From maximum force to inverse square gravity

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The equivalence of maximum force $c^4/4G$ and the field equations of general relativity provides a simple derivation of inverse square gravity – and suggests a lack of gravitational physics beyond general relativity.

Introduction: maximum force

The discovery of the inverse square dependence of gravity by Hooke and Newton unified the description of motion in the sublunar and the translunar realms. Later, Einstein's special and general theories of relativity unified the description of motion for large speeds, for highly concentrated masses, and for the universe at large. Special relativity is based on the invariant maximum speed c that is realized by massless radiation. Similarly, general relativity can be based on the invariant maximum force

$$F_{\text{max}} = \frac{c^4}{4G} \approx 3.0 \cdot 10^{43}\,\text{N} \quad , \tag{1}$$

that is realized by gravitational horizons. Here, G is the gravitational constant. Interestingly, maximum force allows deducing inverse square gravity in a quick way.

It is known since 1973 that *general relativity contains and implies a maximum force* [1–21]. As Gibbons showed, the force between two black holes is never larger that the maximum value, including the factor 1/4 [7]. The maximum force value $c^4/4G$ arises because there is a maximum energy per distance in general relativity: the ratio between the energy Mc^2 of a Schwarzschild black hole and its diameter $D = 4GM/c^2$ is given by the maximum force value, independently of the size and mass of the black hole. Other types of black holes – whether charged, rotating or both – do not allow producing larger ratios. Also the force on a test mass that is lowered towards a gravitational horizon with a string never exceeds the force limit, when the minimum size of the test mass is taken into account. No physical system allows exceeding the force limit. All apparent counter-examples to maximum force disappear when explored in detail [22–26]. Maximum force passes all known experimental and theoretical tests.

Maximum force $c^4/4G$ *implies and contains* Einstein's field equations of general relativity. There are at least two ways to show this [8, 9, 26, 27]. One way starts by showing that maximum force implies a limit on the elastic deformation of space. This limit implies a relation between energy and curvature, which then implies the field equations. The other way uses maximum force to deduce the first law of horizon mechanics, which in turn implies the field equations.

Given that the field equations follow from maximum force, one can see maximum force $c^4/4G$ as a *principle* of nature. Maximum force in general relativity can be compared to maximum speed in special relativity.

Given that maximum force implies the field equations, it must imply inverse square gravity.

A new derivation of inverse square gravity

The definition of maximum force $c^4/4G$ as energy per length can be rephrased by stating that there is a maximum energy in any enclosed spherical area A with circumference πD. This is the hoop conjecture [28, 29]:

$$F_{\text{max}} = \frac{c^4}{4G} \geq \frac{E}{A}\,\pi D \quad . \tag{2}$$

In special relativity, the acceleration a of a test mass around a sphere of diameter D is limited by

$$a \leq \frac{c^2}{D} \quad . \tag{3}$$

The two limits describe the same situation. Setting them equal to eliminate D yields

$$E = \frac{c^2}{4\pi G}\,a\,A \quad . \tag{4}$$

This consequence of maximum force is, at the same time, a version of the first law of horizon mechanics [30, 31]. Inserting the relations $E = Mc^2$ and $A = 4\pi r^2$ – valid for flat space and thus away from any horizon – yields

$$a = \frac{MG}{r^2} \quad . \tag{5}$$

Thus inverse square gravity is, in flat space, a direct consequence of maximum force and maximum speed.

This derivation of inverse square gravity from general relativity is simpler than the derivation usually found in textbooks. The derivation completely avoids the use of tensors. Therefore, this derivation can be useful in teaching.

Testable predictions

Several testable predictions for research on gravitation follow from maximum force. First, maximum force suggests that the hoop conjecture is valid; both concepts are closely tied to horizons.

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Secondly, the simplicity of the principle of maximum force suggests that *no deviations* from general relativity will ever be found, in particular for strong fields. Indeed, the most recent observations about black hole mergers [32] and about the double radio pulsar PSR J0737–3039A/B [33] failed to find any deviation.

Thirdly, defining gravity and general relativity using the force limit $F_{\max} = c^4/4G$ forms a *limit triplet* together with the definition of special relativity using the speed limit $v_{\max} = c$ and that of quantum theory using the action limit $W_{\min} = \hbar$. The limit triplet predicts the lack of any trans-Planckian effect in nature, both macroscopic and microscopic. So far, no such effect was observed, despite intense searches in cosmology, black hole physics and particle physics. Among many other limits, maximum force implies the existence of a smallest length.

Finally, the explanatory power and the simplicity of the limit triplet suggest that it should also hold in any unified theory. This prediction can be tested in the future.

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