took up this suggestion.

In short, the strand tangle model implies the commutation relation between the momentum and the position operators. This topic is worthy of more detailed exploration.

G Many particles, creation and annihilation

The fermion trick shown in Figure 4 describes particle exchange and fermion behaviour using tethers. The approach implies that a system consisting of several fermions is described by several tangles. Every tangle generates its wave function. Thus, the strand tangle model naturally describes many particle systems by using many wave functions. In the most common states of a many-particle system observed in everyday life – i.e., in incoherent, unentangled, separable states – this description of a many-particle system is sufficient, straightforward, and agrees with experiments.

Creation operators are operators that add a tangle to vacuum strands. In the strand tangle model, a creation operator transforms untangled vacuum strands into a particle tangle. Similarly, *annihilation operators* untangle the strands of a particle tangle, yielding a vacuum. Thus, both operators automatically ensure an *integer* number of particles. Both operators are also *almost* local, realizing the observational constraints and theoretical demands.

In short, in the strand tangle model, *many-particle* systems in incoherent, unentangled, separable states are described by many tangles. The case of coherent, entangled, and inseparable states can also be described with tangles: this is done in Section 16.

H From the quantum of action to Hilbert spaces and entanglement

Historically, Schrödinger derived his evolution equation from de Broglie's matter waves. More precisely, he derived his equation from the expression $\lambda = \hbar/mv$ for the wavelength of quantum particles. The study of the solutions of the Schrödinger equation showed that they form a Hilbert space. Since the Schrödinger equation is linear, its solutions also yield superpositions, interference, and entanglement.

Thus, non-relativistic quantum mechanics of free electrons is based on only two quantities: the quantum of action $\hbar$ and the mass m of the electron. These are the only parameters in the Schrödinger equation for free electrons. All quantum effects disappear when $\hbar$ is set to zero.

For relativistic electrons, the finite speed of light c must be taken into account. For electrons in an electric potential, their electric charge e must be taken into account. Finally, quantum field theory arises when $\hbar$ is introduced into the description of the electromagnetic interaction.

In short, quantum theory, in all its mathematical aspects and all its counter-intuitive details, follows from the non-zero value of $\hbar$. Nothing else was necessary to derive quantum theory, and this has not changed in the meantime. The simple basis for quantum theory explains why assigning $\hbar$ to a crossing switch allows deducing quantum theory, including wave functions, Hilbert spaces and the Schrödinger equation.

I What is a quantum field?

Strands propose a specific definition of a quantum field. The definition sums up all the counterintuitive aspects of the strand tangle model encountered so far.

▷ In the strand tangle model, each *quantum field* is a fluctuating *rational tangle* that is made of strands with Planck radius.

Strand shape fluctuations lead to fluctuations of the strand crossings; averaged over time, strand crossings produce *complex wave functions.* At large distances from the tangle core, there are no crossings; indeed, the quantum field has a vanishing amplitude at large spatial distances. Crossing switches define observables and arise at crossings. Therefore, the norm of a wave function is a *probability density*. The emerging continuous distributions of crossings and crossing switches are *fields*. Their tangle structure allows for the *counting* of particles. The strand tangle topology determines the particle type and its quantum numbers. The strand tangle model automatically yields the *indistinguishability* of identical particles, their *spin*, and their *statistics*, including their close connection.

The strand tangle model implies that particles are *rational tangles* of strands. Rational tangles allow for virtual particles, particle transformations, and particle reactions. Particles are countable and localized *excitations* of the (untangled) *vacuum*. The excitation is given by the tangling of untangled strands. Fermions are rational tangles with 4 or 6 tethers. The (rational) tangle model reproduces *particle creation* by tangling, particle *annihilation* by untangling, particle *absorption* by tangle combination and particle *emission* by tangle separation. Vacuum excitation results in both particles and antiparticles. The (rational) tangle model also implies that (certain) *particles can interact*: different tangles can be combined, or a single tangle can be separated into two or three tangles. The (rational) tangle model also implies that (certain) *particles can transform* into each other: tether braiding changes particle type. The strand tangle model reproduces the *three generations* and the *particle spectrum* as a result of tangle classification. The strand tangle model reproduces particle quantum numbers, their mass values, and their mixing angles as a consequence of topological and geometric tangle properties.

The strand tangle model implies that interactions are *deformations* of tangles that lead to crossing changes. Strand deformations imply that interactions are *local* – within Planck dimensions – and that the interaction spectrum is *limited* by the Reidemeister moves to the three known gauge groups of the standard model. The strand tangle model implies, because of virtual particle clouds, that couplings are *unique* and that they *run* with four-momentum.

In short, fluctuating tangles reproduce and visualize both wave functions and quantum fields. Strands provide the foundation for the concept of a quantum field.

J On physics' axioms and Hilbert's sixth problem

The following arguments do not rely on strands, but strands allow visualizing them in a simple way.

Combining maximum speed c, the quantum of action $\hbar$, and maximum force $c^4/4G$ yields the existence of a minimum length in nature. The minimum length $\sqrt{4G\hbar/c^3}$ is twice the Planck length and has a value of about $3 \cdot 10^{-35}\,\mathrm{m}$. This tiny value is both the smallest possible length measurement result and the smallest possible length measurement error. The same applies to time.

The smallest length implies the lack of points in nature. Therefore, the idea that points *exist* in nature needs to be discarded. Using points to describe nature is useful and helpful, but points are approximations. Points and instants of time have no basis in observations.

The lack of points implies the lack of (mathematical) continuity. Continuous space and time are approximations. Continuity, Euclidean space, space-time, Riemannian space, vector spaces and Hilbert spaces are mathematical concepts based on points and continuous structures. All these mathematical concepts describe nature only approximately.

Space is not made of points. Time is not made of instants. Coordinates are approximations. The same limitation holds for any other observable quantity: measurement results without errors are impossible. Measurement results are not real numbers; they are not points on the real line. Measurement results are more like small, fuzzy clouds on the real line.

Also *point particles* do not exist in nature. Singularities do not exist in nature. Both concepts are incompatible with the smallest length. Also locality is an approximation. Local operators are approximations.

The lack of exact measurement results implies the *lack of sharp boundaries.* There is no way to separate space into separate regions. There are no exact boundaries in nature. Nature is inherently fuzzy. There is no way to distinguish exactly.

The lack of precise measurements and sharp boundaries implies the *lack of sets* in nature. It is impossible to distinguish with precision what is inside from what is outside a set. Sets cannot be defined in nature. In addition, the impossibility to measure more precisely than the minimum length also implies the impossibility to distinguish set elements from each other. This implies the impossibility to define elements of sets in nature.

In mathematics, all axioms are based on distinctions, elements, boundaries, and sets. But all these concepts do not exist in nature, because in nature, there is the smallest length and length measurement error. In other words, the minimum length implies the *lack of axioms in nature.*

Thus, physics as a whole cannot be based on axioms. Hilbert's sixth problem, where he asked for an axiomatic system for all of physics, is unsolvable when gravity and quantum theory are combined.

However, *parts* of physics *can* be based on axioms – just not physics as a whole. Axioms are possible in parts of physics because in parts of physics – such as in quantum theory without gravity, or in gravity without quantum theory – the minimum length does *not* arise. In parts of physics, length and time measurements can be (imagined to be) infinitely precise. Only the combination of quantum theory with gravity eliminates the possibility of infinite precision and thus eliminates the possibility to define axioms.

Despite the lack of axioms in physics, a complete description of physics and nature remains possible. A complete description of nature will be *logically circular.* A logical circularity makes describing nature possible, despite the lack of axioms. Strands provide such a logical circular description: it contains (background) space and time in its fundamental principle while stating at the same time that (physical) space and time arise from averaging strands.

Even though space and time are approximations, they *must* be used to talk about nature. There is no way to do physics without them. One reason: all the fundamental constants, c, $\hbar$, G and k, and thus all measured quantities, have the metre and the second in their units and use space and time in their fundamental implementations. Another reason: space and time are required to distinguish and to think; there is no way to do otherwise. Alternatively, space and time are needed to communicate; there is no way to do otherwise. A description of nature without a space-time background is impossible.

In short, every description of relativistic quantum gravity, the strand tangle model included, implies that axioms covering all of physics do not exist. An *axiomatic* theory of relativistic quantum gravity, an *axiomatic* unified theory, is impossible. However, this conclusion does not prevent a *complete* description of nature. Indeed, such a description is provided, at present, by general relativity and by the standard model. This conclusion about axioms does not prevent a *unified* description of nature. Indeed, such a description is provided by the strand tangle model. Finally, although a quantum description of gravity implies that points and instants do not exist, points and instants must be used to describe nature.

K The possible Feynman vertices and the renormalizability of the standard model

Using the particle tangles of the fermions and of the bosons given above, in Sections 31 and 33, only a limited number of interaction vertices are topologically possible. All the possible vertices are listed in Figure 33 and Figure 34. The vertices cover all Feynman vertices of the standard model with massive Dirac neutrinos. No additional vertex arises and no observed vertex is missing [63]. The lack of additional vertices agrees with observations. As a consequence, the full standard model, with massive Dirac neutrinos, arises from tangles of strands.

The figures show that only *triple* and *quadruple* interaction vertices arise. This limitation is due to the limited number of strands in the tangles of elementary particles. This limitation also implies that QED, QCD and the theory of the weak interaction are *renormalizable*. This result is expected, as the strand model is an intrinsically finite description of quantum field theory.

In short, the strand tangle model implies the complete standard model with massive Dirac neutrinos. Strands also imply the finiteness of the standard model, and its renormalizability in the conventional formulation with point particles.

L Strands and superstrings

At first sight, strands differ from superstrings in several respects. Superstrings are described by Planck tension, have a Lagrangian, live in 10 or more dimensions, and carry superfields, which are based in non-commutative space. Superstrings have been generalized to various types of supermembranes, such as D-branes. In contrast, strands have no tension, have no Lagrangian, live in usual $3 + 1$ dimensions, and carry no fields and no observables of any kind. There is only one type of strand; strands have Planck radius and are extended, and they are discrete and countable; strands differ from space and time, which both result from averaging them. At the same time, the entire universe is supposed to be a single closed strand.

At second sight, the differences between strands and superstrings might be much smaller. If one adds the angles of crossings to the usual space-time coordinates, a higher number of dimensions can be said to arise. Crossings yield Lagrangians. Crossing switches can be seen as a weak form of non-commutative space. In addition, both approaches respect the Planck limits. Certain brane types arise in finite numbers in finite volumes of three-dimensional space. Finally, strands can even be thought of as realizing specific forms of AdS/CFT duality and of holography, each taking Planck limits into account.

In short, there might exist a mapping between certain brane types and strands.

M Remarks on the Yang-Mills millennium problem

In the year 2000, the Clay Mathematics Institute asked, in its list of millennium problems, to prove the existence of a non-trivial quantum field theory for every (non-Abelian) compact simple gauge group in continuous flat space-time. It also asked to prove that each such quantum field theory leads to a finite mass gap. In the millennium problem, a quantum field theory is defined as a structure realizing the axioms by Streater and Wightman and those by Osterwalder and Schrader, which are based on perfect locality and continuous quantum fields. This millennium problem has

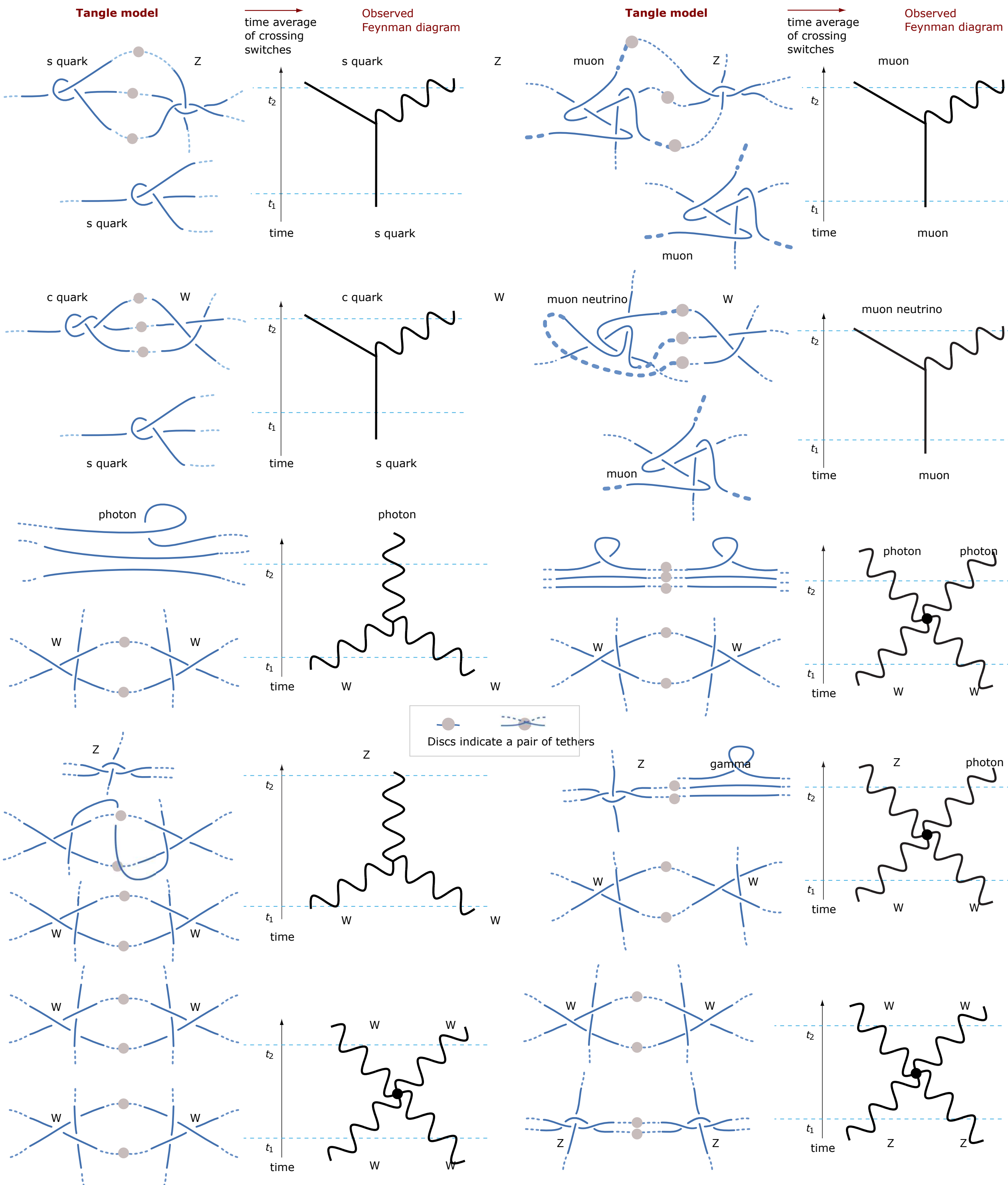

FIG. 33: Part 1: The interaction vertices allowed by fermion and boson topologies imply the complete Lagrangian of the standard model and its renormalizability. (Improved from reference [63].)

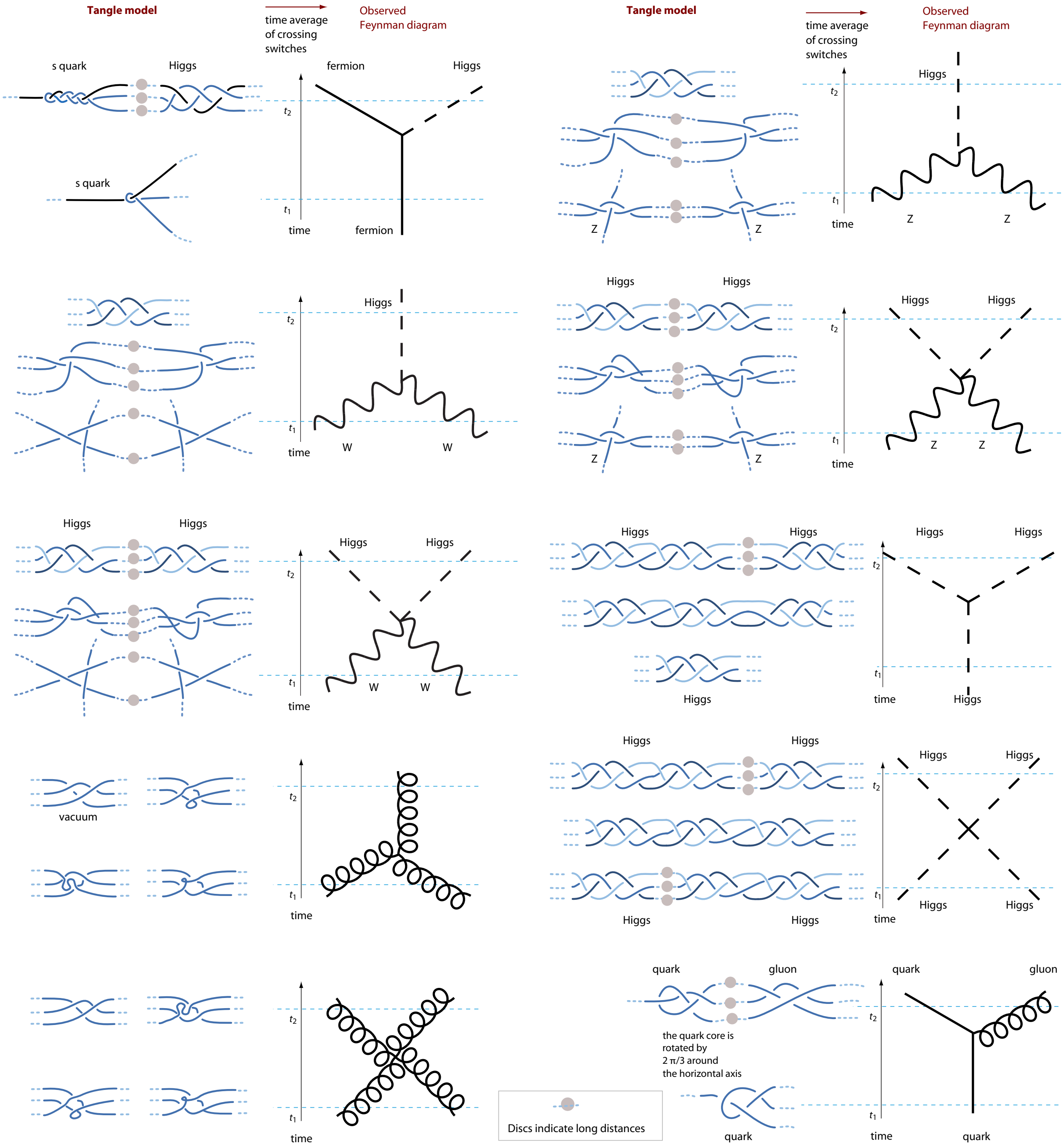

FIG. 34: Part 2: The interaction vertices allowed by fermion and boson topologies imply the complete Lagrangian of the standard model and its renormalizability. (Improved from reference [63].)

two aspects: a physical aspect about nature, and a mathematical aspect about axiomatic quantum field theory.

The physical aspect of the millennium problem is answered negatively both by nature and by the strand model. In nature, continuous space-time does not exist, due to the smallest length. In nature, additional gauge groups do not exist: only the two nuclear interactions are non-Abelian gauge theories. Also strands answer the question in a way that leaves *no* imaginable alternative to the standard model (with massive Dirac neutrinos) at all: due to the existence of just three Reidemeister moves and of the smallest length, additional gauge groups and additional gauge particles are impossible. This result of the strand model, told in the main section of this article, was presented in references [63, 95, 96]. Strands clarify the relation between the three dimensions of space, the three possible gauge groups, and the three fermion generations. Strands also explain the finite entropy of black holes as a consequence of discrete, Planck-sized constituents. In particular, strands imply that the non-continuity of space and the lack of higher gauge groups are two sides of the same coin. In addition, both nature and strands show that other values for the coupling constants or for the particle masses are impossible. Strand deformations provide a precise and mathematically complete description of quantum gauge theory in four-dimensional space-time, thus answering the challenge that motivated the millennium problem [191].

In total, three reasons imply that the physical Yang-Mills millennium problem has no relation to nature. First, the millennium problem explicitly assumes that continuous space, continuous fields, perfect locality, perfectly local operators and perfect point particles exist. Thus, the millennium problem explicitly disregards the smallest length and all the other Planck limits of quantum gravity. Secondly, the millennium problem disregards the way that quantum fields and gauge groups arise in nature. Thirdly, the millennium problem disregards the way that spatial three-dimensionality arises in nature.

All three reasons for the lack of any relation between the Yang-Mills millennium problem and nature are *independent* of the strand tangle model. Whatever the complete description of nature may be, it does not comply with the assumptions of spatial continuity and locality in the Yang-Mills millennium problem. Whatever the complete description may be, it will, by definition, explain why only the observed gauge interactions exist. Whatever the complete description may be, it will, by definition, explain why three-dimensionality arises. Already the statement of the Yang-Mills millennium problem contradicts nature and observations.

In summary, if the Yang-Mills millennium problem is restricted to the physical aspect, the existence of additional quantum field theories *in nature*, the answer is that they do not exist, as known experimentally for roughly a century. Strands confirm and *explain* this observational result, possibly for the first time. The best one can do is to restrict the Yang-Mills millennium problem to proving the existence of a finite mass gap in the special case of the gauge group SU(3) of the strong nuclear interaction (in vacuum). Only in this case is the answer positive, due to the existence of glueballs – both in observations [20] and in the strand tangle model [97].

The mathematical aspect of the millennium problem has various issues that arise independently of strands. *Continuous* space is in contrast with the minimum length in nature. Continuous space has no units of length and time. Units of length and time are needed to discover and determine the quantum of action $\hbar$. Continuous space is *in contrast* with quantum field theory.

In mathematics, one can choose to ignore nature and physics and assume both continuity and quantum field theory at the same time. This leads to a further issue. The statement of the millennium problem defines quantum field theory as a structure realizing specific axioms. This implies the assumption that the quantum of action $\hbar$ and the speed of light c are finite. But exact continuity implies that the Planck length is zero. This implies that G must vanish, in contrast to observations. And a vanishing G implies that mass units cannot be defined and mass values *cannot* be measured.

In mathematics, one can choose to ignore nature and physics and assume continuity, quantum field theory, and non-vanishing G. However, if one insists that G is finite after all, then one has a non-vanishing minimum length. The minimum length implies that there are no points, no sets and no axioms. The lack of axioms, sets and points is deduced in detail in Appendix J. Quantum field theory and finite G, taken together, imply that there is *no* axiomatic quantum field theory.

Of course, in mathematics, one can choose to ignore nature and physics further and assume both continuity and axiomatic quantum field theory at the same time. The assumptions of continuity and of the existence of an axiomatic quantum field theory, taken together, imply the description of particles as exact point particles. These assumptions are purely mathematical and contradict general relativity with its impossibility of objects that are smaller than the Schwarzschild radius determined by their mass. Thus the concepts of 'particle' and 'quantum field' assumed in the millennium problem have no relation to actual particles in nature or to actual quantum fields. All these mathematical assumptions have an unfortunate effect. Different elementary particles differ at least at the Planck scale. Planck scale differences explain the existence of different quarks and the existence of electrons, muons and tau leptons. Eliminating the Planck scale by assuming point particles prevents deriving the existence of any specific elementary particle. Eliminating the Planck scale by assuming point particles prevents deriving the three generations. Eliminating the Planck scale implies that the existence of elementary particles cannot be derived but must be *assumed*. In particular, it is impossible to derive whether additional gauge bosons exist. This implies that it is impossible to derive whether additional gauge interactions with additional gauge groups exist.

In mathematics, one can choose to ignore nature and physics completely and assume continuity, axiomatic quantum field theory, and assume the existence of point-like gauge bosons for gauge groups that do not exist in nature. All these assumptions are suggested by the formulation of the millennium problem. But all these assumptions also prevent deducing the existence of any additional particle asked for in the millennium problem, such as generalizations of glueballs. In particular, the unphysical assumptions imply that particle mass is a mathematical parameter and not the result of an underlying mechanism of nature. Because of all the unphysical and contradictory assumptions, the existence of any mass gap asked for in the millennium problem cannot be deduced. And even if it could, it would not provide any information about nature.

In short, the three limits of nature – the speed limit c, the quantum of action $\hbar$, and the maximum force $c^4/4G$ – imply a minimum length, given by twice the Planck length. In nature, the minimum length implies the lack of continuity of space, and thus the lack of perfect locality. In nature, only this lack of continuity of space allows the appearance of quantum field theory, the definition of measurement units, the definition of physical observables, including mass, and the determination of the dimensionality of space. Instead, the Yang-Mills millennium problem assumes counterfactual continuity of space, counterfactual perfect locality, counterfactual axiomatic quantum field theory, counterfactual properties of gravitation, counterfactual point-like particles, and a counterfactual origin of three-dimensionality. The Yang-Mills millennium problem continues by asking about counterfactual gauge interactions, counterfactual gauge bosons, counterfactual generalized glue balls, and counterfactual mass mechanisms. All these counterfactual concepts have little relation to the corresponding concepts used by physicists. Above all, the assumptions about space, particles, quantum theory, and gravitation used in the millennium problem contradict each other. The Yang-Mills millennium problem uses concepts that are not mathematically consistent. On such a basis, the proof of additional quantum field theories and of mass gaps requested by the Yang-Mills millennium problem is impossible. In contrast, in a description of nature and quantum field theory that agrees with observations and that is mathematically consistent, no Yang-Mills theories beyond SU(3) exist. A finite mass gap appears to arise only for the gauge group SU(3) of the strong interaction.

N On the Nielsen–Ninomiya theorem

When space is a lattice, the Nielsen–Ninomiya theorem implies the necessity of doubling the number of chiral fermions [192–194]. This theorem has generated numerous studies on its extensions and limitations. The theorem is also the reason for various difficulties in the numerical simulations of elementary particles.

In the strand tangle model, space is *not* discretized: space is *not* a lattice. The smallest length in nature, $\sqrt{4G\hbar/c^3}$, twice the Planck length, is also the smallest possible length measurement error. This minimum value prevents the existence of points. Space is not made of points; it is neither discrete nor continuous. The continuity arises at larger length values, due to the averaging by strand fluctuations. Given that space is neither discrete nor a lattice, there is no Brillouin zone in the strand tangle model. Therefore, the strand tangle model does not realize the conditions for the doubling of fermions and for the no-go theorem published by Nielsen and Ninomiya.

In short, the strand tangle model circumvents the Nielsen–Ninomiya theorem.

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