The quadruple gravitational constant, the Bronshtein cube of limits, and the future of fundamental physics

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(Dated:)

It is argued that the quadruple gravitational constant $4G$ can be seen as a fundamental limit of nature. The limit holds across all gravitational systems, and distinguishes gravitationally unbound from bound systems. Describing $4G$ as a limit allows extending the Bronshtein cube of physical theories to a cube of limits, in which all theories of modern physics, including special relativity, general relativity and relativistic quantum gravity, are described by a specific fundamental limit. The existence of these limits suggests intriguing predictions for research in fundamental physics.

I. INTRODUCTION

Special relativity is based on the invariant maximum speed c that is realized by massless radiation. In the past decades, it became clear that general relativity can be based on the invariant maximum force $F_{\max} = c^4/4G \approx 3.0 \cdot 10^{43}$ N which is realized on gravitational horizons. It might be surprising at first that classical gravity can also be based on an invariant limit, the quadruple gravitational constant $4G$, which is realized by parabolic gravitational motion. It is shown that this connection allows describing all of modern physics using invariant limit quantities. In addition, it is argued that the connection strongly restricts the possible options for the future theory of relativistic quantum gravity.

II. MAXIMUM FORCE

In 1973, Elizabeth Rauscher, followed by many others, discovered that general relativity *implies* a maxmimum force c^4/G [1–31]. In 2002, Gibbons included the factor 1/4 and showed that the force between two black holes is never larger that the maximum value $c^4/4G$ [7].

The maximum force value $c^4/4G$ is due to the maximum energy per distance ratio appearing in general relativity. For a Schwarzschild black hole, the ratio between the energy Mc^2 and its diameter $D = 4GM/c^2$ is given by the maximum force value, independently of the size and mass of the black hole. Also the force on a test mass that is lowered with a rope towards a gravitational horizon – whether charged, rotating or both – never exceeds the force limit, *when* the minimum size of the test mass is taken into account. All apparent counter-examples to maximum force disappear when explored in detail [21–24, 26].

In addition, maximum force $c^4/4G$ *implies* Einstein's field equations of general relativity [8, 9, 26, 27]. As a result, the maximum force limit can be seen as the defining *principle* of general relativity. The situation resembles special relativity, of which the maximum speed limit can also be seen as the defining principle.

The maximum force principle for general relativity is not unique. Other quantities that combine c and G, such as maximum power $c^5/4G$ [5, 12, 16, 26, 29, 30, 32–35] or maximum mass flow rate $c^3/4G$ [25, 26], can also be taken as principles of relativistic gravitation. A similar situation arises in special relativity, where, e.g., both c and c^2 can be taken as defining limit.

In short, general relativity can be deduced from the principle of maximum force $c^4/4G$. Likewise, special relativity can be deduced from the principle of maximum speed c. A question arises: can classical inverse square gravity also be deduced from a limit?

III. THE QUADRUPLE GRAVITATIONAL CONSTANT

Traditionally, the gravitational constant G is defined as the proportionality constant appearing in expressions for classical gravity, such as

$$a = \frac{GM}{r^2} \quad . \tag{1}$$

This expression is due to the work by Hooke, Newton, Cavendish and many others. Because the expression unified the sublunar and translunar effects, the inverse square dependence of classical gravity is often called *universal* gravity. Rewriting the equation using the diameter $d = 2r$ yields

$$a = \frac{4GM}{d^2} \quad . \tag{2}$$

In the following, it will be argued that the quadruple gravitational constant $4G$ is a *limit value* for products of observables that yield the result $4G$ in gravitating systems.

IV. $4G$ AS LIMIT DISTINGUISHING UNBOUND FROM BOUND PARTICLES

A particle or test mass is unbound from a large mass M if its kinetic energy is larger than the gravitational

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potential energy. In other terms, a particle is *unbound* or *free* if the doubled centre-to-centre distance $d = 2r$ and the speed v obey

$$\frac{dv^2}{M} \geq 4G \approx 2.7 \cdot 10^{-10}\,\mathrm{m}^3/\mathrm{kg\,s}^2 \ . \qquad (3)$$

The constant $4G$ thus describes the difference between *unbound* and *bound* particles near a mass M. If a particle has a product dv^2/M that is larger than $4G$, it is unbound; otherwise it is bound. For example, a rocket at double distance d from the Earth's centre, flying faster than the escape velocity $v = \sqrt{4GM/d}$, is unbound. In contrast, a stone on the ground is bound to Earth.

Equivalently, a particle is *unbound* from a mass M if

$$\frac{ad^2}{M} \geq 4G \ . \qquad (4)$$

Also if a particle near a mass M obeys

$$\frac{1}{\rho T^2} \geq 4G \ , \qquad (5)$$

where the 'effective' density $\rho = M/d^3$ and the 'effective' particle cycle time around the mass $T = d/v$ are implied, then the particle is unbound. In all other cases, the particle is gravitationally bound.

In short, $4G$ is the *smallest* possible value of any product of quantities containing a nearby mass M that can arise for an *unbound* particle or test mass. Crossing the limit *bounds* the particle or test mass to the attracting mass M.

Approaching the limit $4G$ from below leads to ever increasing cycle or orbital times, until parabolic motion is reached; approaching $4G$ from above leads from *unbound* hyperbolic to parabolic motion, when gravitational potential energy and kinetic energy are exactly equal.

V. TESTING THE LIMIT $1/4G$ IN COSMOLOGY

Using equation (5), every physical system can be described by an effective density times time squared. Limit (5) implies that $1/4G \approx 3.7 \cdot 10^9\,\mathrm{kg\,s}^2/\mathrm{m}^3$ is the *largest* possible value for effective density times time squared that can arise in an *unbound* system.

This also applies to the universe as a whole. The quantity $1/4G$ limits the product ϱT_H^2 of matter density and (Hubble) time squared. Present data [36] shows that the limit is not exceeded. One thus can state that the matter in the universe, seen at a large scale, is generally unbounded or at the limit between bound and unbound state.

In short, the limit $4G$ holds in cosmology.

VI. WHY IS THE LIMIT $4G$ AND NOT SIMPLY G?

The factor 4 arises because of the historical preference of using the radius over the diameter in the definition of gravitationally bound systems. But if radius is used, the limit in equations (2) to (5) would not always be the same multiple of G.

Because of the historical preference for radius over diameter, the factor 4 also appears in the maximum force $c^4/4G$. And as shown in reference [26], this is also the origin of the factor 4 in the expression $S/k = Ac^3/4G\hbar$ for black hole entropy.

VII. TESTING THE LIMIT $4G$ IN ROTATING GALAXIES

There is one group of situations where the validity of the limit $4G$ is uncertain. The rotation of stars orbiting galaxy centers is an intense topic of research. In almost all galaxies the most distant stars are measured to rotate *faster* than predicted from inverse square gravity with the estimated central mass values. Different explanations have been proposed.

In the most common explanation, the deviation is explained with yet unobserved (cold) dark matter [36]. A minority of researchers explores *deviations* from the inverse square dependence of gravitation [37]. They postulate that at distances that would lead to accelerations smaller than a universal constant $a_0 \approx 1.2 \cdot 10^{-10}\,\mathrm{m/s}^2$, the actual acceleration due to gravitation is *larger* than the one predicted by inverse square gravity.

Both explanations can explain the observed rotation curves. The explanation with dark matter claims that stars at the outer edge of galaxies are bound because the actual mass is larger than the luminous mass. Theories with deviations from inverse square gravity claim that at large distances, masses remain bound even if speeds are larger than the conventional escape velocity.

The discussion between the two explanations – and a number of additional ones – is not settled yet. It might even be, for example, that the constant a_0, if it exists, is due to some quantum effect on the cosmological scale, so that the limit $4G$ remains valid. Future research will show which explanation is correct.

In short, so far, there is no definite observation contradicting the limit $4G$.

VIII. PROPERTIES AND LIMITATIONS OF THE LIMIT $4G$

Like in both theories of relativity, also the limit $4G$ is *invariant*. Boosts do not change the limit value. The same applies to $\hbar$ and c.

Like c, $\hbar$ and $c^4/4G$, also the limit $4G$ applies only to *real and free* particles. Bound or virtual particles do not comply with the limits.

Like c and $c^4/4G$, also the limit $4G$ applies only to observables measured at *a single location in space-time*. It does not apply to any sum of observables at different positions, such as any sum of observables for different par-

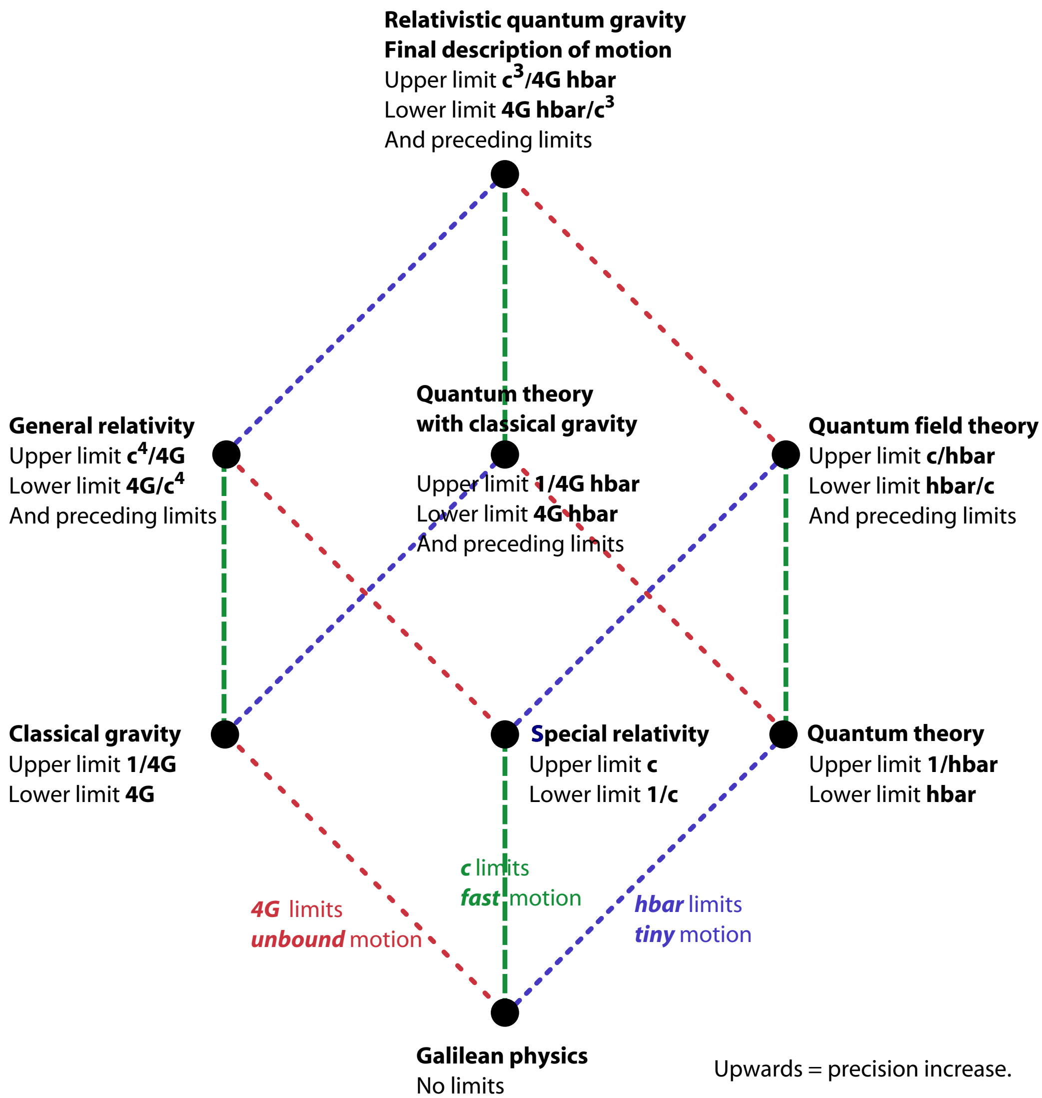

FIG. 1: The Bronshtein *limit* cube of physical theories. Following the arrows leads to an increase of the precision of the description of nature. As explained in the text, several other choices of limits – upper or lower – are also possible at each corner.

ticles in a gas, or sums of observables at different times, or both. All these limits are *local*.

IX. WHAT IF THE LIMIT $4G$ WOULD NOT EXIST?

Measurements are comparisons with standard units. If c would not exist as natural unit, speeds could not be measured. If $c^4/4G$ would not exist as natural unit, forces could not be measured.

If $4G$ would not exist, combinations of observables with the same units as $4G$ could not be measured. In other terms, boundedness and unboundedness could not be distinguished.

In short, a world where $4G$ is not a limit is hard or even impossible to imagine.

X. INVERSE SQUARE GRAVITY

To complete the parallel to special and general relativity, it makes sense to recall that a lower limit $4G$ *implies* universal gravity. Indeed, inverse square gravity is implicit in the definition of $4G$.

Fans of pointed formulations could take this analogy to the extreme and call the limit value $4G$ a *principle* of classical gravitation. However, it seems that the analogy to the other theories of physics is better made with the help of limits, as argued in the following.

XI. THE BRONSHTEIN LIMIT CUBE OF PHYSICAL THEORIES

Since almost a hundred years, the Bronshtein cube is used to structure the different parts of physics [38, 39]. The descriptions of universal gravity and of general relativity explored above allow defining a *limit* at every corner of the cube – except for Galilean mechanics. The origin of this option is the following summary of modern physics:

- Universal gravity is equivalent to $dv^2/M \geq 4G$.
- Special relativity is equivalent to $v \leq c$.
- General relativity is equivalent to $F \leq c^4/4G$.
- Quantum theory is equivalent to $W \geq \hbar$.
- Quantum field theory is equivalent to $ml \geq \hbar/c$.
- Non-relativistic quantum gravity is equivalent to free particles with $Av^3 \geq 4G\hbar$.
- Relativistic quantum gravity is equivalent to $l^2 \geq 4G\hbar/c^3$.

Using these limits, the resulting physics cube is illustrated in Figure 1. The choice of the limit for every field of physics is not unique; at each corner of the cube, each nonzero exponent of $4G$, c and $\hbar$ can be changed at leisure. This allows assigning different upper or lower limits at each corner.

A number of properties of this *Bronshtein limit cube* are worth recalling. First, whenever a limit c, $4G$ or $\hbar$ is added to a given description of motion, a *more precise* description is obtained. It is thus expected that relativistic quantum gravity, even though not known in all details, will be the most precise description possible – because it takes into account *all* limits.

Secondly, apart from the correcting factor 4, all limits are Planck limits. In other terms, one can say that the (corrected) Planck limits define modern physics.

Thirdly, if desired, the lower limit for entropy, defined by the Boltzmann constant k – or another thermodynamic variable – *can* be added to the discussion [38, 40].

In short, fields of physics can be defined with limits. This characterization is useful for the teaching of physics. Learning physics starts at the bottom left of the Bronshtein cube, where no limits are assumed, and proceeds towards the top right, where all physical observables are limited by the corrected Planck values.

In addition, the limits imply several consequences for the future of research in fundamental physics.

XII. PREDICTIONS ABOUT THE FUTURE OF PHYSICS

As long as rotating galaxy curves do not disprove inverse square gravity, the lower limit property of the quadruple gravitational constant $4G$ allows defining *every* theory of modern physics using a *limit* on the motion of unbound particles. This includes the still elusive theory of relativistic quantum gravity.

As long as rotating galaxy curves do not disprove inverse square gravity, each theory at each corner of the Bronshtein limit cube agrees with all observations.

The simplicity, the agreement with experiment and the explanatory power of the fundamental limits at each corner of the Bronshtein cube are intriguing. Above all, the limit cube implies several testable predictions.

First, the Bronshtein cube suggests that there is *no* physics beyond general relativity even for the strongest gravitational fields (e.g., no other terms in the Hilbert action). The cube also suggests that there *no* physics beyond special relativity, even for the largest speeds or for the fastest processes (e.g., no doubly special relativity). Furthermore, the cube suggests that there is *no* physics beyond quantum theory, even for the smallest processes. This implies the lack of additional dimensions, of discrete space-time, and of non-commutative space-time.

These predictions agree with all observations so far. In fact, these predictions eliminate various candidate theories proposed in the research literature.

Describing nature with limits implies that there are no infinite quantities in nature, neither infinitely large nor infinitely small. More precisely, the limits imply that there are no trans-Planckian effects in nature. In particular, the limits imply:

- There are no points in space or instants in time.
- Space-time is effectively continuous, despite the existence of a smallest length.
- There are no higher dimensions.
- There are no lower dimensions.
- There is no space-time foam.
- There is no discrete space-time.
- There is no space-time lattice.
- There are no singularities.

The Bronshtein cube with limits also shows that physics laws are *simple*. Every theory in physics can be defined with a limit only. Every theory of physics is based on extremely simple math. This suggests that also the theory of relativistic quantum gravity is based on a limit, such as the minimum length $4G\hbar/c^3$, and on simple math. (Like for the other theories, there is no unique defining limit; any corrected Planck value that contains $4G$, $\hbar$ and c can be used. This includes a minimum time, a minimum area, a minimum volume, a maximum acceleration, a maximum mass density, etc.)

All arrows converge to the relativistic quantum gravity. The convergence implies that the limit $4G\hbar/c^3$ (or any equivalent limit), *alone by itself*, describes and *implies* relativistic quantum gravity. The Bronshtein cube thus suggests that the complete description of motion is essentially known already. Whatever the microscopic degrees of freedom of space, matter and radiation may be, they follow the corrected Planck limits.

On the other hand, several unsolved issues do not appear in the cube: the origin of particle masses, gauge coupling constants and mixing angles and phases do not arise at all. The Bronshtein cube thus on one hand provides no clue at all for understanding the fundamental constants, but on the other hand provides hope that relativistic quantum gravity, the complete complete description of motion, is not far. Like for all predictions in physics, the future will tell.

There is something fascinating about science.
One gets such wholesale returns of conjecture
out of such a trifling investment of fact.
Mark Twain, *Life on the Mississippi.*

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